The Impact of Device-to-Device Communication on the Capacity of Cellular Systems

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***Abstract*— In this paper, the capacity of underlaying cellular networks is analyzed by considering the impact of Device-to-Device (D2D) mode, which is shown to be capable of substantially enhancing the sum transmission rate of the cellular network. Based on the theory of stochastic geometry, the *successful transmission probabilities* of both the D2D users and the conventional cellular users are derived, with the D2D transmission rate as well as the sum rate given out simultaneously. Furthermore, the optimal D2D-users density that maximizes the sum rate is also derived. In addition, we extend the analysis from single-cell scenario to the multicell environment. Numerical results demonstrate the relationship between D2D-contributed capacity and the D2D-users density. It is shown that the network capacity can be enhanced by increasing the density D2D users when the D2D-users density is relatively low, until a break-even point is approach, in which case further increasing D2D users may even erode the sum rate.**

I. Introduction

With the rapid development of wireless communication techniques, the capacity of existing cellular networks has become insufficient for supporting the customers' exponentially growing demands of throughput [1], [2]. Device-to-Device (D2D) communication, which is capable of substantially enhancing the data rate by allowing proximity users to communicate directly with each other (i.e. without relying on the intervention of base station (BS)) and offloading efficiently the local tele-traffics from the BS [3], has attracted wide attentions in both academia and industry. Furthermore, D2D communication may coexist with the conventional cellular systems, thus enabling the establishment of short-range and low-power wireless links to bring about a range of benefits such as an improved channel capacity and spectral utilization [4].

Despite of that, the implementation of D2D techniques in cellular networks may also give rise to some new challenges due to the severe interference imposed on the cellular users (CUs). In order to avoid the above-mentioned interference issue, some literatures tried to allocate dedicated cellular resources for D2D communications (i.e. in an overlay model [5]). However, the overlay model may invoke an inefficient cellular resource utilization. On the contrary, the alternative solutions relying on an underlay model [6] prefer to reuse the cellular resources when supporting D2D communication, but the interference-management problem becomes one of the most critical issues for the D2D-enabled underlaying cellular networks.

To address the above-mentioned issue, three categories of interference-management techniques, namely *mode selection*, *resource allocation* and *power control*, have been proposed [7]:

- *Mode selection* in D2D communication enables the users to choose an appropriate communication mode between D2D mode (communicate directly to another user) and the conventional cellular mode (as a traditional CU).
- *Resource allocation* is regarded as an efficient method for avoiding the nearby users to communicate over the same resource blocks.
- *Power control* of both DUs and CUs have to be properly regulated in order to ensure that the minimum SINR requirement can be maintained.

Apart from that, there also exist several literatures that focused on the capacity optimization joint two or three of the above-mentioned techniques. For example, authors of [8] optimized the sum-rate of the network by jointly performing power control and resource allocation subject to the constraint of spectral efficiency and power consumption, whilst considering three link-sharing strategies, i.e. non-orthogonal sharing mode, orthogonal sharing mode and cellular mode. In order to enhance the D2D-aided network capacity, authors in [9] proposed a scheme to enable the CUs to allocate a part of their transmission power to assist the D2D communication. The influence of power control on the transmission capacity region of D2D network has been investigated in [10] and shown the different impact of reuse and dedicated mode. Furthermore, the authors in [11] formulated a joint-optimization problem by combining the schemes of mode selection, resource allocation and power allocation in a multi-cell cellular network, in which a distributed sub-optimal heuristic algorithm was proposed for solving the encountered NP-Hard problem.

Although the literatures aforementioned focused on different aspects of D2D communication, all of them reached the following consensus: D2D communication is capable of substantially improving the cellular capacity subject to an efficient interference management. Unfortunately, the existed literatures failed to point out "*to what extent the capacity improvement could be achieved by employing D2D mode*". In other words, is it possible for us to improve the cellular capacity unlimitedly by continuously increasing the percentage of DU-pairs? If not, what is the break-even point that maximize the whole network

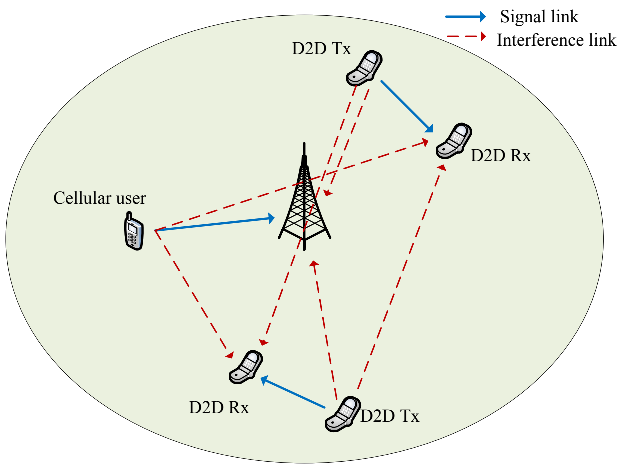

Fig. 1. The model of D2D-based underlay cellular networks.

capacity?

In this paper, we try to answer the above-mentioned questions by analyzing the relationship between the capacity of D2D-based underlaying cellular systems and the DUs density in both single-cell scenario and multicell environment. The remainder of this paper is organized as follows. In Section II, the D2D-based underlaying cellular system model of single cell is described. After that, the network capacity as well as the optimal DU density is derived in Section III. In Section IV, we extend the above-mentioned analysis into multicell environment, followed by numerical results given by Section V. Finally, Section VI concludes this paper.

II. D2D Underlaying Cellular System Models

In this section, we consider a single-cell system comprising D2D pairs in an underlay manner, with the uplink cellular spectrum resources reused by the DUs. The cell radius is assume to be R, with a BS B located at the center of the cell, as shown in Fig. 1. Without loss of generality, the arrangement of both the CUs and D2D transmitters (D2D Txs) are assumed to follow the independent homogeneous Poisson Point Process (PPP). We assume that each DU has a matched DU close to it to constitute a D2D pair, with the average length of the established D2D links represented by R_d. Furthermore, link signaling of each CU is assumed to be orthogonal to each other, thus invoking no interference among CUs in the single-cell scenario. Additionally, in order to simplify the analysis, we only consider the network capacity based on a single-sub-channel analysis. Note that the DUs reuse the same sub-channel still follow PPPs owing to the thinning property of PPP. Therefore, co-channel users comprising one CU locating at R_c away from the BS and DUs form a PPP Π_d, whose density is given by λ_d m^{-2}. Without loss of generality, all the CUs have the same transmit power P_c, whilst all the DUs transmit at the same power P_d.

Wireless channels for the proposed underlay networks can be modelled as path-loss multiplied by the Rayleigh fading coefficients, as defined by

$$P_r = P_t h_{ij} D_{ij}^{-\alpha}, \tag{1}$$

where P_t and P_r represent the transmitter and receiver power, respectively, α stands for the standard path loss exponent, h_{ij} denotes the Rayleigh fading coefficients between node i and j and it has an exponential distribution with unit mean, D_{ij} represents the distance between the transmitter and the receiver. The signal-to-interference-plus-noise ratio (SINR) of CUs and DUs can thus be expressed as

$$\mathrm{SINR}_c = \frac{P_c h_{cB} D_{cB}^{-\alpha}}{\sigma^2 + I_{dc}}, \tag{2}$$

$$\mathrm{SINR}_d = \frac{P_d h_d D_d^{-\alpha}}{\sigma^2 + I_{cd} + I_{dd}}, \tag{3}$$

respectively, where I_{dc} denotes the interference power imposed on CUs by DUs, I_{cd} stands for the interference power imposed on DUs by CUs, I_{dd} represents the interference among DUs, and σ^2 is the covariance of Additive-White Gaussian Noise (AWGN).

III. Optimal Capacity of D2D-based single-Cellular Systems

In this section, we analyze the capacity of D2D-based underlaying cellular systems as a function of DUs density from the perspective of successful transmission probability. We first derive the successful transmission probabilities of both DUs and CUs. After that, we derive the DUs capacity as well as the total system capacity. Finally, we obtain the optimal DUs density that maximizes the D2D network capacity.

A. Successful Transmission Probability

We define the successful transmission probability as the probability that a randomly chosen user can successfully reach its predetermined target SINR T.

From the stationarity of the Poisson process, all the receivers have the same statistics in terms of signal reception [12]. Since the typical D2D receiver may suffer from interference imposed by the other DUs as well as the uplink transmissions of CUs, the SINR at the DU's receiver can be expressed as

$$\mathrm{SINR}_d = \frac{P_d h_d R_d^{-\alpha}}{\underbrace{\sum_{i\in\Pi_d/d_0} P_d h_{i0} D_{i0}^{-\alpha}}_{I_{dd}} + \underbrace{P_c h_{c0} D_{c0}^{-\alpha}}_{I_{cd}} + \sigma^2}, \tag{4}$$

where d_0 denotes the associative D2D transmitter of the desired D2D receiver. The successful transmission probability of the typical D2D receiver can thus be expressed as

$$\begin{aligned}\mathbb{P}_d &= \mathbb{P}(\mathrm{SINR}_d > T) \\ &= \mathbb{P}\left[h_d > T R_d^{\alpha}\left(I_{dd} + I_{cd} + \sigma^2\right)/P_d\right].\end{aligned} \tag{5}$$

Since h_d follows an exponential distribution with unit mean, (5) can be rewritten as

$$\begin{aligned}\mathbb{P}_d &= \mathbb{E}_{I_{dd},I_{cd}}\left\{\exp\left[-TR_d^{\alpha}\left(I_{dd}+I_{cd}+\sigma^2\right)/P_d\right]\right\}\\ &= \mathbb{E}_{I_{dd}}\left[\exp\left(-TR_d^{\alpha}\frac{I_{dd}}{P_d}\right)\right]\mathbb{E}_{I_{cd}}\left[\exp\left(-TR_d^{\alpha}\frac{I_{cd}}{P_d}\right)\right]\\ &\quad\times\exp\left(-TR_d^{\alpha}\frac{\sigma^2}{P_d}\right),\end{aligned} \tag{6}$$

Note that the transformation of (6) can be performed based on the independence between random variables I_{dd} and I_{cd}, as given by

$$\begin{aligned}&\mathbb{E}_{I_{dd}}\left[\exp\left(-TR_d^{\alpha}\frac{I_{dd}}{P_d}\right)\right]\\ &= \mathbb{E}_{\Pi_d,h_{i0}}\left[\exp\left(-TR_d^{\alpha}\sum_{i\in\Pi_d/d_0}h_{i0}D_{i0}^{-\alpha}\right)\right]\\ &= \mathbb{E}_{\Pi_d,h_{i0}}\left[\prod_{i\in\Pi_d/d_0}\exp\left(-TR_d^{\alpha}h_{i0}D_{i0}^{-\alpha}\right)\right]\\ &= \mathbb{E}_{\Pi_d}\left(\prod_{i\in\Pi_d/d_0}\frac{1}{1+TR_d^{\alpha}D_{i0}^{-\alpha}}\right).\end{aligned} \tag{7}$$

Based on the probability generating function (PGF) of PPP [12], we have

$$\mathbb{E}\left[\prod_{x\in\Phi}f(x)\right] = \exp\left(-\lambda\int_{\mathbb{R}^2}(1-f(x))dx\right). \tag{8}$$

Thus, the equation (7) can thus be rewritten as

$$\begin{aligned}&\mathbb{E}_{I_{dd}}\left[\exp\left(-TR_d^{\alpha}\frac{I_{dd}}{P_d}\right)\right]\\ &= \exp\left[-2\pi\lambda_d\int_{R_d}^{\infty}\left(1-\frac{1}{1+TR_d^{\alpha}x^{-\alpha}}\right)xdx\right]\\ &= \exp\left(-\pi R_d^2\lambda_d T^{2/\alpha}\int_{T^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du\right),\end{aligned} \tag{9}$$

where a variable transformation was performed on the last step of (9), i.e. $u=\left(\frac{x}{R_dT^{1/\alpha}}\right)$.

Similarly, we can also get

$$\mathbb{E}_{I_{cd}}\left[\exp\left(-TR_d^{\alpha}\frac{I_{cd}}{P_d}\right)\right] = \frac{1}{1+T\frac{P_c}{P_d}\left(\frac{R_d}{D_{c0}}\right)^{\alpha}}. \tag{10}$$

Combining (9) and (10), we can derive the successful transmission probability of the typical D2D receiver as

$$\mathbb{P}_d = \frac{\exp\left(-\pi R_d^2\lambda_d T^{2/\alpha}\int_{T^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du - TR_d^{\alpha}\frac{\sigma^2}{P_d}\right)}{1+T\frac{P_c}{P_d}\left(\frac{R_d}{D_{c0}}\right)^{\alpha}}. \tag{11}$$

Similarly, the successful transmission probability of the CUs can be derived as

$$\mathbb{P}_c = \exp\left(-\pi D_{cB}^2\lambda_d T^{\frac{2}{\alpha}}\int_0^{\infty}\frac{1}{1+\frac{P_c}{P_d}u^{\alpha/2}}du - \frac{TD_{cB}^{\alpha}\sigma^2}{P_c}\right). \tag{12}$$

B. Maximal Cellular Capacity

The capacity of a cellular system can be defined as "*the maximum spatial density of successful transmissions while guaranteeing the target successful transmission probability*" [13], i.e.

$$\begin{aligned}C_{sum} &= \lambda_d\mathbb{P}_d + \frac{1}{\pi R^2}\mathbb{P}_c\\ &= \frac{\lambda_d\exp\left(-\pi R_d^2\lambda_d T^{\frac{2}{\alpha}}\psi_1 - TR_d^{\alpha}\frac{\sigma^2}{P_d}\right)}{1+T\frac{P_c}{P_d}\left(\frac{R_d}{D_{c0}}\right)^{\alpha}}\\ &\quad+\frac{\exp\left(-\pi D_{cB}^2\lambda_d T^{\frac{2}{\alpha}}\psi_2 - \frac{TD_{cB}^{\alpha}\sigma^2}{P_c}\right)}{\pi R^2}\end{aligned} \tag{13}$$

where $\psi_1 = \int_{T^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du$ and $\psi_2 = \int_0^{\infty}\frac{1}{1+\frac{P_c}{P_d}u^{\alpha/2}}du$. Since CUs form a PPP, the probability that the distance between a CU and the BS is smaller than x satisfies

$$\mathbb{P}(D_{cB}<x) = \frac{\pi x^2\lambda_c}{\pi R^2\lambda_c} = \frac{x^2}{R^2}, \tag{14}$$

where the PDF of D_{cB} is given by $f(x)=\frac{2x}{R^2}$. Within one cell, the expectation of that distance can be derived as

$$\mathbb{E}(D_{cB}) = \int_0^R\frac{2x^2}{R^2}dx = \frac{2}{3}R. \tag{15}$$

Relying on the stationarity of the Poisson process, the average distances from any point in the cell to the CU are statically identical, implying that $\mathbb{E}(D_{c0}) = \mathbb{E}(D_{cB})$. Consequently, (13) can be rewritten as

$$\begin{aligned}C_{sum} &= \frac{\lambda_d\exp\left(-\pi R_d^2\lambda_d T^{2/\alpha}\psi_1 - TR_d^{\alpha}\frac{\sigma^2}{P_d}\right)}{1+T\frac{P_c}{P_d}\left(\frac{3R_d}{2R}\right)^{\alpha}}\\ &\quad+\frac{\exp\left(-\frac{4\pi}{9}R^2\lambda_d T^{\frac{2}{\alpha}}\psi_2 - \frac{2^{\alpha}TR^{\alpha}\sigma^2}{3^{\alpha}P_c}\right)}{\pi R^2}.\end{aligned} \tag{16}$$

By taking partial derivative of λ_d for C and making $\frac{dC_{sum}}{d\lambda_d}=0$, we have

$$\begin{aligned}f(\lambda_d) &= \frac{9\left(1-\pi R_d^2\lambda_d T^{2/\alpha}\psi_1\right)}{4T^{\frac{2}{\alpha}}\psi_2\left[1+T\frac{P_c}{P_d}\left(\frac{3R_d}{2R}\right)^{\alpha}\right]}\\ &\quad\times\ \exp\left[-\pi T^{\frac{2}{\alpha}}\left(R_d^2\psi_1 - \frac{4}{9}R^2\psi_2\right)\lambda_d\right]\\ &\quad\times\ \exp\left[T\sigma^2\left(\frac{R_d^{\alpha}}{P_d}+\frac{\left(\frac{2}{3}R\right)^{\alpha}}{P_c}\right)\right]-1=0,\end{aligned} \tag{17}$$

In theory, the DUs density λ_d satisfying equation (17) is the optimal DU density that maximizes the D2D network capacity. Unfortunately, it is very hard to derive an analytical expression of λ_d from the equation (17), if not impossible. In light of the fact that $f(0) > 0$ and $f(\infty) < 0$ are satisfied and at the same time $f(x)$ is a continuous function of $x \in (0, \infty)$, we can conclude that there must exist a λ_d to make $f(\lambda_d) = 0$ (i.e. the optimal λ_d that maximizes the capacity does exist).

From (16), the DUs' capacity could be much higher than the CUs' capacity, if λ_d is not very small, and we can approximate the transmission capacity of D2D networks as

$$\begin{aligned}\tilde{C}_{sum} =& \frac{\lambda_d}{1 + T\frac{P_c}{P_d}\left(\frac{3R_d}{2R}\right)^{\alpha}} \\ &\times \frac{\exp\left(-\pi R_d^2 \lambda_d T^{2/\alpha}\psi_1 - TR_d^{\alpha}\frac{\sigma^2}{P_d}\right)}{\pi R^2}.\end{aligned} \tag{18}$$

Consequently, we can derive the approximately optimal $\tilde{\lambda_d}$ by taking partial derivative of λ_d for $\tilde{C}$, leading to

$$\tilde{\lambda_d} = \frac{1}{\pi R_d^2 T^{2/\alpha}\psi_1}. \tag{19}$$

IV. Optimal Capacity of D2D-based Networks under Multicell Environment

In this section, we extend the capacity analysis of the proposed D2D-based underlaying networks into the multicell environment. Without loss of generality, we consider a network comprising both the cellular and D2D links, and focus our attention on the uplink capacity. The BSs, DUs and CUs are all randomly distributed within the network and are modelled by independently PPPs, as denoted by Π_B, Π_d and Π_c with density λ_B, λ_d and λ_c, respectively. Furthermore, we assume a more general D2D-pairing scenario, in which one DU is allowed to directly communicate with any other DUs, if their distance (D2D link distance) R_d' is smaller than the D2D communication threshold R_d^t, i.e. $R_d' < R_d^t$ [1].

The SINR at the DU receiver can be expressed as

$$\begin{aligned}\mathrm{SINR}_d =& \frac{P_d h_d R_d^{-\alpha}}{I_{dd} + I_{cd} + \sigma^2} \\ =& \frac{P_d h_d R_d^{-\alpha}}{\sum\limits_{i\in\Pi_d/d_0} P_d h_{i0} D_{i0}^{-\alpha} + \sum\limits_{i\in\Pi_B} P_c h_{B_i0} D_{B_i0}^{-\alpha} + \sigma^2},\end{aligned} \tag{20}$$

from which the successful transmission probability of the typical D2D receiver can be expressed as

$$\begin{aligned}\mathbb{P}_d &= \mathbb{P}(\mathrm{SINR}_d > T) \\ &= \int_0^{\infty} \mathbb{P}\left(\frac{P_d h_d r^{-\alpha}}{I_{dd} + I_{cd} + \sigma^2} > T\right) f_{R_d}(r) dr \\ &= \int_0^{\infty} \exp\left(-\frac{T\sigma^2 r^{\alpha}}{P_d}\right) \exp\left[-\lambda_d T^{\frac{2}{\alpha}} \frac{2\pi^2 r^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)}\right] \\ &\qquad \times \exp\left[-\lambda_B \left(\frac{TP_c}{P_d}\right)^{\frac{2}{\alpha}} \frac{2\pi^2 r^2}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\right] f_{R_d}(r) dr \\ &= \int_0^{D} \exp\left[-T^{\frac{2}{\alpha}} \frac{2\pi^2 r^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)} \left(\lambda_B \left(\frac{P_c}{P_d}\right)^{\frac{2}{\alpha}} + \lambda_d\right)\right] \\ &\qquad \times \exp\left(-\frac{T\sigma^2 r^{\alpha}}{P_d}\right) \frac{2r}{{R_d^t}^2} dr,\end{aligned} \tag{21}$$

where $f_{R_d}(r)$ is the probability density function of the distance between the D2D pair, and this probability can be further derived as

$$f_{R_d}(r) = \begin{cases} \dfrac{2r}{{R_d^t}^2}, & 0 \le r \le R_d^t, \\ 0, & r > R_d^t. \end{cases} \tag{22}$$

Similarly, the successful transmission probability of the CUs can be expressed as

$$\begin{aligned}\mathbb{P}_c = \int_0^{\infty} &\exp\left[-T^{\frac{2}{\alpha}} r^2 \left(\lambda_d \left(\frac{P_c}{P_d}\right)^{\frac{2}{\alpha}} \frac{2\pi^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)} + \pi\lambda_B\rho\right)\right] \\ &\exp\left(-\frac{T\sigma^2 r^{\alpha}}{P_d}\right) e^{-\pi\lambda_B r^2} 2\pi\lambda_B r dr.\end{aligned} \tag{23}$$

where $\rho = \int_{T^{-\frac{2}{\alpha}}}^{\infty} \frac{1}{1+u^{\alpha/2}} du$.

We assume that all the DUs are permitted to simultaneously access the spectrum, whereas the CUs are scheduled in a round-robin fashion, which means that in each time only one uplink CU can be activated within each cell. Therefore, the spectral efficiency of CUs and DUs link (SE_c and SE_d) can be given by [14]

$$SE_d = \mathbb{E}[K_d \log(1 + \mathrm{SINR}_d)] = \int_0^{\infty} \frac{\mathbb{P}_d}{T+1} dT \tag{24}$$

and

$$\begin{aligned}SE_c &= \mathbb{E}[K_c \log(1 + \mathrm{SINR}_c)] \\ &= \frac{\lambda_B}{\lambda_c}\left(1 - e^{-\frac{\lambda_c}{\lambda_B}}\right) \int_0^{\infty} \frac{\mathbb{P}_c}{T+1} dT,\end{aligned} \tag{25}$$

respectively, where K denotes the time-access factor of each link.

For brevity, we use the successful transmission probability as the performance indicator to evaluate the spectral efficiency of each link, and (24) and (25) can thus be simplified as $SE_d = \mathbb{P}_d$ and $SE_c = \frac{\lambda_B}{\lambda_c}\left(1 - e^{-\frac{\lambda_c}{\lambda_B}}\right)\mathbb{P}_c$, respectively. Furthermore, if the thermal noise power is assumed to approach

[1] In the last chapter of single cell scene, we treat the D2D link length R_d as an average value, i.e. a fixed value. In contrast, in this chapter of multicell environment we treat the D2D link length R_d' as a random variable to seek a more general consequence.

TABLE I

KEY PARAMETERS IN THE SIMULATION

Parameter	Physical Mean	Value
P_c	Tx Power of CUs	30dBm
P_d	Tx Power of DUs	20dBm
α	Path loss coefficient	4
R	cellular radius	200m
R_d	The distance between a DU pair	20m
σ^2	Power level of thermal noise	-174dBm/Hz
T	Successful transmission threshold of SINR	10dB

zero, the overall network capacity can be simplified as

$$\begin{aligned} C_{sum} &= \lambda_d \mathbb{P}_d + \frac{\lambda_B^2}{\lambda_c}\left(1 - e^{-\frac{\lambda_c}{\lambda_B}}\right)\mathbb{P}_c \\ &= \frac{1 - \exp\left[-T^{\frac{2}{\alpha}} \frac{2\pi^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)}\left(\lambda_B \left(\frac{P_c}{P_d}\right)^{\frac{2}{\alpha}} + \lambda_d\right)\right] R_d^t}{T^{\frac{2}{\alpha}} \frac{2\pi^2}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)}\left(\frac{\lambda_B}{\lambda_d}\left(\frac{P_c}{P_d}\right)^{\frac{2}{\alpha}} + 1\right) {R_d^t}^2} \\ &+ \frac{\frac{\lambda_B}{\lambda_c}\left(1 - e^{-\frac{\lambda_c}{\lambda_B}}\right)}{T^{\frac{2}{\alpha}}\left(\frac{\lambda_d}{\lambda_B}\left(\frac{P_c}{P_d}\right)^{\frac{2}{\alpha}} \frac{2\pi}{\alpha \sin\left(\frac{2\pi}{\alpha}\right)} + \rho\right) + 1}, \end{aligned} \tag{26}$$

implying that the overall network capacity cannot be increased unlimitedly with the increases of the number of D2D pairs. The capacity upper bound is then given by

$$\lim_{\lambda_d \to \infty} C_{sum} = \frac{\alpha \sin\left(\frac{2\pi}{\alpha}\right)}{2T^{\frac{2}{\alpha}}(\pi R_d^t)^2}. \tag{27}$$

From the above-mentioned analysis, a capacity ceiling appears in the D2D-aided underlaying cellular systems.

V. NUMERICAL RESULTS

In this section, we evaluate the capacity of the D2D-aided underlaying cellular systems as a function of DU density in both the single-cell scenario and the multicell environment. We focus our attention on explaining how the density of DUs impacts the overall cellular capacity. In a single-cell scenario, we only evaluate the cellular capacity of a single sub-channel, implying that we consider only one CU and a group of DUs that form a PPP of density λ_d. Furthermore, the cellular radius is assumed to be $200m$ and the distance between the DU transmitter and the receiver is assumed to be $20m$. The other parameters considered in the simulation are listed in Table I.

The relationship between the D2D-based underlay cellular capacity and the DU density is evaluated in follows. When the DU density is low, the cellular capacity can be improved by simply increasing the DU density, until a break-even point is approached. This result adheres to our intuition: the more the D2D pairs, the higher the overall capacity. However, once the DU density increases to a level that is beyond a critical point, the overall capacity will decrease, if we further increase the DU density. More seriously, the overall capacity may even approach zero, if the DU density is infinitely increased. We can explain the above-mentioned phenomenon as follows: once the DU density is extremely high, the cellular systems becomes interference-limited, and the performance gain brought by increasing DU density cannot counter-balance the capacity loss induced by the severe interference.

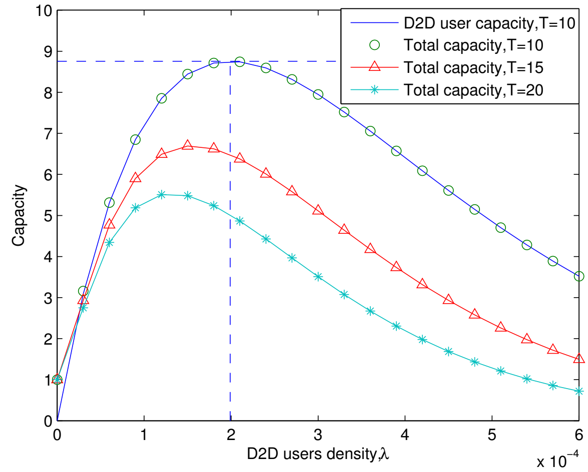

Fig. 2. Cellular Capacity under different successful transmission threshold of SINR.

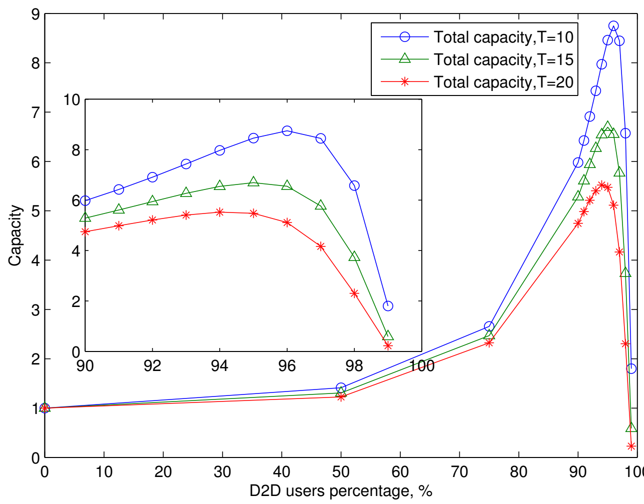

Fig. 3. Cellular Capacity under different percentage of DUs.

In Fig.2, we evaluate the overall network capacity as a function of DU density. It is shown that the capacity contributed by DU pairs dominates the overall capacity, if the DU density is not very low. This finding makes sense, because we consider only one CU in each sub-channel (i.e. the total number of DUs is $N_d = \pi R^2 \lambda_d \mid_{\lambda_d = 2*10^{-4}} \approx 25$), leading to $C_{total} = 8.7522$ and $C_d = 8.7521$, in which case almost all the network capacity is contributed by the DUs. The optimal DU density can be attained either by using numerical analysis (i.e. $\lambda_d = 1.9902 * 10^{-4}$) or relying on theoretical approximation (i.e. $\tilde{\lambda_d} = 1.9901 * 10^{-4}$ by solving equation (18)). It is shown that the approximated solution is almost

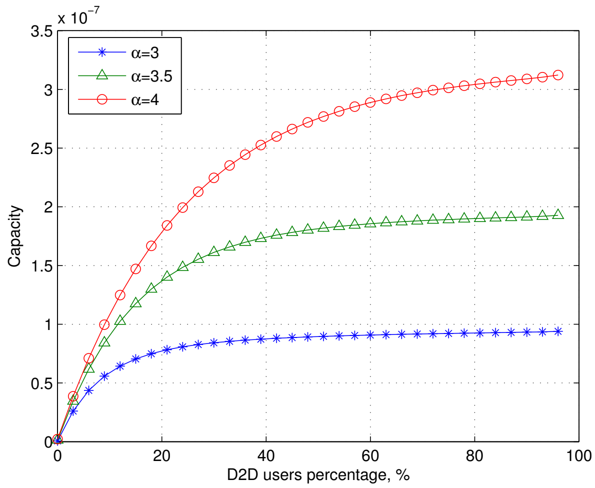

Fig. 4. Cellular Capacity under multicell situation.

identical to that of the numerical solution. Furthermore, the successful transmission probability becomes lower for a higher successful transmission threshold of SINR T, thus eroding the overall network capacity. In Fig.3, on the other hand, we evaluate the network capacity under different percentages of DUs. It is shown that the network capacity will decline sharply with the percentage of DUs increases, if the DUs dominate the cellular users.

In Fig.4, we evaluate the network capacity under multicell environment, with BSs, DUs and CUs modelled as PPPs. Furthermore, the densities of BS and UEs (comprising both DUs and CUs) are set to be $\lambda_B = \dfrac{1}{\pi \cdot 300^2}$ and $\lambda_{c+d} = \dfrac{100}{\pi \cdot 300^2}$, respectively. Numerical results showed that the total capacity increases rapidly as the D2D percentage increases. However, when the D2D percentage exceeds 60%, the overall capacity will almost attain its stable state due to the constraints imposed by the severe interference. A performance ceiling always appears in the D2D-aided underlaying cellular systems, if the D2D pairs tend to infinity.

VI. Conclusions

In this paper, we evaluated the overall cellular capacity as a function of DU density and analyzed how the DU density impacts the networks capacity. We derived the optimal DU density that maximizes the capacity of D2D-aided networks by analyzing the successful transmission probabilities of both DUs and CUs. It was indicated that the benefit brought by D2D communication in terms of channel capacity is never infinite, because the cellular systems become interference-limited as the D2D density increases. Furthermore, the optimal DU density that maximizes the overall network capacity are attainable either by using theoretical approximation or relying on numerical analysis. Once the break-even point is met, it was shown that the overall capacity will be even eroded by further increasing the DU density.

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