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Improvement and analysis of multi-station TDOA positioning algorithm based on MSVD

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***Abstract*—Aiming at the failure of multi-station TDOA positioning caused by factors such as unreasonable sensor distribution and measurement noise, a new robust multi-station TDOA positioning method based on Modified Singular Value Decomposition (MSVD) is proposed. In the new algorithm, we use the condition number to evaluate the ill-conditioned degree of the observation equation, and then use the idea of truncated singular value decomposition to correct the singular value with higher ill-conditioned, so as to suppress the influence of measurement noise. The new algorithm effectively considers the resolution and variance of positioning parameter evaluation, and achieves effective suppression of random observation noise in the observation equation. The simulation results show that, compared with the least squares method, the new algorithm can adapt to various multi-station deployment situations, and the positioning results are more accurate and stable.**

Keywords- TDOA Positioning; Modified Singular Value Decomposition (MSVD); Ill-conditioned; Least Squares; Positioning Algorithm

I. INTRODUCTION

At present, with the escalation of the form of warfare, passive positioning plays an increasingly important role in electronic warfare due to its strong concealment. Passive positioning uses the electromagnetic wave radiation of the target to locate the target. The common multi-station TDOA (Time Difference of Arrive) positioning, which uses the multi-station and multi-target positioning time difference for positioning, can also obtain the direction finding information of the target [1,2] , multi-station TDOA positioning is the earliest and relatively mature positioning method in practical systems [3,4], many scholars have done a lot of research on it, especially in multi-station optimization layout [5], accuracy analysis [6], A lot of research results have been achieved in maneuvering target tracking [7], positioning error registration [8], etc., but with the escalation of interference, multi-station TDOA also faces various challenges [9], such as in the case of unreasonable layout How to improve the positioning accuracy, tracking and filtering, etc., there are still many problems worthy of study.

In this paper, the accuracy of the multi-station TDOA positioning algorithm based on the least squares method is analyzed, and a multi-station TDOA positioning algorithm based on the modified singular value method is proposed for the unreasonable layout, which can adapt to various station layout situations. which can effectively suppress the measurement noise on the observation equation, improve the stability and accuracy of the estimation results, and verify the effect of the algorithm by simulation.

II. MULTI-STATION TDOA SPATIAL POSITIONING ALGORITHM

TDOA is a method of positioning using time difference. By measuring the time when the signal arrives at the monitoring station, the distance to the signal source can be determined. Using the distance from the signal source to multiple radio monitoring stations (with the radio monitoring station as the center, the distance is the radius as a circle), the position of the signal can be determined. By comparing the time difference between the signals arriving at multiple monitoring stations, a hyperbola with the monitoring station as the focal point and the distance difference as the long axis can be drawn, and the intersection of the hyperbolas is the position of the signal. The location figure is shown in Figure 1. TDOA is a multi-station positioning system, so to locate the signal, there must be at least 3 monitoring stations for simultaneous measurement.

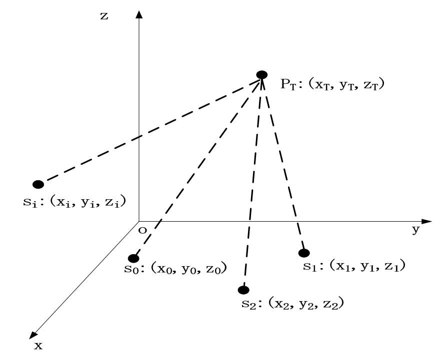

Figure 1. multi station TDOA positioning diagram

In Figure 1, we set S_0 as the master station, S_i (i=1,…,n) as the secondary station, n as the number of secondary stations, and r_i (i=0,1,…,n) as the radiation target and observation the distance between ith stations, Δr_i(i=1,…,n) is the difference between the observation target distance of each secondary ith station and the observation target distance of the master station, so the following relationship can be obtained:

$$\begin{cases} r_0^{\ 2} = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \\ r_i^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \end{cases} \quad (1)$$
$$\Delta r_i = r_i - r_0 = c \cdot \Delta t_i ; i = 1, 2, \cdots, n$$

Among them: the position of the radiation target is (x, y, z), the position of the master station is (x_0, y_0, z_0), the position of the secondary ith station is (x_i, y_i, z_i), the distance between the target and the master station is r_0, the distance between the target and the secondary station is r_i, and the distance between the master station and the slave station is Δr_i. the speed of signal propagation is c, and Δt_i is the time difference between the radiation source signal to the master station and the secondary station.

Equation (1) can be further simplified

$$(x_0 - x_i)x + (y_0 - y_i)y + (z_0 - z_i)z = k_i + r_0 \Delta r_i \quad (2)$$

Where: $k_i = 1/2(\Delta r_i^2 + (x_0^2 + y_0^2 + z_0^2) - (x_i^2 + y_i^2 + z_i^2)), i = 1, 2, \cdots, n$.

Write Equation (2) as a matrix for all station observations:

$$AX = Z \quad (3)$$

Among them: $A = \begin{bmatrix} x_0 - x_1 & y_0 - y_1 & z_0 - z_1 \\ x_0 - x_2 & y_0 - y_2 & z_0 - z_2 \\ \vdots & \vdots & \vdots \\ x_0 - x_n & y_0 - y_n & z_0 - z_n \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $Z = \begin{bmatrix} k_1 + \Delta r_1 \cdot r_0 \\ k_2 + \Delta r_2 \cdot r_0 \\ \vdots \\ k_n + \Delta r_n \cdot r_0 \end{bmatrix}$.

Solve the solution of Equation (3) using least squares:

$$\hat{X} = \left(A^T A\right)^{-1} A^T Z \quad (4)$$

Define the information matrix of X as:

$$J = A^T A \quad (5)$$

III. ILL-CONDITIONED ANALYSIS OF LEAST SQUARES ESTIMATION

In the process of solving Equation (3) by the least squares method, the condition for obtaining an effective solution is that the information matrix J is invertible, that is, the observation matrix A is full rank. However, in actual observations, due to factors such as noise and unreasonable station layout, the column correlation or weak correlation of matrix A will be caused, that is, J is ill-conditioned, and the solution will change drastically at this time with small noise. That is, Equation (3) is ill-conditioned, and it is difficult to obtain a stable solution by solving it directly, and the solution of Equation (4) is invalid. In practice, when the positions of the secondary station and the secondary station are close to each other, it is a common ill-conditioned scenario, and the observation data of each secondary station is basically the same, $(x - x_1, y - y_1, z - z_1) \approx (x - x_i, y - y_i, z - z_i)$, resulting in a linear correlation or weak correlation of the column vectors of the matrix A, which further leads to J irreversibility or ill-conditioned.

In order to effectively evaluate the morbidity in the least squares estimation, it is necessary to quantitatively give an index for evaluation. In the numerical calculation, the condition number is usually used for the ill-conditioned evaluation. For Equation (5), the small disturbance of the estimation result caused by the observation noise has the following relationship:

$$\frac{\|\delta X\|}{\|X\|} \lessapprox \text{cond}\{A\} \left(\frac{\|\delta A\|}{\|A\|} + \frac{\|\delta Z\|}{\|Z\|} \right) \quad (6)$$

For nonsingular matrix A, define the condition number as follows:

$$\text{cond}\{A\} = \max_i |\lambda_i(A)| \big/ \min_i |\lambda_i(A)| \quad (7)$$

It can be seen from the definition that the more scattered the eigenvalues of the matrix A, the larger the cond{A}, and the higher the error upper limit of the further solution vector. To obtain a stable solution, cond{A} must be compressed. However, the condition number is a measure of the ill-conditioned nature of the entire matrix, which only depends on the maximum and minimum eigenvalues. In practice, we also need to quantitatively determine the relationship between the eigenvalues to better determine the ill-conditioned nature of the matrix. To do this, we define conditional index:

$$\eta_k = \frac{\max_j |\lambda_j|}{|\lambda_k|} \quad (8)$$

Define η_k as the k condition index of matrix A, where λ_1 is the maximum eigenvalue, obviously $\eta_k \geq 1$.

It can be seen from the definition that the condition index directly reflects the influence degree of the least squares estimation parameters by the measurement noise. If the condition index is high and the observation matrix A has a small disturbance, the results of the least squares estimation will change considerably. A large number of numerical calculation results show that (Belsley, 1991; Davey, 1959) [12], if the ill-conditioned is weak, the condition index is less than 100; if the ill-conditioned is strong, the condition index is between 100 and 1000; In severe cases, the condition index is above 1000.

IV. MULTI-STATION TDOA POSITIONING BASED ON MSVD

The least squares method is a linear unbiased estimation with minimum variance of equation (3), but if it is affected by noise under ill-conditioned conditions, the result of the least squares estimation will change drastically, and the estimated variance will increase sharply, equation (4) becomes a de facto biased estimate. When the observation equation A is ill-conditioned, in order to make the estimation result more robust, we use the modified singular value decomposition method to modify the eigenvalues of the observation matrix.

According to the definition of the information matrix *J*, *J* is a real symmetric matrix. According to the singular value decomposition theorem, for any real symmetric matrix $J_{m\times m}$, there is an orthogonal matrix $V_{m\times m}$ that satisfies:

$$J = VSV^T \tag{9}$$

Among them: $S = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}$, $\Sigma = diag(\lambda_1, \lambda_2, ..., \lambda_r)$, $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_r > 0$, λ_i is J eigenvalues; $r \leq \min(m,n)$. Matrix V can be represented in column vector form $V = (v_1, v_2, ..., v_n)$

If J is ill-conditioned or irreversible, the Mooer-Penrose generalized inverse of *J* is defined as:

$$J^+ = VS^{-1}V^T \tag{10}$$

Among them: $S^{-1} = diag(\lambda_1^{-1}, \lambda_2^{-1}, ..., \lambda_r^{-1}, 0, ..., 0)$

The variance of the least squares estimate is:

$$\mathrm{D}(\hat{b}) = \sigma_0^2 VS^{-2}V^T \tag{11}$$

Write the above equation in vector form:

$$\mathrm{var}(\hat{b}) = \sigma_0^2 \sum_{i=1}^{r} (v_i / \lambda_i)^2 \tag{12}$$

It can be seen from the variance equation (12) of the least squares estimation result: if the observation equation is ill-conditioned, there is $\lambda_i \to 0$, so $1/\lambda_i \to \infty$. At this time, the measurement noise will cause the variance of the estimation result to amplify sharply, the estimation result will be very unstable, and the result may even be completely wrong. According to the multi-station TDOA algorithm, the eigenvalues of the information matrix are the characteristics of continuous distribution, and the idea of modified singular value decomposition is used to modify the singular values, so that the estimation results are more robust.

The modified singular value decomposition method modifies the eigenvalues to achieve the purpose of reducing the variance of the estimated results, so the total number of eigenvalues will not decrease. There are many ways to modify eigenvalues. A common modification is $\overline{\lambda_i} = \lambda_i + \alpha/\lambda_i$, where α is a number greater than 0. The modified expression of S^{-1} is:

$$S^{-1} = diag\left(\frac{\lambda_1}{\lambda_1^2 + \alpha_1}, ..., \frac{\lambda_r}{\lambda_r^2 + \alpha_r}, 0, ..., 0\right) \tag{13}$$

Write the above equation in vector form:

$$\hat{b}_\alpha = \sum_{i=1}^{r} \frac{\lambda_i}{\lambda_i^2 + \alpha_i} (Z, v_i) v_i \tag{14}$$

The variance of the MSVD estimate is:

$$\mathrm{var}(\hat{X}_\alpha) = \sigma_0^2 \sum_{i=1}^{r} v_i^2 \Big/ (\lambda_i + \alpha / \lambda_i)^2 \tag{15}$$

It can be seen from equations (13) and (14) of the MSVD estimation results that selecting a suitable α can reduce the variance of the estimation results and improve the estimation stability.

V. SIMULATION

Suppose the multi-station TDOA system has one main station and three secondary stations, where the position of the main station is (0km, 0km, 0.1km), and the positions of the three secondary stations are (20km, 20km, 0km), (-20km, 25km, 0km), (15km, 20km, 0km). The time measurement error of each station is 30ns, the position error of each station is 10m, and the time measurement error correlation coefficient is 0.5. It is assumed that the height of the radiation target is 10km, and the target is in the range of x: -100~100km, y: -100~100km for simulation verification. Figures 2 and 3 are the GDOP diagrams of the least-squares and MSVD-based improved algorithm positioning accuracy, respectively. The comparison shows that the MSVD-based positioning accuracy is significantly higher than the least squares method. Figures 4 and 5 are the contour maps of the positioning accuracy of the least squares method and the improved algorithm based on MSVD. The comparison shows that the positioning accuracy based on MSVD is significantly higher than that of the least squares method. Figure 6 is the GDOP contour Figure improved by the MSVD-based algorithm.

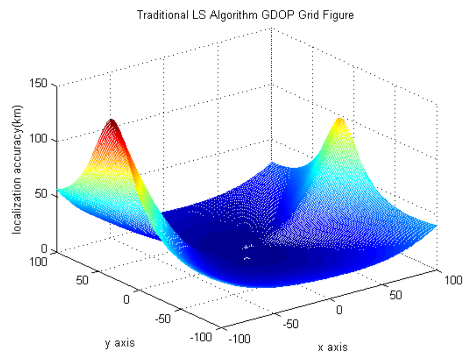

Figure 2. least squares GDOP grids

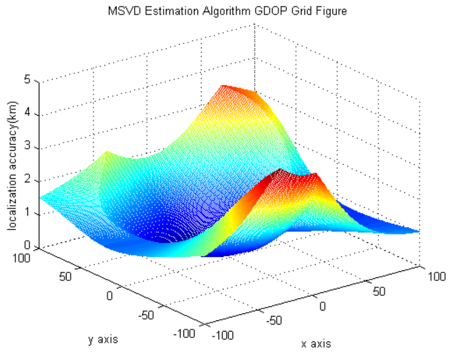

Figure 3. MSVD-based estimation GDOP grids

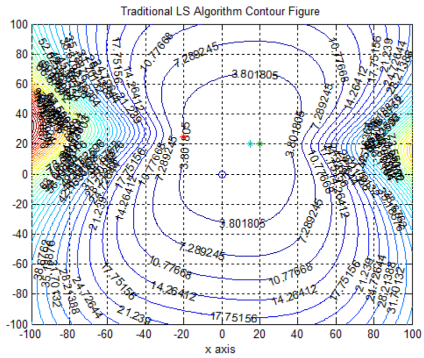

Figure 4. least squares for locating GDOP contour lines

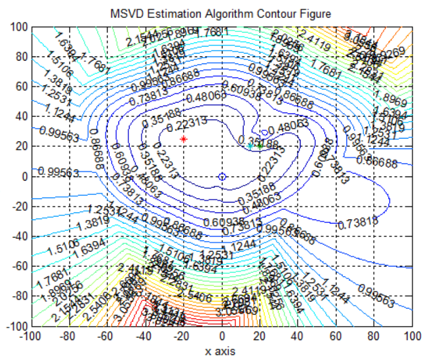

Figure 5. MSVD-based for locating GDOP contour lines

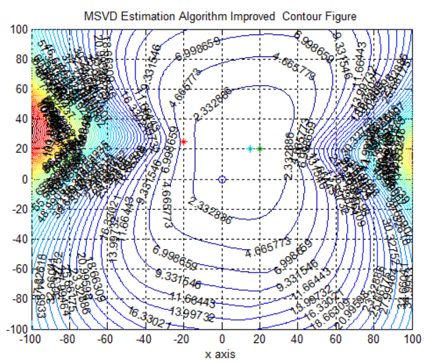

Figure 6. MSVD-based improved the positioning of GDOP contours

VI. CONCLUSION

In the multi-station TDOA positioning system, this paper proposes a positioning method based on Modified Singular Value Decomposition (MSVD) for the ill-conditioned observation equations caused by the unreasonable layout of the observation stations.The method analyzes the cause of ill-conditioned in the least squares algorithm, uses the condition index to quantitatively measure the ill-conditioned degree of the information matrix, and obtains a robust estimation result by modifying the smaller eigenvalues. The new algorithm realizes the comprehensive consideration of the resolution and variance of the positioning parameters, and can adapt to various site distribution layouts. The simulation results show that the GDOP and contour figure of the positioning accuracy are both improved by an order of magnitude compared with the least squares algorithm, and the results are more stable, which has good engineering application value.

AUTHOR

Pan Jianghuai (1982.10-) was born in Gao 'an, Jiangxi Province, China. He graduated from Wuhan University with a bachelor's degree in 2004. In 2007, he graduated from Jiangsu Automation Research Institute of China Ship Research Institute with a master's degree. He is now studying for his PhD in Nanjing University of Aeronautics and Astronautics. He has won 2 first prizes and 1 second prize at provincial and ministerial level. He has published 1 monographs, 9 patents and 21 papers (including 7 in EI and 9 in Chinese core). In 2014, he was employed as a senior engineer. His research interests include short-range anti-missile, maneuvering target tracking, information fusion, spatial registration, etc.

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