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FEA based fast topology optimization method for switched reluctance machines

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Abstract

In this paper, a finite element analysis (FEA) based fast optimization method to optimize a lightweight in-wheel switched reluctance machine is presented. This method speeds up the switched reluctance machine optimization procedure by running the FEA simulations with single-phase constant current excitations for half electrical cycle and estimating the machine performance metrics using the gathered FEA data. Hence, the machine‘s dynamic performance estimation process takes shorter for each design candidate. The optimization algorithm employs designs of experiments (DOE), response surface (RS) analysis method, and differential evolution algorithm (DE). Here, the DOE method is used to reduce the search space by narrowing down the upper and lower boundaries of each design variable based on the RS analysis. Although this process does not guarantee getting the Pareto front, it places the search space close to the actual one. Hence, the multi-objective DE optimization finds the Pareto optimal solution set without requiring a large number of iterations as well as a large number of candidate designs for each iteration. The method is applied to a 24/16 SRM that is intended to be used in a lightweight race car as a hub motor. Six dimensionless geometric variables are optimized to satisfy three objective functions, namely torque ripple, motor mass, and copper loss. While the conventional DE takes at least 3000 candidate designs, the proposed method considers only 559 designs to reach a similar Pareto front. It is observed that the proposed method takes about 6 h 30 min compared to the conventional method that takes 32 h 50 min using the same computer. Therefore, the computation time is reduced almost five times with the proposed method.

Keywords Fast optimization · Switched reluctance machine · Multi-objective differential evolution algorithm · Design of experiment · Response surface analysis · Finite element analysis

1 Introduction

Due to its reliability and low cost, the usage of rare-earth magnet-free switched reluctance machines (SRM) for electric and hybrid vehicles gains widespread popularity in recent years [1]. SRMs are comparable with induction machines (IMs) in terms of power density and efficiency [2]. Since the magnets and windings are absent on the rotor, SRMs are cheap, reliable, robust, and have simple geometry compared to the IMs and permanent magnet synchronous machines (PMSMs). Hence, SRMs are suitable for high-speed and harsh environment applications such as electric vehicles (EVs) [3].

In SRMs, electrical energy is converted to mechanical energy with the help of reluctance torque. By applying current to the stator windings, the nearest rotor pole is aligned with the excited stator pole to minimize the magnetic reluctance. Later, the stator phases are energized sequentially to provide proper rotation [4, 5].

Design of the electrical machines based on the design specifications and the operating conditions is an important matter that should be dealt with carefully. To analyze the influence of the different design variables on machine performance, an appropriate optimization technique is generally utilized. Though a wide range of optimization techniques can be found in the literature, choosing the right technique depends on the nature of the problem [6–8].

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Several optimization studies have been introduced for SRMs in the literature. Design limitations of SRMs are investigated in [9], where the design objectives are maximizing the torque, the torque per ohmic losses and the efficiency. The optimum tooth-width/tooth-pitch ratio, optimum rotor-diameter/stator-diameter split ratio, and yoke-width/half-of-the-tooth-width ratio of the motor are studied to meet the three objectives mentioned above. In [10], the torque and co-energy are selected as objective functions and the flux density is considered as a constraint. A sequential approximate optimization method is used to meet the objectives and to find the optimum values of stator inner diameter, stator pole angle, rotor pole angle, and air gap of a 6/4 SRM. In [11], the objective is to increase the efficiency by shortening the flux path in an outer rotor SRM for electric bus applications. A multi-objective design optimization is proposed in [12] for 8/14 outer rotor SRM, where genetic algorithm (GA) optimization is applied and the objective functions are introduced as a weighted sum of the individual design objectives. Objective functions of the optimization are selected as maximizing average steady-state torque, torque factor, torque quality factor, average torque per active mass, loss factor, and minimizing the torque ripple. In [13], an SRM optimization for EVs is introduced, where some constraints are considered and motor parameters are optimized using the parametric sweep method of the machine with both static and dynamic analyses. Another study using the Kriging model and GA for optimizing the 6/4 SRM is reported in [14]. The objective function is to reduce the torque ripple while finding the minimum Kriging function instead of solving the problem by finite element. A multi-objective optimization with GA is studied and five different SRM designs are compared in [15], where two objectives; maximizing the output torque and minimizing the root mean square (RMS) value of torque ripple are considered. In [16], the design and optimization of a four-phase 16/20 in-wheel switched reluctance motor (IW-SRM) with four indicators; torque, torque ripple, efficiency, and torque density are proposed, where the design objectives are converted into a single objective function via optimized weight functions. In general, optimization objectives are chosen as minimizing the motor mass/volume, minimizing the losses/maximizing the efficiency, and minimizing the torque ripple. In some studies, stack length is kept constant and the first objective is defined as maximizing the torque per mass/volume [17], whereas, in other studies the objective is selected as minimizing the motor mass by calculating the stack length for all candidate designs to get the same torque output. In this paper, the latter approximation is considered.

According to [18, 19], the evolutionary algorithm has great potential for solving a wide range of difficult problems. One of the suitable evolutionary algorithms for solving multi-objective and multi-constrained optimization problems is the differential evolution (DE) algorithm. A comprehensive study of multi-objective optimization algorithms that compares three different optimization algorithms is presented in [20], where DE, GA, and particle swarm optimization (PSO) for the design of a 6/4 SRM are studied. The results state that the DE algorithm has better performance in terms of convergence speed and quality of the final Pareto solution set compared to the others. In [21], a multi-objective DE algorithm is applied to an SRM to minimize the torque ripple of 6/4 SRM while building an active current profile that is integrated into multi-objective optimization using the DE algorithm. Another multi-objective DE algorithm-based optimization study is proposed in [22], where, the optimal design of three different SRM topologies is considered for the direct drive EV applications and reducing the total loss and mass are the two objectives of the optimization. In [23], design of experiment (DOE) and DE based method is considered to identify the designs of an SRM with magnetically disconnected rotor modules. The aim is to minimize the losses and the mass while producing the target torque. In [24], multi-objective optimization of an IW-SRM using the DE algorithm is proposed. The paper only investigates the optimum rotor pole and stator pole arc angles to achieve desired average torque and high torque density. Another design optimization study of an IW-SRM using multi-objective DE algorithm is reported in [17]. Here, the machine is analytically modeled and both static and dynamic analyses are performed, where static torque, efficiency, and torque per volume are considered as objective functions.

One of the biggest drawbacks for majority of the aforementioned optimization studies is that they use weighted sum approach which converts a multi-objective optimization problem to a single objective by taking the weighted average of each individual design objectives [25]. As a matter of fact, determining the weights is a tiresome process and quantifying the importance of the objectives may vary from one designer to another. Furthermore, this approach does not always produce the desired solution, and the optimization algorithm provides only one solution at the end, which makes this approach less flexible. Instead, it is desirable to obtain a set of optimized designs where one of them can be selected as a final design. Hence, the designer has multiple optimal solutions to select. This approximation is called Pareto based multi-objective optimization (PBMO). In this paper, the importance of all objective functions are considered equal, and a solution set is created instead of a single solution.

PBMO algorithms often require a large number of candidate designs. For instance, a typical DE algorithm requires a population of 50 individuals and 60 generations yielding to 3000 total simulations to approximate the Pareto optimal solution set, which leads to a large amount of computation time.

In this study, to minimize the computation time a combination of the DOE/Response Surface (RS) analysis method and the DE algorithm is used to design an IW-SRM for a lightweight electric racing car. During the optimization process, FEA simulations are set up to run a single phase constant current excitation to obtain the static flux linkage and torque curves for four current levels (minimum number of curves to represent the nonlinearity) from zero to the rated current. Hence, the flux linkage and torque curves versus the current and position are obtained over a half electrical period. Since the flux linkage waveform has even and torque waveform has odd symmetry with respect to the mid pole position, these data can be reconstructed for the rest of the period. Using the recorded data, a MATLAB®/Simulink® model is built and steady-state machine performance is analyzed with the optimized turn on/off angles. The results of this simulation are sent back to the main optimization algorithm to be evaluated. This way, the performance data to be extracted from coarsely recorded static curves and fast candidate design performance analysis are performed for each candidate design.

The motor geometry is built in FEA software based on the six dimensionless ratio parameterized variables. First, DOE and RS analyses are performed to narrow down the search space by updating initial optimization variable boundaries. Later, the DE optimization algorithm is performed with the updated boundaries to minimize three design objectives, namely; mass, copper loss, and torque ripple. The FEA simulations are done at a constant speed, constant current, and constant current density in the stator slots. The stack length of the machine is selected in a way to get a constant average torque value; hence all the candidate designs generate the same output torque. Induced winding voltage and rotor/stator pole arc angles are considered as constraints. Constraints are handled with Lampinen's constraint handling approach [26]. The Pareto optimal set is obtained and a suitable design among the set is selected as a final design. During the optimization process, 59 simulations are performed for initial DOE/RS analysis and 500 simulations with 25 individuals in a population and 20 generations for DE optimization. Therefore, the computation time to obtain the performance data for each candidate design, as well as the whole optimization process, is reduced. The total number of simulations is reduced to 559 from typical 3000–10,000 simulations performed in similar studies in the literature [17, 25]. Even if the conventional multi-objective DE runs with 3000 candidates, which is the minimum required number, the proposed method reduced the computation time significantly from 32 h 50 min to 6 h 30 min with a computer that has two Intel Xeon E5-2620V4, 2.1 GHz processors, 64 GB of DDR4 RAM, and 256 GB of SSD hard drive.

This paper is organized as follows: The optimization problem definition and the details of the FEA, optimization parameters selection, objectives, and constraints are presented in Sect. 2, followed by the explanations of the DOE and RS methodology and DE algorithm in Sect. 3 and 4. The optimization of a 24/16 IW-SRM is presented in Sect. 5.

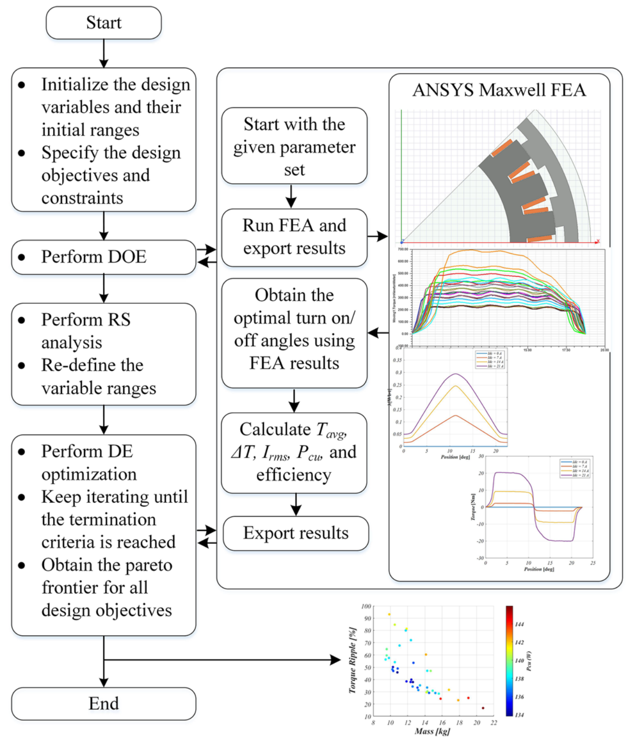

Fig. 1 Flow diagram of the optimization process

2 Optimization problem

The proposed method for the topology optimization of lightweight IW-SRM is performed using the DOE method followed by RS analysis and DE algorithm. The optimization algorithm is executed in MATLAB®, parameter sets are sent to FEA software, Ansys Maxwell, and data are collected back as shown in Fig. 1.

The optimization starts with the initialization of the design variables and their initial upper and lower boundaries followed by the specification of the design objectives and the constraints, which are given in detail in Sect. 2.3. The second step of the algorithm is performing the DOE and RS analysis to find out in what range the parameters should be limited to minimize all the cost functions; hence, the parameter boundaries are refined, narrowed down. Therefore, the search space got smaller and convergence time got shorter. The algorithm continues with the DE algorithm and iterates until the termination criteria is satisfied to find the final Pareto front.

As seen in Fig. 1, in DOE and DE steps there is a sub-function that calculates the performance of the candidate SRM designs and exports these results to the main algorithm.

In order to obtain the performance measures of an SRM with the given parameter set, one should either run dynamic simulations in FEA software or built a model that can give acceptable results using the data taken from the FEA software. The first option requires the time stepping FEA simulation to be run for numerous times to find the optimal angles which is a time consuming process. It should be kept in mind that this process only determines the optimized angles, and should be repeated for all candidate designs during the optimization; hence, the computation time will be massive. Instead, the flux linkage vs position vs current and torque vs position vs current waveforms can be determined from the FEA, and lookup tables (LUT) can be generated. Then, a motor model can be built in a software platform like MATLAB®/Simulink® to run dynamic simulations. The optimal turn on/off angles can be determined through an iterative search or with a simple angle sweep algorithm, and all the performance measures can be calculated using the optimized angle values. One problem with this method, it again requires the FEA simulation to run many times to obtain high-resolution LUTs to represent the candidate machine with high accuracy. Although this method takes less time than calculating everything with time-stepping FEA, it still takes a considerable amount of time.

In this study, this process is even simplified by using coarse LUTs that only have data at four equally distanced current values from zero to the rated current where there is no need to run the simulation at 0 Amp value. Since the flux linkage waveform has even symmetry and torque waveform has odd symmetry with respect to the mid pole position, these data can be taken up to half of the period and whole waveforms can be reconstructed. This way, FEA simulation time for each candidate design is cut in half.

The SRM model is built in MATLAB®/Simulink® R2020b using the LUTs generated in the previous stage and the terminal voltage equation

$$V = iR + \frac{d\lambda}{dt} \tag{1}$$

where V is the terminal voltage, i is the phase current, R is the winding resistance per phase, and λ is the flux linkage.

Along with the SRM model, the MATLAB®/Simulink® simulation contains a reference current generation block, a hysteresis current controller, and an asymmetric bridge inverter as shown in Fig. 2. The simulation runs at constant rated speed and gives the output instantaneous and average torques, torque ripple, phase currents and voltages, and the average copper loss as output. Also, for each candidate design, stator and rotor core volumes, length and the cross section of the copper wire, the average flux density of the stator and rotor poles and yokes, as well as the induced maximum winding voltages are recorded in the FEA.

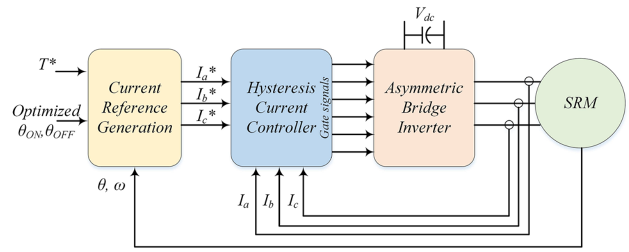

Fig. 2 SRM control model used in the optimization process

Since the FEA is performed with a constant phase current, core loss cannot be accurately calculated with traditional methods. Instead, the core loss is estimated using the catalog data, iron volume, and average flux density at the stator and rotor poles and yokes while friction and windage losses are neglected. Hence, the efficiency of the machine is calculated as follows:

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{cu}} + P_{\text{fe}}} \tag{2}$$

where P_{out} is the output power which is the product of the average torque and angular speed, P_{cu} is the copper loss, and P_{fe} is the core loss that is expressed as follows [17]:.

$$P_{\text{fe}} = K_{\text{fe}}(V_{\text{rotor}} + V_{\text{stator}}) \tag{3}$$

where K_{fe} is the core loss coefficient for the base speed, V_{rotor} and V_{stator} are the volumes of the rotor and stator, respectively. The copper loss calculation is given in Sect. 2.3.

The details of the FEA model and the process of design variable selection are explained in Sects. 2.1 and 2.2 followed by the objectives and constraints of the optimization problem in Sect. 2.3.

2.1 Finite element analysis (FEA)

The cross-sectional view of the machine FEA model, as well as the geometric parameters, are shown in Fig. 3. Parameter definitions and important design elements are provided in Table 1.

The maximum current density is kept constant based on the cooling requirements by calculating the stator slot areas for each individual design and adjusting the number of turns per pole accordingly.

Stack length is also kept constant to acquire the torque and induced voltage waveforms for unit stack length, later these quantities are scaled with the actual stack length calculated for achieving the target average torque value.

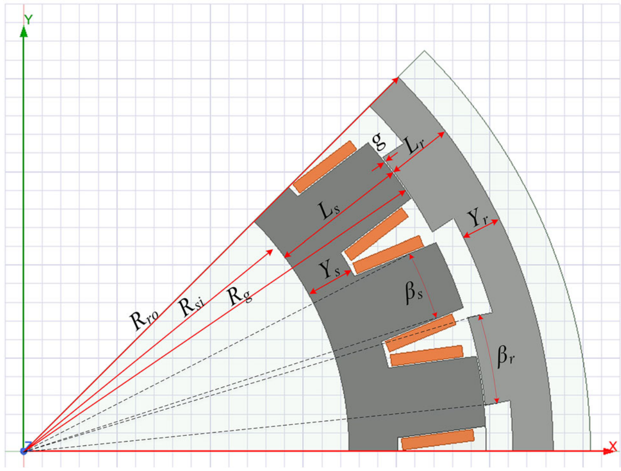

Fig. 3 Geometric parameters of in-wheel SRM

Table 1 Design parameters

Parameter	Definition
R_{ro}	Rotor outer radius
R_{si}	Stator inner radius
R_{g}	Mid-air gap radius
G	Air gap length
L_{s}	Total stator thickness
L_{r}	Total rotor thickness
Y_{s}	Stator yoke thickness
Y_{r}	Rotor yoke thickness
β_{s}	Stator pole arc angle
β_{r}	Rotor pole arc angle
N_{s}	Number of stator poles
N_{r}	Number of rotor poles
N_{t}	Number of turns per pole
L_{stack}	Stack length

2.2 Design variables selection

It is required for a motor optimization system to have a flexible parametric model for each possible design inside the search area. The geometric parameters are based on the stator outer radius, and they are ratio parameterized. For a fixed motor outer diameter, six dimensionless geometric variables are defined as given in Table 2.

Using these parameters as optimization variables allow one to run the algorithm in all the search domain without having geometry errors.

2.3 Objectives and constraints

In this study, the optimization problem has multiple objectives and constraints. The first of the three objectives is minimizing the overall motor mass, which includes the stator and rotor core as well as the winding copper masses.

Table 2 Dimensionless geometric parameters

Variable	Definition	Ratio
K_{rt}	The ratio of rotor inner radius to motor outer radius	$\frac{R_{\text{ri}}}{R_{\text{ro}}}$
K_{sl}	The ratio of stator yoke to total stator thickness	$\frac{Y_{\text{s}}}{L_{\text{s}}}$
K_{ry}	The ratio of rotor yoke to total rotor thickness	$\frac{Y_{\text{r}}}{L_{\text{r}}}$
S_{e}	Stator pole arc embrace	$\frac{\beta_s}{360/Ns}$
R_{e}	Rotor pole arc embrace	$\frac{\beta_r}{360/Nr}$
K_{dsi}	The ratio of stator inner radius to motor outer radius	$\frac{R_{\text{si}}}{R_{\text{ro}}}$

$$f_1 = \min\left(M_{\text{stator}} + M_{\text{rotor}} + M_{\text{windings}}\right) \tag{4}$$

where M_{stator} is the stator mass, M_{rotor} is the rotor mass, and M_{windings} is the winding mass in kilogram. The stator and rotor masses are calculated using the cross-sectional areas, A_{stator} and A_{rotor}, measured in FEA software, stack length, and the mass density of the M19 electrical steel, d_{M19}, which is 7300 kg/m^3, as follows:

$$M_{\text{stator}} = A_{\text{stator}} L_{\text{stack}} d_{\text{M19}} \tag{5}$$

$$M_{\text{rotor}} = A_{\text{rotor}} L_{\text{stack}} d_{\text{M19}} \tag{6}$$

The winding mass is calculated as:

$$M_{\text{windings}} = \left(\frac{A_{\text{slot}}}{2} k_{\text{fill}}\right) l_{1\text{turn}} d_{\text{Cu}} \tag{7}$$

$$l_{1\text{turn}} = 2\left(L_{\text{stack}} + 2L_{\text{ext}} + \left(\pi R_{\text{slot}}\left(\beta_s + \frac{1}{2}\left(\frac{360}{N_s} - \beta_s\right)\right)\right)\right) \tag{8}$$

where A_{slot} is the slot area, k_{fill} is the slot fill factor, $l_{1\text{turn}}$ is the average length of a single turn, R_{slot} is the is the average slot radius, L_{ext} is end winding extension length, d_{Cu} is the mass density of the copper, 8960 kg/m^3.

The second objective is to minimize the copper loss, P_{Cu}.

$$f_2 = \min(P_{\text{Cu}}) \tag{9}$$

The copper loss is calculated as:

$$P_{\text{Cu}} = N_{\text{phase}} I^2_{\text{phase,rms}} R_{\text{phase}} \tag{10}$$

where N_{phase} is the number of phases and R_{phase} is the phase resistance that is expressed as follows:

$$R_{\text{phase}} = \frac{\rho l_{1\text{turn}} N_t}{A_{\text{slot}} k_{\text{fill}}} \times \frac{N_s}{N_{\text{phase}}} \tag{11}$$

where ρ is the resistivity of the copper.

The third objective is to minimize torque ripple.

$$f_3 = \min\left(\frac{\max(T) - \min(T)}{\text{mean}(T)}\right) \quad (12)$$

where T is the electromagnetic torque.

The first constraint is the maximum induced voltage on the phase windings, U, where it should be less than the voltage limit, U_0,

$$\Gamma_1 = U - U_0 \leq 0 \quad (13)$$

and the other constraint is on the rotor pole arc angle, β_r, where it should be greater than or equal to the stator pole arc angle, β_s.

$$\Gamma_2 = \beta_r - \beta_s \leq 0 \quad (14)$$

Here, Γ is the constraint matrix that contains Γ_1, and Γ_2. In the DE optimization algorithm, constraints are handled just like the objectives. This method is called Lampinen's constraint handling method and will be given in detail in Sect. 4.

3 Design of experiments and response surface methodology

DOE and RS techniques are useful statistical tools that provide a full insight into the interaction between the design variables and the design objectives [25]. RS methodology discovers the relationships between several critical design variables and response variables. The idea behind the RS methodology is to use a series of designed experiments to obtain an optimal response [27]. Currently, the RS methodology using proper DOE has become extensively attractive, since DOE reduces the search space by narrowing down the upper and lower boundaries of each design variable based on the RS analysis. Recent studies in electric machine design optimization topics show that these techniques are used more and more to determine the parameter sensitivity over the design objectives [6]. The DOE approach is reported to be more accurate when the number of design variables is small. Even if it is not, the DOE approach gives an insight to the designer for adjusting certain optimization parameters such as boundaries of optimization variables and selecting the significant design parameters. In this study, the DOE method is used to determine the narrowed-down ranges of each individual design variable. Hence, the search domain shrinks down to an area where all the design objectives get close to their minimum values. This process does not guarantee obtaining the Pareto frontier; however, it places the search space close to the actual Pareto front to be used as a starting point for a more sophisticated optimization algorithm, which in our case is DE. Hence, the computation time for the optimization got a further reduction.

Since the number of design variables is only six in our case, an additional elimination process for insignificant design parameters is not necessary. On the other hand, machines having more than seven variables may require such process before proceeding to DE optimization [25].

4 Differential evolution

Differential evolution is one of the evolutionary algorithms which is first introduced in [18, 28]. DE algorithm is a suitable evolutionary algorithm for solving nonlinear and multi-constrained complex optimization problems.

In DE, perturbing the vector population is done by using vector differences. DE has similarities with traditional evolutionary algorithms; however, the DE algorithm runs with real numbers rather than operating in binary form as in the GA and it does not use probability function. Instead, DE performs the mutation-based distribution of the solutions in the current population. In this way, search direction and possible step sizes depend on the location of the individuals selected to calculate mutation values [24].

After specifying the population size, M, both upper, x_u, and lower limits, x_l, must be specified for each design variable. Then, the first generation is initialized based on the normal distribution. Crossover probability factor, C_r, and the scaling vector, F, should also be defined here.

The main DE process includes the procedures of initialization, mutation, crossover, and selection [18, 25].

After deciding the variables to be optimized as,

$$x = \left[K_{rt}, K_{sl}, K_{ry}, S_e, R_e, K_{dsi}\right] \quad (15)$$

the initial values of the first generation design variables of the population is formed. The initial values of the jth ($j = 1,\ldots, M$) design variable of the ith ($i = 1,\ldots, N$) vector in the first generation can be expressed as:

$$x_{j,i,1} = \text{rand}_j([1, N]).\,(x_u - x_l) + x_l \quad (16)$$

where N is the dimension of solution, and the final value of i, $\text{rand}_j([1, N])$ is a $1 \times N$ vector while $0 \leq \text{rand}_j(1 \times N) < 1$.

Then, three different vectors $x_{r1,g}$, $x_{r2,g}$ and $x_{r3,g}$ are randomly selected to generate a mutant vector $v_{i,g}$. This process adds a scaled difference between two randomly selected vectors to a third vector and can be expressed as follows [26, 29]:

$$v_{i,g} = F \times \left(x_{r1,g} - x_{r2,g}\right) + x_{r3,g} \quad (17)$$

where F is the scaled factor which varies between 0 and 1, and g varies from 1 to G where G is the number of generations.

Trial vector $u_{i,g}$ is generated out of two different vector variables, namely; $x_{i,g}$ and $v_{i,g}$ by crossover procedure which is formulated as follows [25, 29]:

$$u_{i,g} = (u_{j,i,g}) = \left[u_{j,1,g},\ u_{j,2,g}, u_{j,3,g} \dots u_{j,i,g}\right] \tag{18}$$

$$u_{j,i,g} = \begin{cases} v_{j,i,g}, & \text{if } \mathrm{rand}_j(0;1) \le C_r \\ x_{j,i,g}, & \text{otherwise} \end{cases} \tag{19}$$

where the user-defined crossover probability factor changes between $0 \le C_r \le 1$. Here, if the random number of the jth variable in the ith vector is less than or equal to C_r, the trial vector $u_{i,g}$ is equal to the mutant vector $v_{i,g}$. Otherwise, the trial variable is copied from vector $x_{i,g}$.

In the selection process of the DE algorithm, the objective cost functions will evaluate the performance of each candidate design. Then, the trial vectors, $u_{\mathrm{i,g}}$, are compared to the target vectors $x_{i,g}$, including the constraints and design objectives in the current generation. It should be noted that the constraints are handled as objectives, which is called Lampinen's selection criterion, and is adopted here as described as follows:

$$x_{i,g+1} = \begin{cases} u_{i,g}, \ \text{if} \begin{cases} \begin{cases} \Gamma_m(u_{i,g}) \le 0 \text{ and } \Gamma_m(x_{i,g}) \le 0, \\ f_n(u_{i,g}) \le f_n(x_{i,g}); \end{cases} \\ \text{or} \\ \begin{cases} \Gamma_m(u_{i,g}) \le 0, \\ \Gamma_m(x_{i,g}) > 0; \end{cases} \\ \text{or} \\ \begin{cases} \Gamma_m(u_{i,g}) > 0\,, \\ \max\left(\Gamma_m(u_{i,g}), 0\right) \le \max\left(\Gamma_m(x_{i,g}), 0\right); \end{cases} \end{cases} \\ x_{i,g}, \ \text{otherwise} \end{cases} \tag{20}$$

where f and Γ are the cost and constraint vectors, respectively.

The optimization process starts from a random parameter set within the specified parameter boundaries and iterates until the pre-defined criteria which are to reach the maximum number of generations is satisfied. Hence, the design performance will be improved each time until achieving the best set of results, Pareto frontier.

5 In-wheel SRM topology optimization with DOE/RS and DE

5.1 Machine specifications

The aim of this paper is to develop an optimized IW-SRM that produces the average desired torque while minimizing the active motor mass, losses, and torque ripple. 24/16 outer rotor SRM is built-in FEA software, ANSYS/Maxwell. The designed IW-SRM runs at 1000 r/min speed with 21 A rated current, 300 V voltage limit, and 0.4 slot fill factor. The air-gap, g, should be selected as small as possible within the manufacturing limits; hence, it is selected as 0.35 mm. The specifications to be met for the in-wheel SRM and fixed design values are given in Table 3.

Table 3 Design specifications

Variable	Definition	Value
D_o	Machine outer diameter	284 mm
S/R	Stator and rotor pole numbers	24/16
T	Average electromagnetic torque	20 N m
η	Efficiency	>90%
k_s	Slot fill factor	0.4
J	Slot current density	7 A/mm^2
n_b	Base speed	1000 r/min
U_0	Voltage limit	300 V
g	Air gap length	0.35 mm
I_{rated}	Rated phase current	21 A

The computer that the optimization algorithm runs have two Intel Xeon E5-2620V4, 2.1 GHz processors (each of them has 8 cores, 16 threads, and 20 MB cache), 64 GB of DDR4 RAM, and 256 GB of SSD. The operating system is Windows 10 Professional, the FEA software is ANSYS/Electronics Desktop 2019-R3 with 16 high-performance computing license (HPC) (allows 16 simulations run at the same time), and the optimization algorithm runs on MATLAB® R2020b.

5.2 DOE and RS analysis

The initial upper and lower boundaries of the dimensionless variables are selected as wide as possible based on mechanical limitations and initial FEA results.

In the DOE method, the central composite design (CCD) [18] method is used for determining the experiments to perform the RS analysis. CCD has generated 59 designs for 6 parameters. This parameter set is simulated and the relationship among the three design objectives for each individual design are obtained as shown in Fig. 4. In this figure, the results that violate the design constraints are marked with x, and the ones that do not are marked with dots.

For RS analysis, the second-order polynomial model is used and the regression coefficients are determined using the least-squares method. Previous studies show that the second-order polynomial model provides flexibility for various functional forms to approximate the response surface more accurately and to ease of estimation of the regression coefficients.

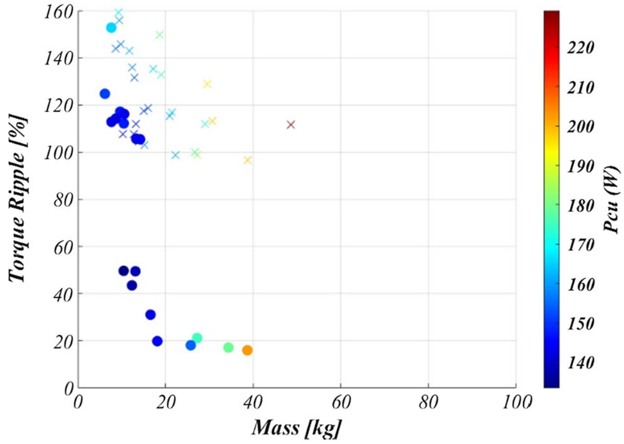

Fig. 4 Results of the DOE analysis

The parameter sensitivity analysis is performed and the effects of the design variables on all design objectives are observed as shown in Fig. 5a–c; hence, upper and lower boundaries of the optimization variables are updated as given in Table 4.

As it is mentioned earlier, the RS analysis guides the designer to the areas that should be focused on; however, it does not find the Pareto frontier parameter sets.

5.3 DE optimization and results

The method used in this paper reduces the number of simulations significantly by performing the DE optimization with 20 generations and 25 individuals in each generation. Therefore, the total number of simulations is reduced to 559 including 59 simulations required for DOE and RS analysis. Whereas, a typical DE optimization task runs with 50 to 100 individuals in each generation and iterates 60 to 100 generations to determine the Pareto-optimal solutions which take 3000 to 10,000 simulations to be performed until the optimization satisfies the termination criteria. Figure 6 shows the Pareto set of the design objectives with the conventional DE and the proposed method. It can be seen from the figure that both methods converged to a similar Pareto front. Measured time for the proposed optimization method including 59 DOE, 500 DE FEA simulations, and processing time in MATLAB® and

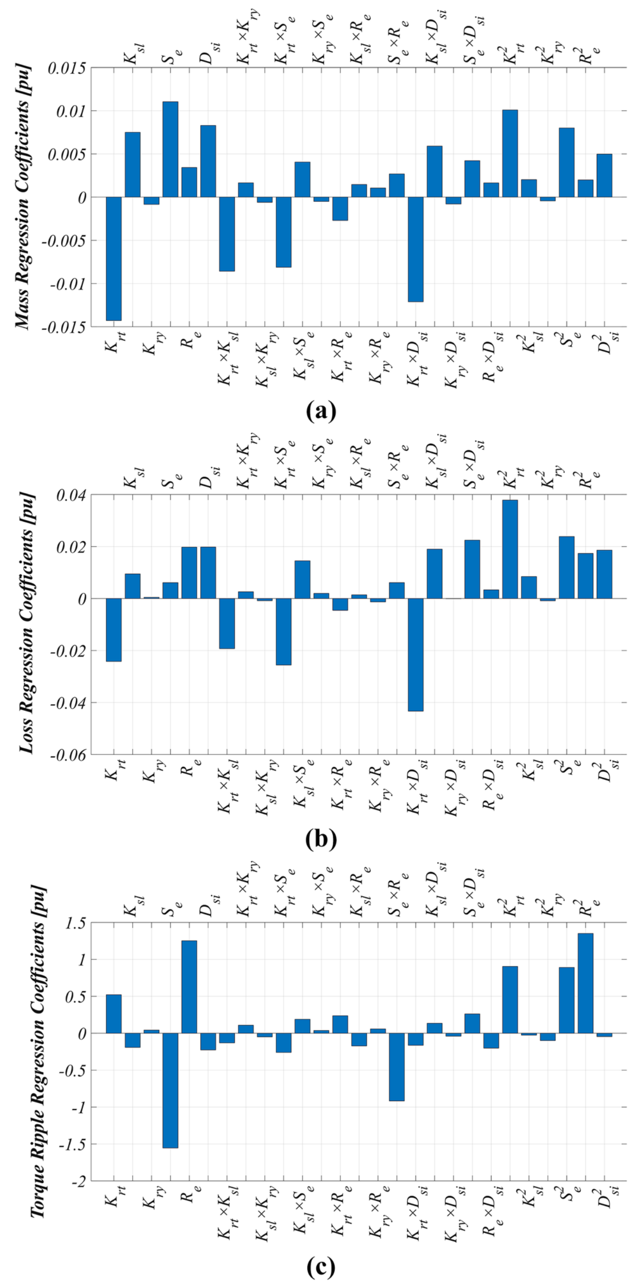

Fig. 5 **a** Sensitivity analysis for motor mass, **b** for copper loss, and **c** torque ripple

Table 4 Initial and updated upper and lower boundaries of the dimensionless geometric parameters

Parameter	Initial minimum	Initial maximum	Updated minimum	Updated maximum
K_{rt}	0.75	0.92	0.85	0.91
K_{sl}	0.25	0.60	0.35	0.40
K_{ry}	0.25	0.60	0.25	0.40
S_{e}	0.40	0.70	0.58	0.70
R_{e}	0.40	0.70	0.45	0.52
K_{dsi}	0.49	0.71	0.58	0.65

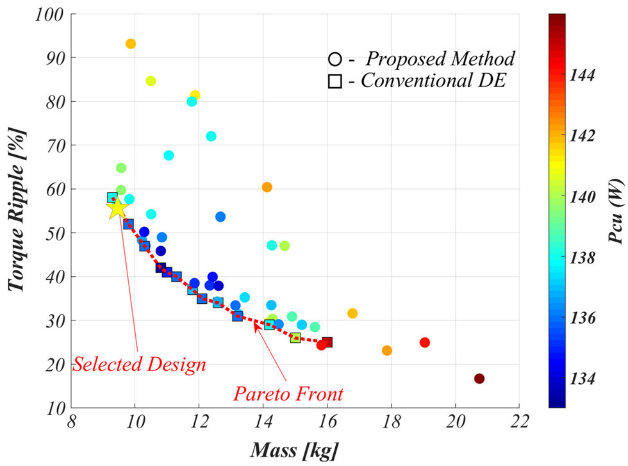

Fig. 6 Pareto set of design objectives

Table 5 Optimized design parameters

Parameter	Optimized value
K_{rt}	0.8767
K_{sl}	0.3883
K_{ry}	0.2749
S_e	0.6124
R_e	0.4207
K_{dsi}	0.5961
N_t	27 turns
L_{stack}	41.06 mm

Simulink® is about 6 h and 30 min, which is about five times faster than a conventional DE optimization that takes about 32 h and 50 min with 3000 FEA simulations.

A Pareto frontier solution set contains multiple candidate designs that are not being dominated by other designs, i.e., no objective can be improved without deteriorating the other objective/s. Therefore, it is not possible to have a candidate machine having the highest efficiency, lowest mass, and torque ripple. Accordingly, there is not a universal best machine but there is the best machine for a specific application. In this study, the motor is intended to be used in a small electric race car, the mass and the efficiency of the motor are more important parameters to be considered for the final design.

Hence, the candidate design that has the parameters given in Table 5 is selected as a final design. Based on the values given in Table 5 and the calculations given in Table 2, the final geometrical parameters are calculated and presented in Table 6. The final design's flux linkage and torque versus current and position curves are presented in Fig. 7.

The dynamic performance of the final design at the rated conditions where the speed is 1000 r/min is simulated in the MATLAB®/Simulink® model given in Fig. 2. The phase currents and resulting torque waveforms with a hysteresis band of ± 0.4 A obtained from the simulations are presented in Fig. 8.

Table 6 Optimized geometrical parameters

Parameter	Optimized value
R_{ro}	142 mm
R_{si}	84.65 mm
R_g	124.32 mm
g	0.35 mm
L_s	39.50 mm
L_r	35.02 mm
Y_s	15.34 mm
Y_r	12.69 mm
β_s	9.186 deg
β_r	9.465 deg

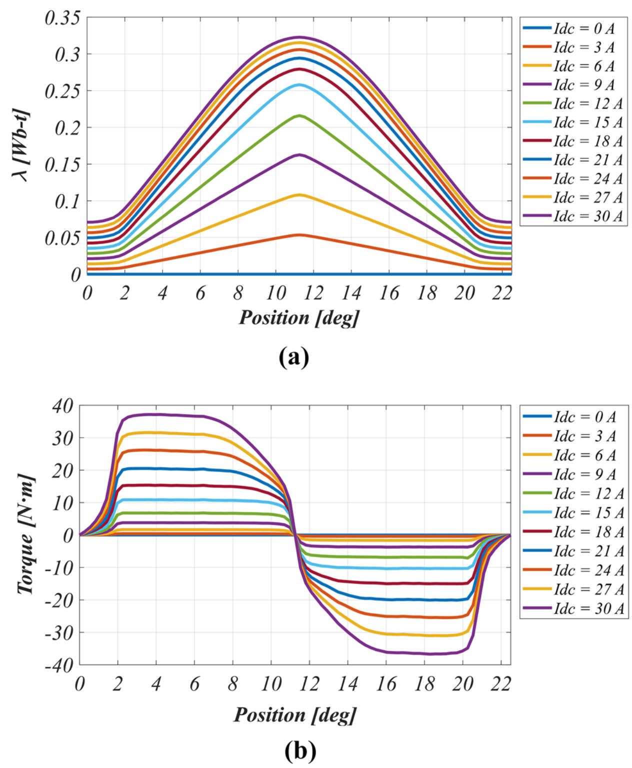

Fig. 7 **a** Flux linkage versus position and current, and **b** torque versus position and current plots

The efficiency of the final design is obtained with (2) by calculating the output power, core loss, and copper loss using (3) and (10) in the Simulink model. The RMS value of the phase current is calculated as 12.81 A for the optimized turn on/off angles, the phase resistance and the copper loss are calculated as 0.27 Ω and 134 W, respectively. The core loss is estimated as 36 W for the rated frequency and 1.6 T peak flux density. The final design has 92.5% efficiency, 9.4 kg motor mass, and 56.4% torque ripple.

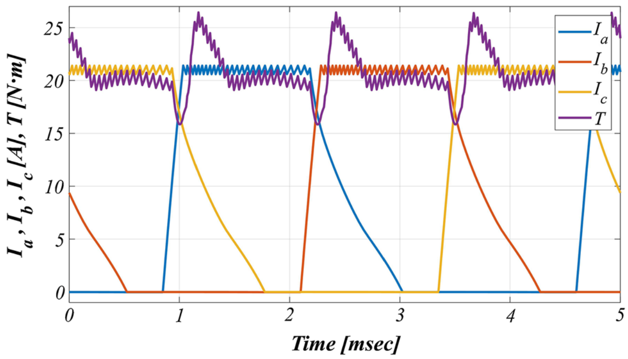

Fig. 8 Phase currents and torque waveforms of the optimized design

6 Conclusion

In this paper, a finite element analysis (FEA) based systematic topology optimization of a lightweight In-Wheel Switched Reluctance Motor (IW-SRM) is considered. The optimization process is completed in two stages to overcome the extensive computation time problem of the existing SRM optimization methods. In the first stage, the design of experiments (DOE) with central composite design (CCD) method and response surface (RS) analysis are performed to make a sensitivity analysis and determine the optimization variables' upper and lower boundaries to ensure to minimize all three design objectives, namely motor mass, copper loss, and torque ripple. Later in the second stage, the differential evolution (DE) algorithm takes over the optimization process with a smaller search space where the high-performance designs become available. During the optimization process, static flux linkage and torque curves obtained from the FEA are used to determine the steady state performance of the SRM with the help of a MATLAB®/Simulink® model. Therefore, the performance data are extracted from the recorded static curves, and fast candidate design performance analysis is evaluated in the optimization process. Owing to this practical engineering approach, the computation time to obtain the performance data for each candidate design and obtaining the Pareto based multi-objective optimization (PBMO) results is reduced significantly.

The proposed method is used to optimize a 24/16 IW-SRM to compare conventional DE optimization. The number of candidate designs and simulations is reduced to 559 from typical 3000–10,000 presented in related studies in the literature. Even with the minimum required number of 3000 candidate designs, the conventional multi-objective DE (MODE) takes 32 h 50 min, while the proposed method only takes 6 h 30 min on a computer that has two Intel Xeon E5-2620V4, 2.1 GHz processors, 64 GB of DDR4 RAM, and 256 GB of SSD hard drive. As a result, at least five times faster optimization process is achieved.

With these results, it is shown that a PBMO of an SRM with a large number of design variables can be done with a much lower computation cost without sacrificing the quality of the final design set. This study can be extended to the design optimization of the SRMs for various target driving cycles, which require even larger number of time stepping FEA simulations at various loading conditions and iterations.

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Availability of data and materials Not applicable.

Code availability Not applicable.

Declarations

Conflict of interest Authors have no conflicting interests to declare.

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