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# Taxonomy of DEVS Subclasses for Standardization 

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## Background

- Schedule-Preserving DEVS (SPDEVS) is a subclass of Finite\& Deterministic DEVS (FDDEVS). [6][5]
- FDDEVS is a subclass of Alur's Timed Automaton (TA) [4].
- Some papers have attempted to convert DEVS into TA for verification [2], [3], [1]
- Q1. Is this conversion DEVS into TA always possible?
- Q2. What are subclasses, super-classes or equivalent classes?
- Q3. Which classes are sub, super, or equivalent classes of DEVS?
- By answering these questions, this paper enables us to standardize DEVS classes.


## Contents I



Figure 1: Presentation Organization

### 1.1 Example of Toaster Trajectories



Figure 2: Trajectories of a Toaster (a) A Toaster, (b) Piecewise Linear Trajectory, (c) Piecewise Constant Trajectory, (d) Event Trajectory

### 1.2 Event Segments

- A timed event: $(z, t)$ of $z \in Z, t \in \mathbb{T}$.
- The null event segment: $\epsilon_{\left[t, t_{u}\right]}$ where $\epsilon \notin Z$ and $\left[t_{l}, t_{u}\right] \subseteq \mathbb{T}$.
- An unit event segment is either a timed event or a null event segment.
- A multi-event segment $\left(z_{1}, t_{1}\right)\left(z_{2}, t_{2}\right) \ldots\left(z_{n}, t_{n}\right)$ over $Z$ and $\left[t_{l}, t_{u}\right] \subseteq \mathbb{T}$ is concatenations of unit event segments $\epsilon_{\left[t, t_{1}\right]},\left(z_{1}, t_{1}\right), \epsilon_{\left[t_{1}, t_{2}\right]},\left(z_{2}, t_{2}\right), \ldots,\left(z_{n}, t_{n}\right)$ and $\epsilon_{\left[t_{n}, t_{u}\right]}$ where $t_{l} \leq t_{1} \leq t_{2} \ldots \leq t_{n-1} \leq t_{n} \leq t_{u}$.
- Example: $\omega_{[0,120]}=(?$ push,25)(!pop,50)(?push,80)(!pop,105).


### 1.3 Universal Timed Language

## Definition 1 (Universal Timed Language)

The universal timed language over an event set $Z$ and a time interval $\left[t_{l}, t_{u}\right] \subseteq \mathbb{T}$, is denoted by $\Omega_{Z,\left[t, t_{u}\right]}$, and is defined as the set of all possible event segments. Formally,

$$
\Omega_{z,\left[t, t_{u}\right]}=\left\{(z, t)^{*}: z \in Z \cup\{\epsilon\}, t \in\left[t_{l}, t_{u}\right]\right\}
$$

where $(z, t)^{*}$ denotes a none or multiple concatenations of null or timed events.

- Note that if $L$ is a language over $Z$ and $\left[t_{l}, t_{u}\right]$, then $L \subseteq \Omega_{Z,\left[t, t_{u}\right]}$.


### 2.1 Timed Event Systems

## Definition 2 (TES)

$$
G=\left(Z, Q, q_{0}, Q_{A}, \Delta\right)
$$

- $Z$ is the set of events;
- $Q$ is the set of states;
- $q_{0} \in Q$ is the initial state variable;
- $Q_{A} \subseteq Q$ is the set of accept states;
- $\Delta: Q \times \Omega_{Z, \mathbb{T}} \rightarrow Q$ is the state trajectory function that defines how a state $q$ changes to another $q^{\prime}$ along with an event segment $\omega \in \Omega_{Z, \mathbb{T}}$.

If $\omega$ is concatenation of two event segments, i.e. $\omega=\omega_{1} \omega_{2}$, then $\Delta(q, \omega)=\Delta\left(\Delta\left(q, \omega_{1}\right), \omega_{2}\right)$. In general if $\omega$ is concatenation of $n$-event segments, i.e. $\omega=\omega_{1} \omega_{2} \ldots \omega_{n}$, where $n>1$ then

$$
\begin{equation*}
\left.\Delta(q, \omega)=\Delta\left(\ldots \Delta\left(\Delta\left(q, \omega_{1}\right), \omega_{2}\right) \ldots\right), \omega_{n}\right) \tag{1}
\end{equation*}
$$

### 2.2 Determinism and Nondeterminism of Timed Event

 Systems
## Example 1 (Deterministic and Nondeterministic Functions)

For example, assume that $A$ and $B$ are real numbers, then $f(a)=a+5$ is deterministic. Given two sets $A=\{$ coin, dice $\}$ and $B=\{$ head, tail, $1,2,3,4,5,6\}$, if the function $f$ indicates outcomes of tossing a coin or a dice, $f$ is non-deterministic. If $r \in\{$ head, tail $\}$ represents the outcome of tossing coin, $r$ is a nondeterministic (or random) variable.

Definition 3 (Deterministic and Non-Deterministic TESs)
A TES $G=\left(Z, Q, q_{0}, Q_{A}, \Delta\right)$ is deterministic if $(1) q_{0}$ is a constant variable, and (2) $\Delta$ is deterministic. Otherwise, $G$ is non-deterministic.

## 2.3 $L(G)$ : Behaviors of a TES $G$

## Definition 4 (Non-infinite length language)

If $0 \leq t<\infty, t$-length observation language of $G, L(G, t)$, is

$$
\begin{equation*}
L(G, t)=\left\{\omega \in \Omega_{Z,[0, t]}: \exists \text { the case }: \Delta\left(q_{0}, \omega\right) \in Q_{A}\right\} . \tag{2}
\end{equation*}
$$

## Definition 5 (Infinite length language)

The infinite length observation language of $G, L(G, \infty)$ is
$L(G, \infty)=\left\{\omega \in \lim _{t \rightarrow \infty} \Omega_{z,[0, t]}: \exists\right.$ the case s.t. $\left.\inf \left(\Delta\left(q_{0}, \omega\right)\right) \subseteq Q_{A}\right\}$.
where $\inf \left(\Delta\left(q_{0}, \omega\right)\right) \subseteq Q$ denotes the states where $\omega$ visits infinitely many times or stays infinitely long.

We would use just $L(G)$ instead of $L(G, t)$ or $L(G, \infty)$ if $t$ is not important.

## 2.4 $E(A)$ : Expressiveness of a formalism $A$

Given a formalism $A$ that is a subclass of TES, it's expressiveness is denoted by $E(A)$.

## Definition 6 (Expressiveness Inclusion)

Suppose that $A$ and $B$ are two TES classes.

- $E(A) \subseteq E(B)$, if for a given instance $a$ of $A, \exists$ an instance $b$ of $B: L(a)=L(b)$.
- $E(A) \subset E(B)$, if $E(A) \subseteq E(B)$ but for a given instance $b$ of $B, \nexists$ an instance $a$ of $A: L(a)=L(b)$.
- $E(A)=E(B)$, if $E(A) \subseteq E(B)$ and $E(B) \subseteq E(A)$.

We use this expressiveness inclusion when showing $E(T A) \subset E(D E V S)$, and
$E(F D E V S) \subset E(F G D E V S) \subseteq E(F C D E V S)$.

### 2.5 Hierarchy of Formalisms

The hierarchy of difference formalism can be defined based on their expressiveness.

## Definition 7 (Subclass, Equivalent class, and Superclass)

Suppose that $A$ and $B$ are two TES classes. Then

- $A$ is called a subclass of $B$ and $B$ is called a superclass of $A$ if $E(A) \subset E(B)$.
- $A$ is called a subclass or equivalent class of $B$ and $B$ is called a superclass or equivalent class of $A$ if $E(A) \subseteq E(B)$.
- $A$ and $B$ are called the equivalent classes if $E(A)=E(B)$.


### 2.6 Homomorphic Timed Event Systems



Figure 3: $H$ is called a homomorphic system of $G$ if such a mapping $f$ exists. If $H$ is a homomorphic system of $G, L(G) \subseteq L(H)$. We use this property when showing $\mathrm{E}(\mathrm{DEVS})=\mathrm{E}(\mathrm{CDEVS})$.

### 3.1 Discrete Event System Specification(DEVS)

## Definition 8 (DEVS)

$$
M=\left(X, Y, S, s_{0}, t a, \delta_{e x t}, \delta_{i n t}, \lambda\right)
$$

- $X$ and $Y$ are the set of input events and the set of output events, respectively;
- $S$ is the set of states; $s_{0} \in S$ is the initial state variable;
- ta : $S \rightarrow \mathbb{T}^{\infty}$ is the time advance function;
- $\delta_{\text {ext }}: Q \times X \rightarrow S$ is the external transition function where $Q=\{(s, e) \in Q, e \in(\mathbb{T} \cap[0, \operatorname{ta}(s)])\}$ is the set of total states, and $e$ is the piecewise linear elapsed time since last event;
- $\delta_{\text {int }}: S \rightarrow S$ is the internal transition function;
- $\lambda: S \rightarrow Y^{\phi}$ is the output function where $Y^{\phi}=Y \cup\{\phi\}$ and $\phi \notin Y$ is a silent event or an unobservable event.


### 3.2 Behaviors of the DEVS class

Let $M=\left(X, Y, S, s_{0}\right.$, ta $\left., \delta_{\text {ext }}, \delta_{\text {int }}, \lambda\right)$ be a DEVS model. Then the behavior of $M$ is explained by a TES $G(M)=\left(Z, Q, q_{0}, Q_{A}, \Delta\right)$ where the event set $Z=X \cup Y^{\phi}$; The state set $Q=Q_{A} \cup Q_{\bar{A}}$ where $Q_{A}=M . Q$ and $Q_{\bar{A}}=\{\bar{s} \notin S\}$ is called the non-accept state in which $\bar{s}$ is piecewise constant.
The initial state variable $q_{0}=\left(s_{0}, 0\right) \in Q_{A}$.
The state trajectory function $\Delta: Q \times \Omega_{Z, \mathbb{T}} \rightarrow Q$ is defined for a total state $q=(s, e) \in Q$ at time $t \in \mathbb{T}$ and an event segment $\omega \in \Omega_{Z,[t, t+d t]}, d t \in \mathbb{T}$ as follows.
For a null event segment, i.e. $\omega=\epsilon_{[t, t+d t]}$,

$$
\Delta(q, \omega)=q \oplus d t= \begin{cases}(s \oplus d t, e+d t) & \text { if } q \in Q_{A}  \tag{4}\\ \bar{s} & \text { otherwise }\end{cases}
$$

which is a timed passage.

For a timed input event, i.e. $\omega=(x, t)$ where $x \in X$

$$
\Delta(q, \omega)= \begin{cases}\left(\delta_{e x t}(s, e, x), 0\right) & \text { if } q \in Q_{A}  \tag{5}\\ \bar{s} & \text { otherwise }\end{cases}
$$

For a timed output or silent event, i.e. $\omega=(y, t)$ where $y \in Y^{\phi}$

$$
\Delta(q, \omega)= \begin{cases}\left(\delta_{\text {int }}(s), 0\right) & \text { if } q \in Q_{A}, \underline{e=\operatorname{ta}(s)}, \underline{y=\lambda(s)}  \tag{6}\\ \bar{s} & \text { otherwise }\end{cases}
$$

If $\omega$ is a multi-event segment, we can apply Equation (1) using above three primitive cases described in Equations (4), (5), and (6).

### 3.3 Definition of Clock-based DEVS Structure

## Definition 9 (CDEVS)

$$
M_{C}=\left(X, Y, S, s_{0}, \delta_{x}, \delta_{y}\right)
$$

- $X$ and $Y$ are the input and output events sets, respectively.
- $S=S_{d} \times \prod_{c \in C}\left(\mathbb{T}^{\infty} \times \mathbb{T}\right)_{c}$ is the set of states that consists of two disjoint sets
- $S_{d}$ is the set of piecewise constant states which is called the set of discrete states.
- $C$ is the set of clock names. Each clock $c \in C$ has two clock variables
- $\sigma_{c} \in \mathbb{T}^{\infty}$ : the schedule of clock $c \in C$, which is piecewise constant.
- $e_{c} \in \mathbb{T} \cap\left[0, \sigma_{c}\right]$ : the elapsed time of clock $c \in C$, which is piecewise linear.
Thus $s=\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right)$ denotes at phase $s_{d} \in S_{d}$, each clock $c$ 's schedule $\sigma_{c}$ and the elapsed time $e_{c}$.


### 3.3 Definition of Clock-based DEVS Structure

## Definition 10 (CDEVS (continued))

$$
M_{C}=\left(X, Y, S, s_{0}, \delta_{x}, \delta_{y}\right)
$$

- $s_{0}=\left(s_{d 0}, \ldots, \sigma_{0 c}, 0, \ldots\right) \in S$ is the initial state variable
- $\delta_{x}: S \times X \rightarrow S$ is the external transition function.
- $\delta_{y}: S \rightarrow Y^{\phi} \times S$ is the output and internal transition function;

Let the remaining time function tr $: S \rightarrow \mathbb{T}^{\infty}$ be

$$
\begin{equation*}
\operatorname{tr}\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right)=\min _{c \in C}\left\{\sigma_{c}-e_{c}\right\} \tag{7}
\end{equation*}
$$

for $\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right) \in S$.

### 3.4 A Example of CDEVS Toaster



Figure 4: A Toaster CDEVS Model where $t \in[20,30]$

### 3.5 Behaviors of the CDEVS class

Given a CDEVS $M_{C}=\left(X, Y, S, s_{0}, \delta_{x}, \delta_{y}\right)$, there exists a TES $G\left(M_{C}\right)=\left(Z, Q, Q_{A}, q_{0}, \Delta\right)$ defining the behavior of $M_{C}$ as follows. The set of events is $Z=X \cup Y$. The set of states is $Q=Q_{A} \cup Q_{\bar{A}}$ where $Q_{A}=\left\{\left(s, t_{s}, t_{e}\right): s \in S, t_{s} \in \mathbb{T}^{\infty}, t_{e} \in \mathbb{T} \cap\left[0, t_{s}\right]\right\}$ and $Q_{\bar{A}}=\{\bar{s} \notin S\}$ in which $t_{s}$ and $\bar{s}$ is piecewise constant, and $t_{e}$ is piecewise linear.
The initial state variable is given

$$
q_{0}=\left(s_{0}, t_{s 0}, t_{e 0}\right)=\left(\left(s_{d 0}, \ldots, \sigma_{0 c}, 0, \ldots\right), \operatorname{tr}\left(s_{0}\right), 0\right) .
$$

The state trajectory function $\Delta: Q \times \Omega_{Z, \mathbb{T}} \rightarrow Q$ is given for $q \in Q$ and an unit segment $\omega$ as below.
For a null segment $\omega=\epsilon_{[t, t+d t]}$ and $t, d t \in \mathbb{T}$,

$$
\Delta(q, \omega)= \begin{cases}\left(\left(s_{d}, \ldots, \sigma_{c}, e_{c}+d t, \ldots\right), t_{s}, t_{e}+d t\right) & \text { if } q \in Q_{A}  \tag{8}\\ \bar{s} & \text { otherwise }\end{cases}
$$

### 3.5 Behaviors of the CDEVS class

For a timed input event $\omega=(x, t), x \in X$, and $t \in \mathbb{T}$,

$$
\Delta(q, \omega)= \begin{cases}\left(\delta_{x}(s, x), \operatorname{tr}\left(\delta_{x}(s, x)\right), 0\right) & \text { if } q \in Q_{A}  \tag{9}\\ \bar{s} & \text { otherwise }\end{cases}
$$

For a timed output event $\omega=(y, t), y \in Y^{\phi}$, and $t \in \mathbb{T}$,

$$
\Delta(q, \omega)= \begin{cases}\left(s^{\prime}, \operatorname{tr}\left(s^{\prime}\right), 0\right) & \text { if } \frac{t_{e}=t_{s}}{}, \delta_{y}(s)=\left(y, s^{\prime}\right)  \tag{10}\\ \bar{s} & \text { otherwise }\end{cases}
$$

## 3.6 $\mathrm{E}(\mathrm{DEVS})=\mathrm{E}(\mathrm{CDEVS})$

## Theorem $1(E(D E V S)=E(C D E V S))$

DEVS and CDEVS are equivalent classes to each other.

$\mathrm{L}(\mathrm{CDEVS})=\mathrm{L}(\mathrm{DEVS})$
Figure 5: Proof of $\mathrm{E}(\mathrm{DEVS})=\mathrm{E}(\mathrm{CDEVS})$ is available at https://sites.google.com/site/moonhohwang/publications

### 4.1 Finite Clock-based DEVS(FCDEVS)

## Definition 11 (FCDEVS)

A Finite CDEVS (FCDEVS) is a subclass of CDEVS
$M_{F C}=\left(X, Y, S, s_{0}, \delta_{x}, \delta_{y}\right)$ where the sets of $X, Y, S_{d}$, and $C$ are
finite. Note that $S=S_{d} \times \prod_{c \in C}\left(\mathbb{T}^{\infty} \times \mathbb{T}\right)_{c}$

### 4.2 Timed Automaton(TA)

## Definition 12 (Timed Automaton(TA))

$$
T A=\left(Z, C, P, p_{0}, I, T\right)
$$

- $Z$ and $C$ are the finite sets of events and the finite set of clocks, respectively.
- $P$ and $p_{0} \in P$ are the finite set of phases which are piecewise constant, and the initial phase variable, respectively.
- I : P $\rightarrow \Phi(C)$ is the phase clock-constraint function where $\Phi(C)=\left\{C \rightarrow \mathbb{I}_{Q}\right\}$ is the set of partial clock constraints.
- $T \subseteq P \times Z^{\phi} \times \Phi(C) \times \mathcal{P}(C) \times P$ is a set of transitions. $A$ transition $\left(p, z, \varphi, C_{R}, p^{\prime}\right) \in T$ can be also inter-changeably represented by the notation $p^{z,\{(c, i n v(c))\}, C_{R}} p^{\prime}$, requires the enabling condition of $I(p)$ and $\varphi$ as a precondition, and the resetting clocks in $C_{R}$ as a postcondition.


### 4.3 A Example of TA Toaster



Figure 6: A Toaster TA Model

### 4.4 Behaviors of the TA class

Given a TA $A=\left(Z, C, P, p_{0}, I, T\right)$, there exists a corresponding FCDEVS $B=\left(X, Y, S, s_{0}, \delta\right)$ that defines the behavior of $A$. We consider all events in $Z$ of $A$ as output events of $B$ so $X=\varnothing$ and $Y=Z$. The state set $S=P \times \prod_{c \in C}\left(\mathbb{T}^{\infty} \times \mathbb{T}\right)_{c}$.
The initial state variable $s_{0}=\left(p_{0}, \ldots, s u\left(p_{0}, c\right), 0, \ldots\right)$ where su : $P \times C \times \mathbb{T} \rightarrow \mathbb{T}^{\infty}$ is called the clock-schedule update function that is given for a phase $p \in P$ and a clock $c \in C$

$$
\begin{gather*}
s u(p, c)=\min \left\{t_{S}\left(\left.(M(I(p)) \cap M(\varphi))\right|_{c} \cap\left[e_{c}, \infty\right)\right)\right. \\
\left.:\left(p, z, \varphi, C_{R}, p^{\prime}\right) \in T\right\} \tag{11}
\end{gather*}
$$

where $M$ is defined in Equation (??) and $t_{S}: \mathcal{P}\left(\mathbb{T}^{\infty}\right) \rightarrow \mathbb{T}^{\infty}$ is the sampling function that is given for a set of time values $\mathbf{t} \subseteq \mathbb{T}^{\infty}$ which can be an time interval,

$$
t_{S}(\mathbf{t})= \begin{cases}\infty & \text { if } \mathbf{t}=\varnothing  \tag{12}\\ t & \text { otherwise } t \in \mathbf{t}\end{cases}
$$

### 4.4 Behaviors of the TA class (continued)

The output and internal transition function $\delta_{y}: S \rightarrow Y^{\phi} \times S$ is given for $s=\left(p, \ldots, \sigma_{c}, e_{c}, \ldots\right), y \in Y^{\phi}:$ If $\exists\left(p, y, \varphi, C_{R}, p^{\prime}\right) \in T$, then

$$
\delta_{y}(s)=\left(p^{\prime}, \ldots, \sigma_{c}^{\prime}, e_{c}^{\prime}, \ldots\right)
$$

where $e_{c}^{\prime}=t_{R}\left(c, C_{R}\right)$ where $t_{R}: C \times \mathcal{P}(C) \rightarrow \mathbb{T}$ is called the resting function that is defined for $c \in C$ and $C_{R} \subseteq C$,

$$
t_{R}\left(c, C_{R}\right)= \begin{cases}0 & \text { if } c \in C_{R}  \tag{13}\\ e_{c} & \text { otherwise }\end{cases}
$$

and $\sigma_{c}^{\prime}=s u\left(p^{\prime}, c\right)$.
If $\nexists\left(p, y, \varphi, C_{R}, p^{\prime}\right) \in T$, then nothing changes because there is no such a transition from $p$, thus $\delta_{y}(s)=\left(p, \ldots, \sigma_{c}, e_{c}, \ldots\right)$.

## Proposition $1(E(T A) \subset E(F C D E V S))$

$E(F C D E V S) \nsubseteq E(T A)$ because TA does not allow clock boundaries of real numbers which are allowed by FCDEVS.

### 4.5 Finite-Graph DEVS

## Definition 13 (FGDEVS)

$$
M_{F G}=\left(X, Y, S, s_{0}, \delta\right)
$$

- $X, Y$ and $S$ are the same as those of CDEVS but they are finite sets; and $s_{0} \in S$ is the initial state.
- $\delta \subseteq S_{d} \times Z^{\phi} \times \Psi(C) \times \mathcal{P}(C) \times S_{d}$ is the finite set of transition relations where $Z=X \cup Y^{\phi}$. A transition $\left(s_{d}, z, \psi, C_{R}, s_{d}^{\prime}\right)$ or its graphical notation $s \xrightarrow{z, \psi, C_{R}} s^{\prime}$ denotes that the discrete state changes $s_{d}$ to $s_{d}^{\prime}$ associated with an event $z$, together with two post-conditions: updating the schedule $\sigma_{c}=\psi(c)$ if $\psi(c)$ is defined for a clock $c \in C$, and resetting the elapsed time $e_{c}$ of each clock $c \in C_{R}$.


### 4.6 A Example of FGDEVS Toaster



Figure 7: A Toaster FGDEVS Model where $t \in[20,30]$

### 4.7 Behavior of FGDEVS

The behaviors of an FGDEVS $M_{F G}=\left(X, Y, S, s_{0}, \delta\right)$ model are given through an FCDEVS $M_{F C}=\left(X, Y, S, s_{0}, \delta_{x}, \delta_{y}\right)$ as follows.
The initial state variable is $s_{0}=\left(s_{d 0}, \ldots, \sigma_{0 c}, 0, \ldots\right)$.
The external transition function $\delta_{x}: S \times X \rightarrow S$ is given for $s=\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right) \in S$ and $x \in X$, if $\exists s_{d} \xrightarrow{x, \psi, C_{R}} s_{d}{ }^{\prime} \in \delta$, then

$$
\delta_{x}(s, x)=\left(s_{d}^{\prime}, \ldots, \sigma_{c}^{\prime}, e_{c}^{\prime}, \ldots\right)
$$

where

$$
\sigma_{c}^{\prime}= \begin{cases}t_{S}(\psi(c)) & \text { if } \psi(c) \text { is defined } \\ \sigma_{c} & \text { otherwise }\end{cases}
$$

and $t_{S}$ is the sampling function defined in Equation (12), and

$$
e_{c}^{\prime}=t_{R}\left(c, C_{R}\right)
$$

where $t_{R}\left(c, C_{R}\right)$ is the resetting function defined Equation (13). If $\nexists s_{d} \xrightarrow{x, \psi, C_{R}} \boldsymbol{s}_{d}{ }^{\prime} \in \delta$, nothing changes by $x$, thus
$\delta_{x}(s, x)=\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right)$.

### 4.7 Behavior of FGDEVS (continued)

The output and internal transition function $\delta_{y}: S \rightarrow Y^{\phi} \times S$ is given for $s=\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right) \in S$ and $y \in Y^{\phi}$, if $\exists s_{d} \xrightarrow{{ }^{Y}, \psi, C_{R}} s_{d}{ }^{\prime} \in \delta$, then

$$
\delta_{y}(s)=\left(y,\left(s_{d}^{\prime}, \ldots, \sigma_{c}^{\prime}, e_{c}^{\prime}, \ldots\right)\right)
$$

where $\sigma_{c}^{\prime}=\psi(c)$ if $\psi(c)$ is defined, otherwise, $\sigma_{c}^{\prime}=\sigma_{c}$; and $e_{c}^{\prime}=t_{R}\left(c, C_{R}\right)$. If $\nexists s_{d} \xrightarrow{y, \psi, C_{R}} s_{d}{ }^{\prime} \in \delta$, nothing changes by an internal transition from $s$ so $\delta_{y}(s)=\left(\phi,\left(s_{d}, \ldots, \sigma_{c}, e_{c}, \ldots\right)\right)$.

## Proposition $2(E(F G D E V S) \subseteq E(F C D E V S))$

It is given by the definition. We still don't know if $E(F C D E V S) \subseteq E(F G D E V S)$ so $E(F G D E V S)=E(F C D E V S)$.

### 4.8 Finite \& Deterministic DEVS (FDDEVS)

## Definition 14 (FDDEVS)

$$
M_{F D}=\left(X, Y, S, s_{0}, \tau, \delta_{x}, \delta_{y}\right)
$$

- $X$ and $Y$ are the same as those of FCDEVS.
- $S$ is the finite discrete states which are piecewise constant.
- $s_{0} \in S$ is the constant initial state.
- $\tau: S \rightarrow \mathbb{Q}_{[0, \infty)}$ is the time schedule function where $\mathbb{Q}_{[0, \infty)}$ is the none negative rational numbers plus infinity.
- $\delta_{x}: S \times X \rightarrow S \times\{0,1\}$ is the external transition function.
- $\delta_{y}: S \rightarrow Y^{\phi} \times S$ is the output and internal transition function.
As the name explains, $\tau, \delta_{x}$ and $\delta_{y}$ of FDDEVS are deterministic.


### 4.9 Behavior of FDDEVS

Given an FDDEVS model $M_{F D}=\left(X, Y, S, s_{0}, \tau, \delta_{x}, \delta_{y}\right)$, there is a corresponding FGDEVS $M_{F G}=\left(X, Y, S_{G}, s_{0 G}, \delta\right)$ can describe the behavior of the original model $M_{F D}$ as follows. The events sets of $M_{F G}$ are the same those of $M_{F D}$. The state set of $S_{G}=\left\{\left(s, \sigma_{c}, e_{c}\right): s \in S, c \in C\right\}$ where $C=\left\{{ }^{\prime} c\right.$ ' $\}$. The initial state $s_{0 G}=\left(s_{0}, \tau\left(s_{0}\right), 0\right)$. The state transition relation $\delta$ of $M_{F G}$ is defined corresponding to each state transition.

$$
\begin{gathered}
\delta_{x}(s, x)=\left(s^{\prime}, 0\right) \text { implies } s \xrightarrow{x, \varnothing, \varnothing} s^{\prime} \in \delta, \\
\delta_{x}(s, x)=\left(s^{\prime}, 1\right) \text { implies } s \xrightarrow{x,\left\{\left(c, \tau\left(s^{\prime}\right)\right)\right\},\{c\}} s^{\prime} \in \delta, \\
\delta_{y}(s)=\left(y, s^{\prime}\right) \text { implies } s \xrightarrow{y,\left\{\left(c, \tau\left(s^{\prime}\right)\right)\right\},\{c\}} s^{\prime} \in \delta .
\end{gathered}
$$

### 5.1 Contributions

- Provided a formal framework that clarifies expressiveness of different formalisms.
- Expressive inclusion among DEVS equivalent and subclasses:


## DEVS=CDEVS

## FCDEVS



### 5.2 Future Directions

- The question whether $E(F C D E V S) \subseteq(F G D E V S)$ or not is still an open problem.
- In addition to TA, expressiveness comparison among other popular formalisms like Colored (timed) Petri-Nets, UML Start-Charts are possible in the same way of timed language approaches.
- Similarity (or Distance) of Two models: Given two DEVS instances $M_{1}$ and $M_{2}$, the distance of $M_{1}$ and $M_{2}$ can be done by their (1) event segments, or (2) states. Then we will have a metric space of discrete event systems using DEVS. That may be answer of simulation model validity for closeness or similarity of two given systems (one can be a target system, the other its simulation model).


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