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Queuing Network Approach for Building Evacuation Planning

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Abstract. The complex behavior of pedestrians in a limited space layout can explicitly be modeled using an *M/G/C/C* state dependent queuing network. This paper implements the approach to study pedestrian flows through various corridors in a topological network. The best arrival rates and their impacts to the corridors' performances in terms of the throughput, blocking probability, expected number of occupants in the system and expected travel time were first measured using the *M/G/C/C* analytical model. These best arrival rates were then fed to its Network Flow Programming model to find the best arrival rates to source corridors and routes optimizing the network's total throughput. The analytical results were then validated using a simulation model. Various results of this study can be used to support the current Standard Operating Procedures (SOP) to efficiently and safely evacuate people in emergency cases.

Keywords: *M/G/C/C* state dependent; network flow model; queuing system; performance evaluation; topological network
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INTRODUCTION

Queuing systems are systems where a number of queues are connected by entities (e.g., customers and products) routing and competing for services. These scenarios are common in service (e.g., bank, cafeteria and healthcare), telecommunication (e.g., data packet, call center and computer network), transportation (e.g., airport and seaport) and manufacturing (e.g., production line) systems. In these systems, the service times fluctuate according to a statistical distribution regardless of the number of residing entities. Other systems dynamically adjust their service times according to the current number of entities. The scenarios are typical for entities flowing through a limited space layout; e.g., pedestrians walking through a corridor and vehicles travelling on a road. The entities' behaviour can be modelled using an *M/G/C/C* state dependent queuing network [1-3]. In this Kendall notation, *M* refers to Markovian arrival processes, *G* for general state dependent service rates, *C* for parallel servers and *C* for the total capacity of a circulation space.

This paper implements the *M/G/C/C* approach to measure the pedestrian flows in a complex hall consisting of series, merging and splitting networks. Our objective is to find the best strategy to flow pedestrians out of the hall to support the current Standard Operating Procedures (SOP) during emergency cases. In order to this, we first developed an *M/G/C/C* analytical model specifically for the hall. Using the model, we then identified the best arrival rate for each available corridor maximizing its throughput. Utilizing the best arrival rate and considering the whole corridors as a network, we then used the network flow programming approach [4] to find the best arrival rates to source corridors maximizing the hall's total throughput. To validate the analytical results, we then developed a discrete event simulation (DES) model [5-7] using Arena software [8-10]. How this commercial software can specifically be used to model the *M/G/C/C* state dependent queuing network has been discussed in detail in [11].

This paper is organized as follows. We first discuss the mathematical background governing the *M/G/C/C* approach and how it can be used to measure pedestrian traffic and congestion. We then briefly describe the structure of the hall used in our case study and present the best arrival of each available corridor maximizing its throughput. We next discuss the strategy to achieve the best performance of the hall to efficiently flow its occupants. Finally, some concluding remarks are given in the last section.

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M/G/C/C ANALYTICAL MODEL

In the *M/G/C/C* approach, a circulation space in a building or an environment acts as servers for its requesting entities. The number of servers is equal to the capacity of the space. Pedestrians arriving to a full space are not allowed to seize the space and they therefore have to queue before being serviced. Additionally, the current number of pedestrians determines the current service time; i.e., their walking speed.

The effect of the number of pedestrians to the current walking speed was formulized by Yuhaski and Smith [3]. They presented linear and exponential models of walking speed as follows:

Linear:
$$V_n = \frac{V_1}{c}(c+1-n) \tag{1}$$

Exponential:
$$V_n = A\exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right] \tag{2}$$

Where
$$\gamma = \frac{\ln\left[\frac{\ln(V_a/V_1)}{\ln(V_b/V_1)}\right]}{\ln\left(\frac{a-1}{b-1}\right)},$$

$$\beta = \frac{a-1}{\left[\ln\left(\frac{V_1}{V_a}\right)\right]^{1/\gamma}} = \frac{b-1}{\left[\ln\left(\frac{V_1}{V_b}\right)\right]^{1/\gamma}},$$

γ, β = Shape and scale parameters for the exponential model,
V_n = Average walking speed for n pedestrians in a corridor,
V_a = Average walking speed when crowd density is 2 peds/m^2 = 0.64 m/s,
V_b = Average walking speed when crowd density is 4 peds/m^2 = 0.25 m/s,
V_1 = Average walking speed for a single pedestrian = 1.5 m/s,
n = Number of pedestrians in a corridor,
$a = 2 \times l \times w$,
$b = 4 \times l \times w$,
$c = 5 \times l \times w$,
l = corridor length in meters, and
w = corridor width in meters.

Based on the models, Yuhaski and Smith [3] developed the limiting probabilities for the number of pedestrians in an *M/G/C/C* model as follows:

$$P_n = \frac{[\lambda E(S)]^n}{n!f(n)f(n-1)\ldots f(2)f(1)}P_0 \quad n = 1, 2, 3, \ldots, c \tag{3}$$

Where,
$$P_0^{-1} = 1 + \sum_{n=1}^{C}\left[\frac{[\lambda E(S)]^i}{i!f(i)f(i-1)\ldots f(2)f(1)}\right].$$

In this model, λ is the arrival rate to a corridor, $E(S)$ is the expected service time of a single pedestrian in the corridor; i.e., $E(S) = l/1.5$, P_n is the probability when there are n pedestrians in the corridor, P_0 is the probability when there is no pedestrian in the corridor, and $f(n)$ is the service rate and is given by $f(n) = \frac{V_n}{V_1}$. c meanwhile refers to the capacity of the corridor. Any pedestrians attempting to enter the full capacity corridor will be blocked. The probability of such blocking (P_{balk}) is equal to P_n where n equals to c. Since Cheah and Smith [12] showed that *M/G/C/C* networks are equal to *M/M/C/C* networks, various performance measures of the corridor can then be computed as:

$$\theta = \lambda(1 - P_{balk}), \quad E(N) = \sum_{n=1}^{C} nP_n \quad \text{and} \quad E(T) = \frac{E(N)}{\theta} \tag{4}$$

where θ is the throughput of the corridor (in pedestrians per second; i.e., peds/s), *E(N)* is the expected number of pedestrians in the corridor and *E(T)* is the expected service time in seconds.

KUISAS's NETWORK AS A CASE STUDY

We consider the KUISAS (Kolej Universiti Islam Sultan Azlan Shah)'s hall as an implementation environment for the *M/G/C/*C approach. The graphical structure of the hall is shown in Figure 1. The numbers represent the corridors, the alphabets *A* until *I* represent the entrance doors to corridors and *A'*, *B'* and *C'* are the exits to open spaces.

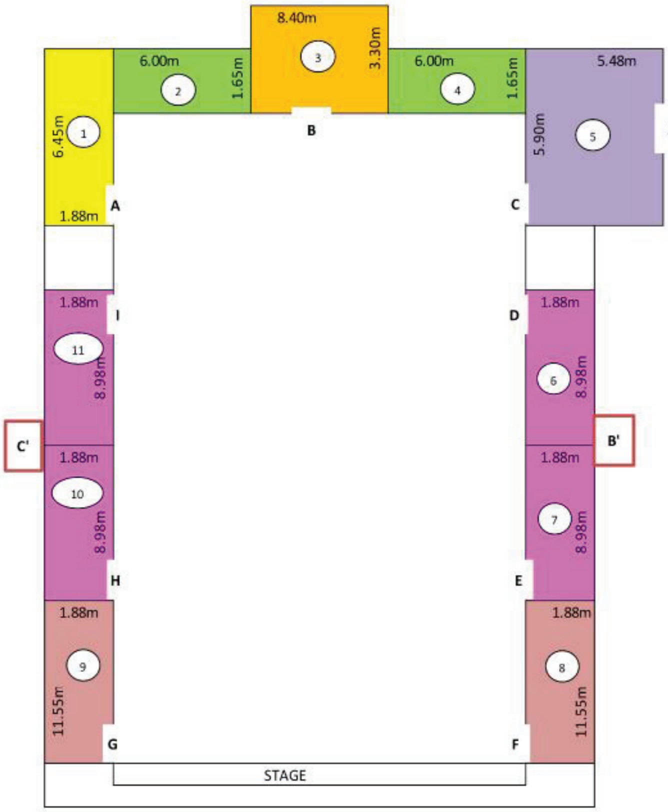

FIGURE 1. KUISAS's Hall Structure

Table 1 represents the dimensions of the corridors in terms of their lengths, widths (at entrances and exits) and capacities. A larger capacity supports more occupants. Corridor *C'* has the biggest space, followed by corridors 3, 8, 9 and 5.

TABLE 1. Corridor Dimensions and Capacities

Corridor	Length (m)	Width		Capacity (occupant)
		Entrance (m)	Exit (m)	
1	6.45	1.88	-	61
2	6.00	1.65	-	50
3	8.40	3.30	-	139

4	6.00	1.65	-	50
5	5.48	1.77	5.90	105
6	8.98	1.88	-	84
7	8.98	1.88	-	84
8	11.55	1.88	-	109
9	11.55	1.88	-	109
10	8.98	1.88	-	84
11	8.98	1.88	-	84
B'	3.60	4.00	-	72
C'	10.00	3.00	-	150

There are 13 corridors in the hall. Corridors 1, 3, 5, 6, 7, 8, 9, 10 and 11 are source corridors. Corridors 2 and 4 are intermediate corridors. Occupants choose the source corridors which are nearest to them to exit the hall using doors *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* and *I*. They then travel to surrounding open spaces through exits *A'*, *B'* and *C'*. Occupants from doors *A*, *B* and *C* choose exit *A'*. Occupants from doors *D*, *E* and *F* choose exit *B'* while occupants from doors *I*, *H* and *G* choose exit *C'*. Exits *A'* and *B'* are the exit corridors while exit *C'* is a staircase to the open spaces. The relationship between the arrival rates (λ) and throughputs (θ) of the corridors when the facility is considered as a network is presented in Table 2. The table also shows the best arrival rates and their impacts to the throughputs, blocking probability (p(c)), expected number of occupants (L) and average travelling time (W).

TABLE 2. Arrival Rates and Throughputs of Corridors

Corridor		λ	**Total λ**	θ	**Best λ**	**Best θ**	**p(c)**	**L**	**W**
Source Corridors	1	λ_1	λ_1	θ_1	1.99718	1.95116	0.02304	19.60526	10.04801
	3	λ_3	$\lambda_3 = \lambda_3 + \theta_2$	θ_3	3.55649	3.52221	0.00964	37.26319	10.57950
	5	λ_5	$\lambda_5 = \lambda_5 + \theta_4$	θ_5	4.12374	4.07036	0.01294	29.74330	7.30729
	6	λ_6	λ_6	θ_6	2.01882	1.98558	0.01647	25.26585	12.72465
	7	λ_7	$\lambda_7 = \lambda_7 + \theta_8$	θ_7	2.01882	1.98558	0.01647	25.26585	12.72465
	8	λ_8	λ_8	θ_8	2.01792	1.99291	0.01240	30.33084	15.21941
	9	λ_9	λ_9	θ_9	2.01792	1.99291	0.01240	30.33084	15.21941
	10	λ_{10}	$\lambda_{10} = \lambda_{10} + \theta_9$	θ_{10}	2.01882	1.98558	0.01647	25.26585	12.72465
	11	λ_{11}	λ_{11}	θ_{11}	2.01882	1.98558	0.01647	25.26585	12.72465
Intermediate Corridor	2	λ_2	$\lambda_2 = \theta_1$	θ_2	1.76335	1.71053	0.02995	17.61650	10.29886
	4	λ_4	$\lambda_4 = \theta_3$	θ_4	1.76335	1.71053	0.02995	17.61650	10.29886
Exiting to Open Spaces	A'	$\lambda_{A'}$	$\lambda_{A'} = \theta_5$	$\theta_{A'}$	-	-	-	-	-
	B'	$\lambda_{B'}$	$\lambda_{B'} = \theta_6 + \theta_7$	$\theta_{B'}$	4.30450	4.21867	0.01994	22.84601	5.41545
	C'	$\lambda_{C'}$	$\lambda_{C'} = \theta_{10} + \theta_{11}$	$\theta_{C'}$	3.25133	3.22194	0.00904	40.39662	12.53799

Using the information in Table 2, we then used the network flow programming approach to find the best arrival rates of source corridors and routing strategies maximizing the hall's total throughput. The objective function is to maximize the total flow out of the exit corridors subject to flow out of each corridor is equal to its flow in and the maximum arrival rate of each corridor must less than or equal to its best arrival rate.

RESULTS AND DISCUSSION

The performance measures of the source corridors for the arrival rates maximizing the throughputs of source corridors are presented in Table 3. These analytical results were then validated with its simulation model. The simulation model was run using *Process Analyzer* built in Arena 14.0. All simulation results reported were based on 30 replications, each of which was run 20000 seconds.

TABLE 3. Analytical and Simulation Arrival Rates Maximizing the Total Throughput of the Hall

Model	Corridor	λ	L	W	p(c)	θ
Analytic	1	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	2	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	3	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	4	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	5	4.1237	29.7403	7.3065	0.0129	4.0704
Simulation			39.9528	11.0777	0.0494	3.9158
Analytic	6	2.0188	25.2627	12.7231	0.0165	1.9856
Simulation			44.2097	25.9429	0.1013	1.8100
Analytic	7	2.0188	25.2627	12.7231	0.0165	1.9856
Simulation			44.2097	25.9429	0.1013	1.8100
Analytic	8	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	9	0.0000	0.0000	0.0000	0.0000	0.0000
Simulation			0.0000	0.0000	0.0000	0.0000
Analytic	10	2.0188	25.2627	12.7231	0.0165	1.9856
Simulation			44.2097	25.9429	0.1013	1.8100
Analytic	11	1.2325	9.3071	7.5514	0.0000	1.2325
Simulation			9.3031	7.5534	0.0000	1.2316
Analytic	*B'*	3.9712	15.4011	3.8792	0.0003	3.9702
Simulation			22.8379	6.2682	0.0295	3.8537
Analytic	*C'*	3.2181	37.0272	11.5391	0.0029	3.2088
Simulation			35.7511	11.1171	0.0000	3.2158
Analytic	**Total throughput of the network**					**11.2493**
Simulation						**10.9853**
Difference (%)						2.3468

For evacuation planning, we should flow the occupants directly to corridor 5 with the arrival rate of 4.1237 ped/s to exit through corridor *A'*. Occupants who are near to exit *B'* should go to corridors 6 and 7 with 2.0188 ped/s. We should block occupants from using door *F* to enter corridor 8 since they have to compete with other occupants from corridors 6 and 7. The same concept goes to corridor 9. Occupants who are near to exit *C'* should only use corridors 10 and 11 to exit to open spaces with 2.0188 ped/s and 1.2325 ped/s respectively. The maximum total throughput of the hall is 11.2493 ped/s. As we can see, the throughput difference between the analytical and simulation results was only less than 2.3468%.

CONCLUSION

This paper shows that an *M/G/C/C* state dependent queuing network can be used as a tool for efficiently evacuating occupants from a complex topological network. As a case study, we chose the KUISAS's hall and calculated the best arrival rates to its available source corridors maximizing its throughput. Any higher arrival rates than the suggested values will cause congestions rather than improving its throughput. Any lower arrival rates than the values smoothly flow pedestrians but this will eventually cause less throughput. Thus, appropriately control the arrival rates and guide the pedestrians during their travel from source to exiting corridors are important to ensure the best throughput.

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