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A causal encounter model of traffic collision avoidance system operations for safety assessment and advisory optimization in high-density airspace

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ABSTRACT

The traffic collision avoidance system (TCAS) acts as a proverbially accepted last-resort means to resolve encounters effectively, while it also has been proven to potentially induce a collision in the hectic air traffic. Thus, new research considering the impact on safety is required to increase the airspace capacity based on a comprehensive analysis and accurate flight evaluation. In this paper, a causal encounter model is proposed to extend the TCAS logic considering the horizontal resolution manoeuvres, which could be used as the auxiliary supports when a potential collision is predicted in the vertical dimension. Based on the generated state space, the model developed in the graphical modelling and analysis software (GMAS), not only provides a better comprehension of the potential collision occurrences for risk assessment by representing the cause-effect relationship of each action, but also aids the pilots in the involved aircraft to make a cooperative and optimal option. Quantitative simulation results are conducted to validate the feasibility and effectiveness of the encounter model with horizontal resolution. The resulting collision scenarios are further investigated to illustrate that the risk rate of TCAS logic failures is expected to reduce by shortening the pilot's response delay, and the computational efficiency is competent in dealing with multi-threat scenarios.

1. Introduction

As stressed in Leva et al. (2009), “accidents are dramatic examples, among other less critical events, pointing out how prospective assessment methods often poorly represent human and organizational aspects and hence limit their value for accident prevention”. We must accept that the existent aircraft collision risk needs some feasible policies and methods to deal with the trade-off between flight efficiency and air traffic management (ATM) capacity. Thus the research of air collision risk should consider both level of safety figures and new useful safety metrics to identify “system weaknesses” that need to be resolved or at least mitigated. These new metrics should provide a better understanding of several micro-level dynamics such as the estimation variation of collision risk achieved by new risk mitigation policies considering the analysis of different interdependent scenarios in the same time-period.

ATM is universally considered to be a “high reliability” service industry in which accidents are infrequent (Kim and Hansen, 2015). However, in the beginning of the 21st century, several factors have contributed to an incremental focus on measuring and managing the safety flight. The collision of two aircraft in 2002 over Überlingen carried away the lives of 71 passengers and crew

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(Brooker, 2008a,b). Furthermore, all 114 people on a MD-87 and a Cessna CJ2 were killed in the 2001 Linate Airport disaster, as well as four ground staffs (Busby and Bennett, 2007). These accidents strengthened all services related to air navigation specially air traffic flow management (ATFM) (Lulli and Odoni, 2007) which supports the use of available airspace effectively, including airport capacity, and therefore its importance has been increased significantly. In the main, the introduction of new technologies and procedures (e.g., high density, remotely piloted aircraft (RPA), flight level capping and free flight among others) would evidently promote the improvement of the air side capacity, but further studies on safety are required to understand new scenarios that could emerge in high density traffic areas. It is widely accepted the importance of research in future ATM scenarios that could be characterized by high-density areas with some flights under free routing procedures coexisting with RPA (Lee, 2006). Present technology allows own aircraft to broadcast its state information such as the position and velocity to neighbouring traffic, and also to receive similar state information from an intruder aircraft. Because of the increasing air traffic density and technological development, the fundamental concept of ATM is under consideration by different ATM stakeholders (Lulli and Odoni, 2007): transfer the control from centralise to distribution, transfer responsibility for conflict avoidance from ground to air, and introduce new technologies to replace the fixed air traffic routes.

In the high-density scenarios, the unpredictable behaviour that emerges in a system-wide range suddenly arises based on the integrated result of successive dynamical operations and pilots' possible reactions which would make an important effect on the surrounding traffic (i.e., safety issues). As it is quite unpredictable, several novel complementary techniques and systems are required to estimate the rigorous safety of new ATM in the congested traffic situations. Thus, despite all the procedures are properly analysed in the new paradigm shift, it is very important to enhance the last-resort of safety (i.e., traffic collision avoidance system (TCAS)) in case an error could be produced and propagated from the different hierarchical safety procedure levels.

TCAS is designed to be the last resort airborne system to prevent midair collisions (MACs) and significantly reduce near midair collisions (NMACs) between aircraft (Brooker, 2005; Furini et al., 2016; FAA, 2010). The main functions of TCAS are to communicate the detected threat to the pilot and to assist in resolving the threat by recommending an avoidance manoeuvre. However, the increased airspace usage can induce a secondary threat as a result of an aircraft manoeuvre following a resolution advisory issued by TCAS, which may issue an inappropriate suggested resolution that resolves a one-on-one encounter with the first threat. This secondary threat may deteriorate to be an induced collision. The main topic of this research focuses on the final phases of an encounter (in TCAS course), in which the risk of an induced collision may exist.

This research discusses the mathematical model for TCAS logic with horizontal resolution capability, that is designed to handle multi-threat encounters in which a potential collision is predicted. The proposed model is constructed in the graphical modelling and analysis software (GMAS), generating the state space to provide a global perspective on the scenario dynamics and a better understanding of the induced collision occurrence for risk assessment, as well as the modified advisory for the pilots to make a cooperative and optimal option. It would offer auxiliary supports in the analysis of hectic traffic scenarios, e.g., terminal manoeuvring area (TMA) and hot spots (Nosedal et al., 2014), and increase the airspace capacity while safely and efficiently manage a higher amount of flights.

This paper further develops the research of potential TCAS induced collisions (Tang et al., 2016) and they are significantly different. The subject of previous research is the design of an encounter model to identify all of the potential collision scenarios that are representative of possible hazardous situations that may occur in the actual airspace. Yet this paper aims to construct a causal encounter model based on the improved TCAS logic considering the horizontal resolution manoeuvres, and it could be used as the auxiliary supports when a potential collision is predicted in the multi-threat scenario. For the research approach, the previous paper proposes a novel scenario generation process of potential collisions and the model inputs are a given initial trajectory and a general specification of the surrounding traffic, while this paper exhibits a process of domino conflict detection and resolution and the model inputs are the initial states of all involved aircraft for the analysis of particular traffic geometries. Besides, (Tang et al., 2016) only provides a mathematical model for the TCAS II algorithm potentially used in a TCAS-TCAS encounter, and this paper expands the mathematical model with the horizontal resolution capability.

As a brief outline of the remainder of this paper, Section 2 summarizes the existing conflict detection and resolution approaches and states the art of collision risk model; Section 3 provides a detailed description of the TCAS operations with horizontal RA capability; Section 4 depicts the proposed GMAS causal model and explains its construction process; Section 5 represents the results of the multi-threat collision scenarios and illustrates the analysis; finally, the conclusions and future work are detailed in Section 6.

2. Motivation

The current ATM system is in the fleetly extensive development from the relatively structured airspace and mainly human-operated system framework (Lee, 2006). In line with the requirements of the future ATM concepts proposed by the Single European Sky ATM Research (SESAR) (Brooker, 2008a,b) (launched by the European Community) and the Next Generation Air Transportation System (NextGen) (Darr et al., 2010) (launched by the US government), the air-traffic flow needs to be more predictable to offer the possibility of more effective use of airspace and airport capacity. Furthermore, to provide air traffic controller (ATCo) and aircrew with more valuable information about the traffic flow, especially the accurate states of the nearby aircraft, various decision support tools (DSTs) in different acting levels are in the development.

2.1. Conflict detection and resolution approaches

With the growth of airspace congestion, there is an extensive need to implement DSTs to assist the human operators in handling conflicts while improving flow efficiency (Alam et al., 2009). The fundamental functions of the DSTs system are conflict detection

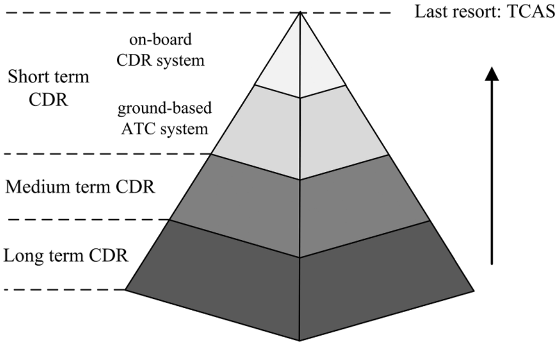

Fig. 1. Major categories of CDR approaches.

(CD) which is to predict a threat that would occur in the future, communicating with human operators to inform the detected threats, and conflict resolution (CR) which is to provide assistance in the process of resolving threat. The complete survey of models and approaches to the conflict detection and resolution (CDR) problem is presented in Kuchar and Yang (2000). On account of the prediction horizon, generally, most CDR techniques and methods can be classified into three major categories, shown in Fig. 1.

-Long term CDR, is useful for airspace planning at a strategic level and roughly handles the horizons above 30 min. Their main concern is a typical management problem of air traffic flow, including the planning of all aircraft trajectories within a relatively longer look-ahead time. Predictions are made from several days up to a few (> 30) minutes before the flight execution phase. Their main goal is to maximize the network route efficiency while minimizes the global operational costs, taking into account the airspace restrictions such as the available capacity at the airports and sectors (Bertsimas and Patterson, 1998; Krozel and Peters, 1997). The EuroControl long-term forecast (LTF) (EuroControl, 2008) is developed by growing baseline traffic using a model of economic and industry developments, taking into account factors related to economic growth, passenger demand, prices, air network structure and fleet composition. Constrained by annual airport capacities, specific models are utilized to address cargo, passenger, business aviation and military general air traffic (GAT).

-Medium term CDR, works at the tactical level and possesses prediction horizons up to 30 min. These planning systems make impossible to improve and perfect the proposed flight plans of Long term CDR during the execution phase, generally thinking about prediction look-ahead time of several minutes. These systems are often used by ATCo due to the presence of disturbances caused by unforeseen events that cannot be predicted beforehand with enough accuracy (i.e., during the flight planning of strategic level) and that usually makes impossible to accomplish with the long term CDR's proposed flight plans during the execution phase (Irvine, 2002; Zuniga et al., 2013). The look-ahead time is large enough (comparing the short term CDR), to allow a tactical control for the flight safety and there is no risk of any imminent collision between aircraft. Adopting the usage of spatial data structures (SDS) constructed based on four-dimensional (4D) trajectories (trajectories defined in the three spatial dimensions together with a timestamp), our research group has developed an efficient medium term CDR approach to detect and resolve conflicts in a TMA (Ruiz et al., 2013).

A weakness in the long term CDR is usually solved by medium term CDR, and a medium term CDR failure scenario can be dealt with by short term CDR. Thus, by discussing and improving the performance of short term CDR, it could be possible to avoid failures that could be hidden in the lack of integration mechanism between trajectory management – separation management – conflict management when applied to future ATM scenarios.

-Short term CDR, works at operational level to avoid the upcoming conflicts, and takes effect horizons up to 10 min. Since they are not planning systems which are different from Long term and Medium Term CDR, there is no need to consider the fuel and flight optimization. Normally it mainly includes two kinds of systems: one is the ground-based safety net intended to assist the controller in preventing collision between aircraft by generating, in a timely manner, an alert of a potential or actual infringement of separation minima (Everson and Fieldsend, 2006; Prandini et al., 2000) (e.g., short term conflict alert (STCA) (EuroControl, 2009)); the other one is a family of airborne devices that function independently of the ground-based ATC system (DoT, 2011) (e.g., TCAS).

The STCA system comprises alert mechanisms for ATCo which provide a warning of airspace infractions between aircraft. It monitors aircraft locations from ground radar, raising a warning to remain a short time to redirect the aircraft when there is a developing threat between dangerously approaching aircraft. Because of the input of STCA systems is from ground radar, they cannot be aware of the intentions of the pilots or ATCo, who may know a potential encounter and already taking measures to resolve it. Thus the alerts issued maybe are not always necessary and the predictions of STCA usually are considered conservatively.

Normally, TCAS, as an alert system operates quietly in the background most of the time. Until now, TCAS I and its improved version, TCAS II, have been defined and approved by the ICAO, and they differ primarily in their alerting capability. TCAS I provide traffic advisories (TAs) to assist the pilot in the visual acquisition of intruder aircraft, whereas TCAS II provide both TAs and resolution advisories (RAs), in other words, recommended escape manoeuvres (DoT, 2011). Various literatures have been published to represent the operating mechanism of TCAS and increase its capability (Abdul-Baki et al., 2000; Altman et al., 1994; Livadas et al., 2000).

2.2. Collision risk models: state of the art

The estimation of MAC/NMAC risk in airspace and its mathematical modelling for processes leading to possible collisions have been in progress for more than 50 years (Machol, 1995). The study of aircraft collision risk was primitively initiated in the early

Table 1
Typical improved TCAS methods and respective character.

Methods	Horizontal escape	Added instrument	Number of aircrafts considered	Other necessary information
Peng and Lin, 2010	✓	✗	2	GPS, ADS-B
Romli et al., 2008	√̸	✗	2	ADS-B
Gallego and Nieto, 2016	√̸	✗	2	✗
Achachi and Benatia, 2015	√̸	✗	2	ADS-B, Satellite data
Munoz et al., 2013	√̸	✗	2	✗
Smith and Fajen, 1999	✗	dual bandwidth receiver	2	✗
Xu, 2013	✗	✗	2	ADS-B
Bakare and Junaidu, 2013	✗	✗	2	Radar data
Woodell and Smoak, 2001	✗	✗	2	Higher resolution radar data
Tang et al., 2015	✗	✗✗	multiple	✗

✓-fully supported, ✗ – not supported, √̸ – partially supported.

1960s by Marks (1963) and Reich (1966). In particular, the Reich model mainly estimates the collision risk for an airway structure, including more than one parallel trajectory. Besides, note that the Lincoln Laboratory of Massachusetts Institute of Technology (MIT) has carried on the long-term research on TCAS performance to estimate collision risk and the development of collision avoidance techniques (DoT, 2011). Their involvement in TCAS dates back to 1974, when the Federal Aviation Administration (FAA) tasked them to participate in the development of an on-board collision avoidance system, and in the mid-1970s, this laboratory began TCAS-related monitoring of aircraft in the Boston airspace, using their own prototype Mode S sensor. In the mid-1990s, Lincoln was tasked with analysing the performance of the TCAS threat logic of that time. Note that since the early 2000s, Lincoln Laboratory has supported safety assessment and evaluation of proposed changes to the TCAS algorithms.

The TCAS has been proven effective to reduce the risk of MAC/NMAC and is currently mandated on all large transport aircraft. Although there are technical advances in equipping electronic systems which are used as DSTs, accidents still occur when the minimum safe separation provision is violated because of the human/technical errors. Any failures of the separation standard have the possibility of collision risk, and therefore several researchers try to improve the TCAS by different methods. The overall features are listed in the Table 1.

Some researchers consider that the next generation TCAS has superior CDR performance. It could deliver both vertical and horizontal RAs. For instance, in an emergent situation, one aircraft may be instructed, "turn right, climb" while the other would be instructed "turn right, descend". Obviously it could take effect on further increasing the total separation between aviation vehicles, in both horizontal and vertical planes. Horizontal instruction is significant in an encounter which is close to the ground, i.e., not enough vertical manoeuvring airspace. The idea is typically illustrated in Peng and Lin (2010). It studies a new generation of TCAS based on the Global Positioning System (GPS) and Automatic Dependent Surveillance Broadcast (ADS-B). Two kinds of horizontal escape manoeuvres are represented: change speed without amending flight direction, and change flight direction without amending speed. In some other contributions, researcher consider combining the horizontal and vertical RAs. The research implemented to determine the positive effect of ADS-B on TCAS is illustrated in Romli et al. (2008). However, the simulation results show that in-apparent improvements in avoidance performance of TCAS could be achieved through the integration with ADS-B, with some other modular factors, e.g., the aircraft dynamic performance and input data, possessing more impacts. A simple encounter model is introduced in Gallego and Nieto (2016) to ease the evaluation of the indicators for pair-wise encounters. The aim is to assess the coherence between threat detection indicators, provided by the complexity metric and TCAS indicators, and to determine whether or not the proposed complexity metric can allow an operational integration in the detection process between separation management and the collision avoidance layers. It has been shown that the proposed horizontal complexity metric and variable in TCAS behaves similarly, in terms of the range between aircraft when alert thresholds are reached or all relative angles and speeds. A novel structure of TCAS is proposed in Achachi and Benatia (2015) to remedy some limitations of ATC performance, and reduce the possibility of unnecessary alerts. It aims to represent visual information based on ADS-B satellite data in the aircraft cockpit, which permits crews to verify of the right operations and assist them to operate reliably with less stress, particularly in Control Terminal Area (CTA). In general, methods allowing horizontal escape usually need other information, such as ADS-B satellite data, GPS information, Radar system and so on. Consequently, the resolution manoeuvre would be totally different with the current system, just providing advisories in altitude, and need more accurate information provided by more advanced hardware.

Other methods and technologies improve the original collision avoidance logic, and do not consider the horizontal resolution manoeuvre. An RA detection algorithm on account of the TCAS II mathematical model is represented in Munoz et al. (2013). It is similar to a CD algorithm, but not calculating the loss of safe interval, it detects RAs of multiple aircraft. In addition, the algorithm has been accurately verified with a kinematic model of routes. It exactly characterizes all threat geometries between each pair of aircraft that initiate an RA within a given time in advance. A dual bandwidth receiver is utilized in Smith and Fajen (1999) to realize the extended operation range of TCAS. During conventional operations, the radio signals of current equipped TCAS are screened out by a band pass filter that passes all TCAS relevant radio signals. When the extended range is expected, the band pass filter is more narrow, which permits only designative TCAS radio signals and ameliorates the noise ratio of signals. The TCAS equipped on own aircraft changes the range mode to detect intruder aircraft with different distances. Besides, some methods also combine other information, such as ADS-B information and radar system. It integrates ADS-B broadcasting information with original TCAS (Xu, 2013), which broadcasts and receives states of the neighbouring aircraft. Fusing the data of TCAS and ADS-B could decrease the interruption ratio of TCAS radio, extend the range surveillance and improve its precision. The combination of a radar with a GPS-based TCAS is put forward in Bakare and Junaidu (2013) for separation guarantee between all instrument flight rules (IFR) routes and visual flight rules (VFR) routes executed as steerable flights. It proposes an improved TCAS logic and device, wherein the input data are augmented by higher resolution radar data (Woodell and Smoak, 2001). By using radar to search out targets, altitude information can be developed for those aircraft not equipped with altitude reporting transponders. The improved accuracy also allows angle/angle perspective display of air traffic and thus provides enhanced situational awareness. In Tang et al. (2015), this causal encounter model generates all the reachable downstream states to strengthen the subsequent decision making of the crew, via integrating state data which are relative to collision information. In addition, some innovative techniques, e.g., removing the scenarios in which the involved aircraft is separating without the possibility of initiating new secondary threat, are adopted to raise the calculative efficiency and effectively resolve the common expansive problem in state exploration.

3. TCAS coordinated operations with the horizontal RA capability

TCAS II was designed to operate in traffic densities of up to 0.3 aircraft per square nautical mile (NM), i.e., 24 aircraft within a 5 NM radius, which was the highest traffic density envisioned over the next 20 years (DoT, 2011). The influence of TCAS on a safety flight has been effective, beneficial, and significant in reducing the collision probability (DoT, 2011). However, the increased airspace usage can induce a secondary threat which may deteriorate to be an induced collision. Induced risk is the potential for the TCAS to cause a collision that did not exist in its absence (Chludzinski, 2009). Therefore, research that explores and avoid such potential induced collision scenarios is needed to enable a robust, reliable and safe ATM system (Tang et al., 2016; Kochenderfer et al., 2010).

This section describes the first step in developing an GMAS model of the TCAS operations, i.e. to specify a mathematical description of TCAS II logic, which is mainly based on the general process of collision avoidance shown in DoT (2011). The booklet (DoT, 2011) only provides the background for a better understanding of the TCAS II by personnel involved in the implementation and operation of TCAS II. And this paper specifies the implementation and operation of TCAS II, using the mathematical equations which are applied as the computation basis of the constructed causal encounter model. In addition, to provide most feasible alert information when a potential induced collision exists, the current model following TCAS logic would be expanded to account for the additional horizontal RA capability, resulting in optimizing advisory and strategy.

3.1. TCAS protection volume

For real applications the CDR system should consider the curvature of the Earth (Fig. 2A), while the geometrical problems in this research have been characterized utilizing a Euclidean three-dimensional (3D) space (not curved space) as shown in Fig. 2B (Ruiz et al., 2013). Since TCAS executes with local scope and within a short period, the hot spot to be analysed by the proposed model will be called basic volume. The definition criteria of this sectional airspace (Euclidean space) are based on the range of reliable surveillance that the aircraft supports (generally 14 NM (DoT, 2011)). The construction of the basic volume is simply made and a planar projection of the Earth has been considered by using a Cartesian coordinate system and with a minimum distortion.

Considering each TCAS-equipped aircraft, a temporal-spatial synthesized protection volume of airspace surrounds, as illustrated in Fig. 3. For the caution area and warning area, the horizontal Tau and DMOD criteria described above shape the horizontal boundaries of this volume, while the vertical Tau and ZTHR thresholds determine the vertical dimensions of the protected volume. Here, the time to the closest point of approach (CPA) is called the horizontal Tau and the time to co-altitude is called the vertical Tau. And at close ranges and at slower closure rates the modified Tau boundaries converge to a non-zero range called DMOD, i.e., Distance MODification. There is a similar problem when the vertical closure rate of the TCAS and the intruder aircraft is low, or when they are close but diverging in altitude. To address that problem, TCAS uses a vertical threshold value, referred to as ZTHR. In summary, the size of the protected volume mainly depends on Tau calculated by the speed and heading of the aircraft involved in the encounter, plus an estimate of the protected miss separation. In order to determine whether a hazardous threat exists, i.e., to issue a TA or an RA, both the range and vertical criteria must be satisfied. If one of them does not meet at the same time, TCAS will not issue a TA or an RA. For checking whether the range and vertical criteria are satisfied, range tests and altitude tests are constantly performed during flight. The criteria used for making a decision about TA and RA issuance depend on the sensitivity level (SL) (DoT, 2011). In the collision area, the TCAS logic fails to resolve the conflict and a terrible collision occurs when the D_{cl} and H_{cl} volumes of both aircraft

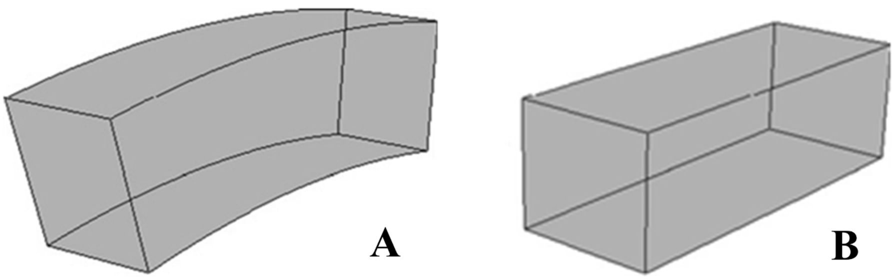

Fig. 2. Sectional airspace model considering the curvature of the Earth A and simplified Euclidean model B.

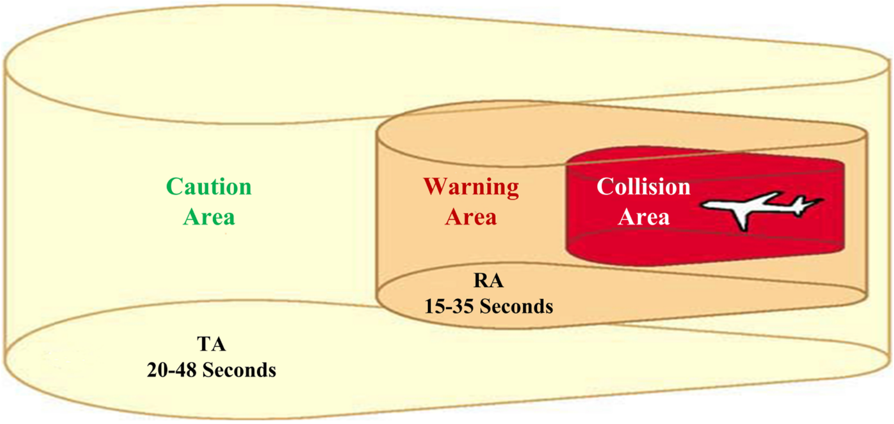

Fig. 3. TCAS temporal-spatial synthesized protection volume.

overlap. Normally, D_{cl} is double the average length of the aircraft fleet operating in the basic volume (or wingspan if longer) while H_{cl} is double the average height of the aircraft fleet operating in the basic volume.

3.2. Threat detection and resolution algorithm

TCAS is designed to take effect autonomously without the support of the aircraft navigation equipment, and generally monitors the airspace around the own aircraft especially paying close attention to the presence of intruders. In order to mathematically model the TCAS logic that is used in a one-on-one encounter for TCAS-equipped aircraft, the operations are divided into the following phases.

3.2.1. Conflict-free flight

In general, during the normal evolution of a flight, the aircraft receives instructions from ATCo and flies in accordance with them (manually or using the autopilot). Meanwhile, TCAS is constantly surveying the surrounding airspace, broadcasting interrogations and receiving state information from nearby aircraft. The dynamic characteristics of *Aircraft i (i = 1, 2,…, n)* at time t includes its position and speed, i.e., $s_t^i = (p_t^i, v_t^i) = [(x_t^i, y_t^i, z_t^i), (v_{t,x}^i, v_{t,y}^i, v_{t,z}^i)]$. If an intruder *Aircraft j* is closing, let $p_t^{ij} = p_t^i - p_t^j$ and $v_t^{ij} = v_t^i - v_t^j$ be their distance and relative velocity.

3.2.2. Threat detection algorithm

Keeping the survey for nearby aircraft, range and altitude tests are performed on each altitude-reporting target. For a closing target to be declared an intruder, the range test is based on TA Tau $T_{tCPA,h}^{ij}$ which must be less than the corresponding criteria $Time_{TA}$, and meanwhile the vertical separation at CPA $T_{tCPA,z}^{ij}$ also must be within the TA altitude threshold $Time_{TA}$ shown in DoT (2011).

$$T_{tCPA,h}^{ij} = \frac{-\sqrt{(x_t^i - x_t^j)^2 + (y_t^i - y_t^j)^2}}{\sqrt{(v_{t,x}^i - v_{t,x}^j)^2 + (v_{t,y}^i - v_{t,y}^j)^2} \cdot \cos\left(\arctan\left(\frac{v_{t,x}^i - v_{t,x}^j}{v_{t,y}^i - v_{t,y}^j}\right) - \arctan\left(\frac{x_t^i - x_t^j}{y_t^i - y_t^j}\right)\right)} \tag{1}$$

$$T_{tCPA,z}^{ij} = -\frac{z_t^i - z_t^j}{v_{t,z}^i - v_{t,z}^j} \tag{2}$$

$$\left(0 < T_{tCPA,h}^{ij} < Time_{TA}\right) \wedge \left(0 < T_{tCPA,z}^{ij} < Time_{TA}\right) \tag{3}$$

For the slow closure encounter that is to be declared a threat, the Tau threshold values are not appropriate, and the horizontal and vertical distances $D_{t,h}^{ij}$ and $D_{t,z}^{ij}$ should severally be within the dimensions of the protected airspace $DMOD_{TA}$ and $ZTHR_{TA}$.

$$D_{t,h}^{ij} = \sqrt{(x_t^i - x_t^j)^2 + (y_t^i - y_t^j)^2} \tag{4}$$

$$D_{t,z}^{ij} = |z_t^i - z_t^j| \tag{5}$$

$$(D_{t,h}^{ij} < DMOD_{TA}) \wedge (D_{t,z}^{ij} < ZTHR_{TA}) \tag{6}$$

3.2.3. Valid threat filter

Depending on the geometry of the encounter, an RA may be delayed or not selected at all (DoT, 2011). The criteria for RA issuance are much more rigorous than TA issuance. For the purpose of discussing potential induced collision, the invalid detected threat should be automatically filtered to improve the calculation efficiency of the TCAS encounter model.

For a TA that would deteriorate to RA, the horizontal distance between *Aircraft i* and *Aircraft j* at CPA in the horizontal plane $D^{ij}_{tCPA,h}$ and the vertical distance between *Aircraft i* and *Aircraft j* at CPA in vertical plane $D^{ij}_{tCPA,z}$, should respectively be within the dimensions of the protected airspace $DMOD_{RA}$ and $ZTHR_{RA}$.

$$D^{ij}_{tCPA,h} = \sqrt{[(x^i_t + v^i_{t,x}\cdot T^{ij}_{tCPA,z})^2-(x^j_t + v^j_{t,x}\cdot T^{ij}_{tCPA,z})^2] + [(y^i_t + v^i_{t,y}\cdot T^{ij}_{tCPA,z})^2-(y^j_t + v^j_{t,y}\cdot T^{ij}_{tCPA,z})^2]} \tag{7}$$

$$D^{ij}_{tCPA,z} = \sqrt{[(z^i_t + v^i_{t,z}\cdot T^{ij}_{tCPA,h})^2-(z^j_t + v^j_{t,z}\cdot T^{ij}_{tCPA,h})^2]} \tag{8}$$

$$\left(0 < T^{ij}_{tCPA,h} < Time_{TA}\right) \wedge \left(0 < T^{ij}_{tCPA,z} < Time_{TA}\right) \wedge \left(D^{ij}_{tCPA,h} < DMOD_{RA}\right) \wedge \left(D^{ij}_{tCPA,z} < ZTHR_{RA}\right) \tag{9}$$

3.2.4. Coordinated threat resolution

In a TCAS/TCAS encounter, the two aircraft declare each other as threats simultaneously when

$$[(0 < T^{ij}_{t,h} < Time_{RA}) \wedge (0 < T^{ij}_{t,z} < Time_{RA})] \vee [(D^{ij}_{t,h} < DMOD_{RA}) \wedge (D^{ij}_{t,z} < ZTHR_{RA})] \tag{10}$$

the RA alerts will be issued and then both aircraft will select their RA sense based on the encounter geometry. The basic rule for sense selection in a TCAS/TCAS encounter is that each TCAS must check to see if it has received an intent message from the other aircraft before selecting an RA sense. In terms of practicality, each aircraft has three choices in sense selection $s^i_{t_{RA}} = (1, 0, -1)$ corresponding to climb, maintain and descend. Coordination interrogations contain information about an aircraft's intended RA sense to resolve the encounter with the other TCAS-equipped intruder. If an intent message has been received, TCAS chooses the opposite sense from that selected by the other aircraft and communicated via the coordination interrogation. Therefore, for *Aircraft i* and *Aircraft j* which are involved in the same threat, their response combination can be $[(1, -1), (1, 0), (0, 1), (0, -1), (-1, 0), (-1, 1)]$. In encounters where either of the senses results in the TCAS aircraft crossing through the intruder's altitude, TCAS is designed to select the non-altitude crossing sense due to the rules of aviation safe even if the altitude crossing sense provides greater separation. Besides, in the TCAS/TCAS encounter, both aircraft should respond RA positively (i.e., climb, descend) not negatively (i.e., maintain) for the coordinated threat resolution. Thus, in summary, the RA sense (upward or downward) is set based on the altitude at t_{RA}, which means that the higher aircraft climbs while the other descends.

The next step is to choose the RA strength which is designed to be the least disruptive to the existing flight path, while still providing the desired vertical separation called Altitude Limit (ALIM) at CPA. Here, it is $ALIM_{RA}$ between *Aircraft i* and *Aircraft j* at CPA. Thus, for fairness their own disruption $\Delta^i_{t,z}$ and $\Delta^j_{t,z}$ are:

$$\Delta^i_{t,z} = -\Delta^j_{t,z} = \frac{1}{2}\left[ALIM_{RA} + (z^i_t-z^j_t) + \left(v^i_{t,z}\cdot T^{ij}_{tCPA,h}-v^j_{t,z}\cdot T^{ij}_{tCPA,h}\right)\right] \tag{11}$$

3.2.5. Least acceleration time

Note that in modelling the aircraft response to RAs, the expectation is that the pilot will begin the initial $a_0 = 0.25g$ acceleration manoeuvre within five seconds (DoT, 2011). Therefore in this research, when an encounter appears, both the aircraft accelerate to appropriate speeds with opposite acceleration (0.25 g and − 0.25 g) in vertical plane and the response time of pilot in *Aircraft i* is assumed as $\widetilde{t}_i (0 \leqslant \widetilde{t}_i \leqslant 5s)$. Under no circumstances the pilot delay can be set as 0 s due to an inevitable delay inherent to human behaviour. Here no reaction time describes the benefits of automating the manoeuvres in the presence of a collision when RPA is involved in the scenario. The least acceleration time $t^i_{a_0}$ of *Aircraft i* could be calculated:

$$t^i_{a_0} = T^{ij}_{tCPA,h}-\sqrt{\left(T^{ij}_{tCPA,h}\right)^2-2\Delta^i_{t+\widetilde{t}_i,z}/a_0} \tag{12}$$

Similarly, the acceleration time $t^j_{a_0}$ of *Aircraft j* could be obtained. And normally, the accelerated vertical rate $v^i_{t_a,z} = v^i_{t+\widetilde{t}_i,z} + a_0 t^i_{a_0}$ is less than 1500 fpm (DoT, 2011).

In addition, pilot response with $0.35g$ acceleration to an achieved vertical rate of 2500 fpm is expected within 2.5 s for subsequent RAs (DoT, 2011). Therefore, in this research, if *Aircraft i* is currently in the RA process and encounters another vehicle *Aircraft k*, *Aircraft i* would take an increasing acceleration $a_0' = 0.35g$ with the response time $\widetilde{t}_i' (0 \leqslant \widetilde{t}_i' \leqslant 2.5s)$.

3.2.6. Clear of conflict

When the RA is removed, a "Clear of Conflict" would be announced to inform the flight crew that the encounter has been successfully resolved. The TCAS manoeuvre would be terminated at t_{CoC} if the following condition is satisfied:

$$(t_{CoC} > t_{CPA}) \vee \left(D^{ij}_{tCoC,h} > DMOD_{RA}\right) \vee \left(D^{ij}_{tCoC,z} > ZTHR_{RA}\right) \tag{13}$$

3.2.7. Potential induced collision

A domino conflict would deteriorate into an induced collision if the diameter and height of the collision area of both aircraft overlap. Assume that *Aircraft i* has a secondary threat with *Aircraft k*. If at moment $\ddot{t}_{cl}$ their horizontal and vertical distances $D^{ik}_{\ddot{t}cl,h}$ and $D^{ik}_{\ddot{t}cl,z}$ are separately less than the D_{cl} and H_{cl} criteria (EuroControl, 2001), TCAS fails to resolve the encounter.

$$D^{ik}_{\hat{t}cl,h} = \sqrt{(x^i_{\hat{t}cl}-x^k_{\hat{t}cl})^2 + (y^i_{\hat{t}cl}-y^k_{\hat{t}cl})^2} \tag{14}$$

$$D^{ik}_{\hat{t}cl,z} = |z^i_{\hat{t}cl}-z^k_{\hat{t}cl}| \tag{15}$$

$$(D^{ik}_{\hat{t}cl,h} < D_{cl}) \wedge (D^{ik}_{\hat{t}cl,z} < H_{cl}) \tag{16}$$

3.3. Horizontal RA improvement

The expansion of collision avoidance (CA) technology will be able to issue not only vertical RAs, but also horizontal RAs to pilots on how to manoeuvre to avoid collision, incorporating the horizontal manoeuvring of the aircraft to increase the CA capability. If TCAS could advise the pilot to turn left or right rather than climb or descend, it provides the advantage that multiple resolution dimensions add significant flexibility to TCAS's choice of resolution actions. Especially, more options would be offered in the rare event of a multi-threat conflict, i.e., TCAS may resolve the domino conflict in the horizontal dimension avoiding the occurrence of terrible collision accident.

The extended horizontal RA capability totally refers to the generation of vertical advisory. TCAS chooses the opposite sense from that selected by the other aircraft and communicated via the coordination interrogation; both aircraft should respond RA positively (i.e., left, right) not negatively (i.e., maintain) for the coordinated threat resolution. The RA strength which is designed to be the least disruptive to the existing flight path, while still providing horizontal restrict (HRES) between *Aircraft i* and *Aircraft j* at the CPA in the vertical plane. Thus, the RA sense (left or right) is set based on the relative position at t'_{RA} in vertical plane, which means that the shortcut to achieve the least safe separation $HRES_{RA}$ would be chosen. Thus, for fairness their own disruption $\Lambda^i_{t,h}$ and $\Lambda^j_{t,h}$ are:

$$\Lambda^i_{t,h} = -\Lambda^j_{t,h} = \frac{1}{2}(HRES_{RA}-D^{ij}_{tCPA,z}) \tag{17}$$

Similarly, both the aircraft accelerate to appropriate speeds with opposite acceleration $\hat{a}_0 = 0.25g$ in the horizontal plane and the response time of pilot in *Aircraft i* is assumed as $\hat{t}_i (0 \leqslant \hat{t}_i \leqslant 5s)$. The least acceleration time $t^i_{\hat{a}_0}$ of *Aircraft i* could be calculated:

$$t^i_{\hat{a}_0} = T^{ij}_{tCPA,z} - \sqrt{(T^{ij}_{tCPA,z})^2 - 2\Lambda^i_{t+\hat{t}_i,h}/\hat{a}_0} \tag{18}$$

Similarly, the acceleration time $t^j_{\hat{a}_0}$ of *Aircraft j* could be obtained.

Let $L^i_{tCPA,z} = (x^i_{tCPA,z}, y^i_{tCPA,z})$ and $L^j_{tCPA,z} = (x^j_{tCPA,z}, y^j_{tCPA,z})$ be the horizontal position coordinates of *Aircraft i* and *Aircraft j* at the CPA in the vertical plane. Thus the horizontal position coordinates of *Aircraft i* at the time that the least safe separation $HRES_{RA}$ is achieved could be calculated:

$$L^i_{tHRES,z} = (x^i_{tHRES,z}, y^i_{tHRES,z}) = \left(x^i_{tCPA,z} + \Lambda^i_{t,h}\cos\left(\arctan\frac{y^i_{tCPA,z}-y^j_{tCPA,z}}{x^i_{tCPA,z}-x^j_{tCPA,z}}\right), y^i_{tCPA,z} + \Lambda^i_{t,h}\sin\left(\arctan\frac{y^i_{tCPA,z}-y^j_{tCPA,z}}{x^i_{tCPA,z}-x^j_{tCPA,z}}\right)\right) \tag{19}$$

the recommended turning angle φ^i_t of *Aircraft i* at time *t*:

$$\varphi^i_t = \arctan\left(\frac{x^i_t-x^i_{tCPA,z}}{y^i_t-y^i_{tCPA,z}}\right) - \arctan\left(\frac{x^i_t-x^i_{tHRES,z}}{y^i_t-y^i_{tHRES,z}}\right) \tag{20}$$

Meanwhile, the recommended turning angle φ^j_t of *Aircraft j* at time *t* could be obtained.

4. Development of the GMAS-based TCAS model

Most systems can be roughly classified considering the time evolution of the properties of interest as continuous or discrete (Jensen and Kristensen, 2009). It is well accepted that a continuous system can be described using a discrete representation, while a discrete system can be described by a continuous model. The choice of employing a continuous or a discrete representation depends on the purpose of investigation (particular objectives) of each study rather than the characteristics of the system. In this research, to explicitly sense the effect of each TCAS action, i.e., TA, vertical RA and horizontal RA, the dynamics of TCAS-equipped aircraft encounters are modelled as a series of discrete events from which the different states of the system can be evaluated. Discrete event system (DES) is defined as "a discrete-state, event-driven system, that is, its state evolution depends on the occurrence of asynchronous discrete events over time" (Geng et al., 2016). In many situations, the system under consideration can be modelled as a DES and the problems can be translated into state estimation problems in a DES framework (Shu and Lin, 2011). In the discrete event-based models, events (i.e., the state changes) can be depicted by a graph-based notation with several nodes and the relations between those events are represented using the links (Wainer, 2009). Thus, a series of discrete events that form the model record the dynamics of a system to perform the state changes, and the links define the relations between events. These DES representations aim to describe the occurrence of finite number events in a discrete time base, (i.e., events occur in a continuous time base, but during a bounded time-span, only a finite number of relevant events occur) (Basile et al., 2009).

In this research, TCAS operations are modelled using the GMAS formalism which is specialized in modelling interactions between different TCAS functions and powerful mathematical basis for conducting state space of discrete events. For multi-threat scenarios, using the range, altitude, and bearing of nearby aircraft that are provided to TCAS by the surveillance function, the TCAS logic initiates and maintains a track on each aircraft. Using the tracks of nearby aircraft, range and altitude tests are performed for each altitude-reporting target. If the RA Tau and either the time to co-altitude or relative altitude criteria associated with the current SL are met, the intruder is declared a threat. When an intruder is declared a threat, a coordinated threat resolution process is used to select the appropriate RA for the encounter geometry. Note that a secondary threat as the result of issuing RA may be induced, and there is the risk which deteriorates to be a collision. Through generating the state space of multi-threat scenario, the positive effects of importing the horizontal RA capability can be quantitatively analysed in order to synthetically apply both the vertical and horizontal manoeuvres more profitably. The optimized advisory would be generated and employed to resolve the negative domino threat.

4.1. Graphical modelling and analysis software

For the experiments, we previously used the state space analysis tool called TIMSPAT, developed at the Logistics and Aeronautics Unit of the Autonomous University of Barcelona. The tool has been shown to be effective for the performance analysis of very demanding and flexible industrial systems (Baruwa et al., 2015, 2016). Yet it is failing to provide the graphical modelling interface faced industrial systems; therefore the errors are difficult to detect in the developing process and TIMSPAT model constituted by a set of text files is not easy to understand the model architecture. Besides, as one of the most commonly used tools for modelling and simulating discrete-event systems, though CPN Tools stands out as an industrial strength software that provides both a graphical editing interface and an interactive simulator for constructing and analysing models, it has some limitations. Its earlier version supports extraordinarily simple calculation only and even with this extension, the up-to-date version is still difficult to integrate complex operation. In addition, it has a state space analysis plug-in, but the absence of efficient search algorithms has limited its applicability and it cannot scale up to industrial-sized problems (Baruwa and Piera, 2016). To overcome the above-mentioned shortcomings of TIMSPAT and CPN Tools, the graphical modelling and analysis software (GMAS) has been developed. GMAS is a powerful graphical and mathematical modelling tool, which has been extensively used to model, simulate, and analyse DESs characterized by concurrency, parallelism, causal dependency, resource sharing and synchronization. In the research project (Feng et al., 2016), it was used as the modelling tool to simulate and analyse various cluster behaviours of unmanned aerial vehicles (UAVs), e.g., synergic control, formation reconfiguration, situation prediction, etc. Besides, based on the proposed GMAS-based model (Tang et al., 2018), a typical three-aircraft scenario was elaborately depicted to clearly represent the different evolution with/without wind disturbance.

The graphical components of GMAS model include start component, data component, function component, nested function component, link component and end component.

Definition 1. The GMAS model can be defined as the following nine-tuple:

$$GM = (S, D, H, H', F, E, F_W, M, M_0) \tag{21}$$

where

$S = \{s_1\}$ represents the set of start component, and the element is unique.
$D = \{d_1, d_2, \cdots, d_a\}$ represents the set of data components, and a is the amount.
$H = \{h_1, h_2, \cdots, h_m\}$ represents the set of function components, and m is the amount.
$H' = \{h'_1, h'_2, \cdots, h'_n\}$ represents the set of nested function components, and n is the amount.
$F = \{f_1, f_2, \ldots f_u\}$ represents the set of link components, and u is the amount.
$E = \{e_1\}$ represents the set of end component, and the element is unique.
F_W: $F \rightarrow \{f_{1,w}, f_{2,w}, \ldots f_{u,w}\}$ is the set of functions on each link component.
M: $S \cup E \cup D \rightarrow \{s_1, e_1, d_1, d_2, \ldots, d_a\}$ is the set of state identifications, which are the state data of start component, end component and data components during the model operation.
M_0: $S \cup D \rightarrow \{s_{1,0}, d_{1,0}, d_{2,0}, \ldots, d_{a,0}\}$ is the set of initial identifications, which are the initial state data of start component and data components before the model operation.
$D \cap (H \cup H') = \varnothing$ (set D does not intersect with the union of set H and H'), and $D \cup (H \cup H') \neq \varnothing$ (set D and the union of set H and H' are not empty at the same time).
$F \subseteq [(S \cup D \cup E) \times (H \cup H')] \cup [(H \cup H') \times (S \cup D \cup E)]$ indicates that link component connects start component, data component or end component with function component or nested function component, and it is the set of directed arcs.

Among them, $G = (S, D, H, H', F, E, F_W)$ forms the physical structure of the GMAS model, (G, M) is called identified GMAS model, and its feature is the introduction of state identification M which is a vector set of the data in start component, end component and data components at the corresponding time. (G, M, M_0) expresses the complete GMAS model in which the initial states M_0 have been provided with the input data in start component and the initial data in data components at the beginning.

4.2. Model representation

The causal model should be informed with the original aircraft state involved in the same scenario. The aircraft trajectories are discrete to be a sequence of 3D waypoints that the corresponding aircraft will follow, with the sequence containing the positions and

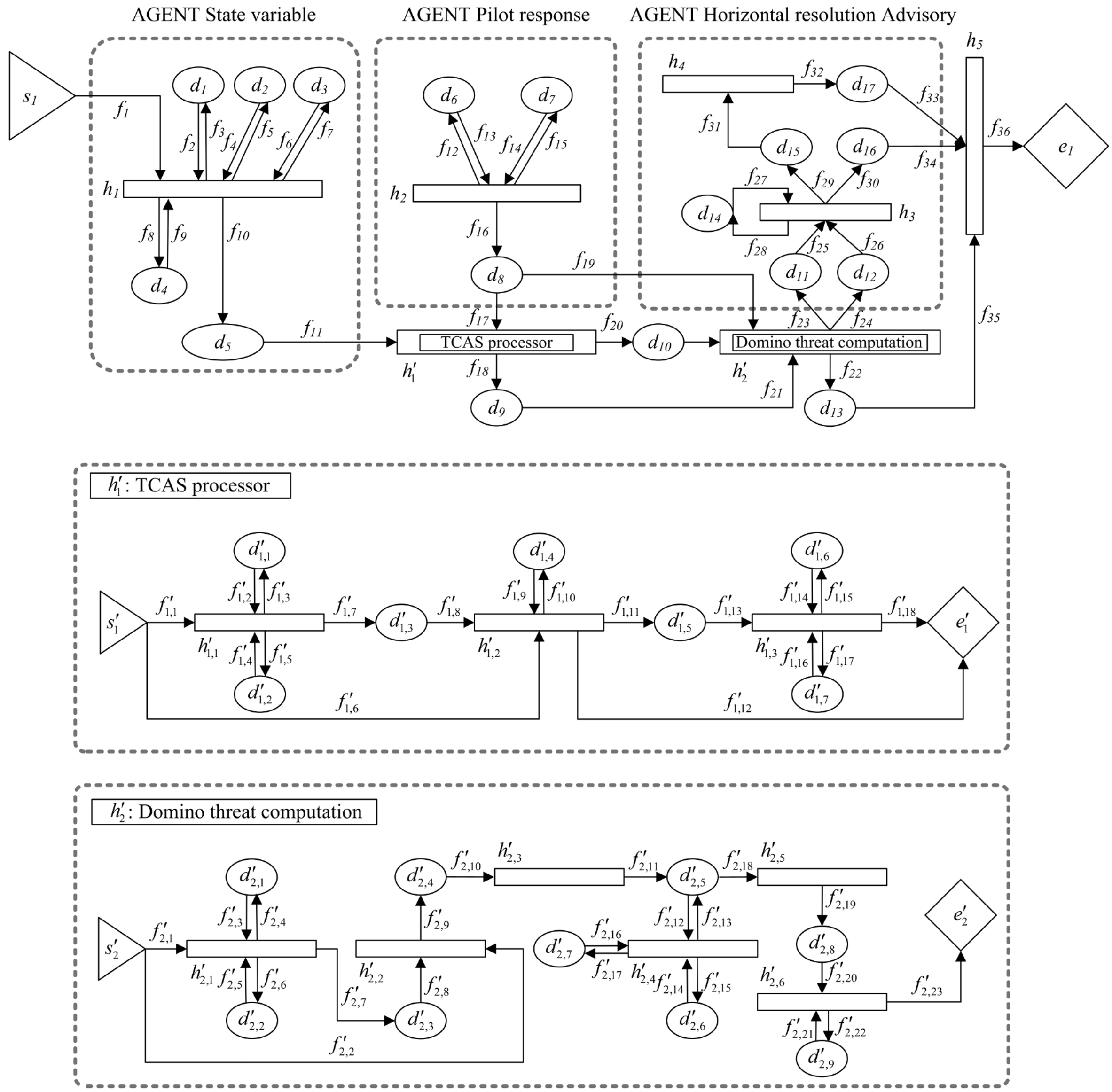

Fig. 4. GMAS-based causal model of improved TCAS.

speeds. It tries to detect and resolve the threat based on the TCAS logic, and determines whether a negative domino threat occurs in the CA process of the previous threats. Based on the mathematical description presented in Section 3, the purpose of the model that results in the GMAS formalism is not only to resolve encounters based on step-by-step logic, but also focuses on exploring the emergent dynamics between the resolution trajectories and the neighbouring trajectories. The developed causal model illustrated in Fig. 4 is based on the aircraft tracking waypoints consisting of three kinds of agents (Agent State variable, Agent Pilot response, and Agent Horizontal resolution advisory) and two nested function components (h_1': TCAS processor and h_2': Domino threat computation) that model for the improved TCAS operations.

- Agent State variable contains one function component (h_1) which aims to improve robustness through considering the uncertainty of motion state. Several typical disturbances should be introduced in simulations to test the robustness of the amended trajectories suggested by TCAS advisories under conditions of uncertainties in operational level. Specially, the speed variation owing to wind instability discussed in this model is identified as the most common factor affecting the en-route trajectory predictions.
- Agent Pilot response owns one function component (h_2) that is used to provide probabilistic pilot reaction for the TCAS advisories. TCAS as a last-resort means that it is a portion of the safety net in the fully complex socio-technical ATM system. Therefore, as a complement to TCAS, some other factors (typically, the pilot response time) can play a considerable role in the process of collision avoidance, and this agent covering the interactions in the sophisticated socio-technical system TCAS is essential.

Table 2
Start, data and end components specification of the main model.

Num.	Components	Description
s_1	Initial waypoints	input the original state information of involved aircraft
d_1	Vx	hold the uncertainty of initial velocity in x bearing
d_2	Vy	hold the uncertainty of initial velocity in y bearing
d_3	Vz	hold the uncertainty of initial velocity in z bearing
d_4	Control	take the subsidiary condition cooperating with h_1 to realize its control function
d_5	Next waypoints	keep the data of serious waypoints in the normal flight without conflict
d_6	Response delay	preserve the pilot's reaction time to the TCAS advisory (within 5 s)
d_7	Possible reaction	contain the choices of sense selection (climb, maintain and descend)
d_8	Potential manoeuvre	summarize the pilot's potential manoeuvre
d_9	RA waypoints	record the RA states of aircraft involved in this scenario
d_{10}	CPA position-time	keep the CPA position and time of each threat
d_{11}	RA collision waypoints	record the RA states of aircraft involved in a potential collision
d_{12}	Collision state	seek out the potential collision state of negative domino effect
d_{13}	Possible future states	indicate all the possible future states, including the potential collision state
d_{14}	Least safe separation	provide the least horizontal safe separation
d_{15}	horizontal position coordinates	preserve horizontal RA states of corresponding aircraft
d_{16}	Acceleration time	keep the horizontal acceleration time of corresponding aircraft
d_{17}	Turning angle	represent the recommended turning angle of corresponding aircraft
e_1	Optimal advisory	output the optimal advisory for the multi-threat scenario

- The nested function component h_1' TCAS processor performs airspace surveillance, intruder tracking, own aircraft altitude tracking, threat detection, RA manoeuvre determination and selection, and generation of advisories. The threat aircraft is also assumed to be equipped with TCAS II in this research, and the avoidance manoeuvre will be coordinated with the threat aircraft.
- The nested function component h_2' Domino threat computation is proposed to identify TCAS weaknesses (potential induced collision) by predictably generating the flyable situations for a specific scenario. The relationship between potential collisions and different pilot behaviours could be displayed.
- Agent Horizontal resolution advisory possesses two function components (h_3 and h_4) and it tries to take the horizontal manoeuvring into consideration when a domino conflict could not be resolved and a potential collision exists in the multi-threat scenario. It is utilized as an auxiliary strategy to ensure the absolute safety of air traffic.

4.3. Specification and description

This causal model includes one start component (s_1), seventeen data components (d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11}, d_{12}, d_{13}, d_{14}, d_{15}, d_{16}, d_{17}), one end component (e_1), depicted in Table 2, as well as five function components (h_1, h_2, h_3, h_4, h_5) and two nested function components (h_1', h_2').

- h_1: Generate the variable speed for each involved aircraft in a scenario. In this block, the event that generates the motion state of each aircraft through randomly selecting the variables in spatial coordinates (x, y, z).
- h_2: Represent a core pilot entity in the collision avoidance to provide a probabilistic pilot response, and it connects with d_6 reserving the reaction time and d_7 containing the sense choices.
- h_1': Contain the part of carrying out the TCAS operations and integrates with h_1 to obtain the initial states of the involved aircraft, and h_2 to receive the impact of pilot behaviour. Its detailed description will be explained below.
- h_2': Explore the domino threat of the possible future situations and integrates with h_1' to obtain the related aircraft state which are in the respective CA process, and h_2 to receive the impact of pilot behaviour. Its detailed description will be explained below.
- h_3: Calculate the least acceleration time to appropriate speed in the horizontal plane, and it connects with d_{11} recording the RA states of aircraft involved in a potential collision, d_{12} seeking out the potential collision state of negative domino effect, and d_{14} providing the least horizontal safe separation.
- h_4: Compute the recommended turning angle, and it connects with d_{15} offering horizontal RA states of corresponding aircraft.
- h_5: Generate the final state space without negative domino states, and it connects with d_{13} supplying the possible future states, including the potential collision state and the event "Horizontal Resolution Advisory" resolving the potential collisions.

This nested function component h_1' includes one start component (s_1'), seven data components ($d_{1,1}'$, $d_{1,2}'$, $d_{1,3}'$, $d_{1,4}'$, $d_{1,5}'$, $d_{1,6}'$, $d_{1,7}'$), one end component (e_1'), depicted in Table 3, as well as three function components ($h_{1,1}'$, $h_{1,2}'$, $h_{1,3}'$).

- $h_{1,1}'$: Evaluate the SL of involved aircraft. Different SLs correspond to different TA and RA thresholds, and it is determined based on their flight altitudes.
- $h_{1,2}'$: Detect the threat. The TA is issued when another aircraft approaches and a collision is would emerge within 20–48 s (variables are provided in $d_{1,4}'$) on account of the SL. It tries to draw the pilot's attention and calculates the CPA to inform h_2' "Domino threat computation".

Table 3
Start, data and end components specification of the sub-model “TCAS processor”

Num.	Components	Description
s_1'	Aircraft state information and potential manoeuvre	input the states of involved aircraft and the pilot's potential manoeuvre
$d_{1,1}'$	Sensitivity level	provide criteria of different levels
$d_{1,2}'$	Control1	take the subsidiary condition cooperating with $h_{1,1}'$ to realize its control function
$d_{1,3}'$	Aircraft in SL	preserve the aircraft states with corresponding flight level
$d_{1,4}'$	$Time_{TA}$-$Distance_{TA}$	keep the time and distance criteria of TA
$d_{1,5}'$	Threat involved aircraft	indicate the states of aircraft involved in a threat
$d_{1,6}'$	$Time_{RA}$-$Distance_{RA}$	keep the time and distance criteria of RA
$d_{1,7}'$	Control2	take the subsidiary condition cooperating with $h_{1,3}'$ to realize its control function
e_1'	CoC and data output	output the RA waypoints and record the CPA information

- $h_{1,3}'$: Resolve the threat. With the communication of the event “ Pilot response”, TCAS issues the RA when a collision would emerge within 15–35 s (variables are provided in $d_{1,6}'$) that is depending on the SL. In order to explore the possible future situations, this transition also serves to transmit RA waypoint information to inform h_2' “Domino threat computation”.

This nested function component h_2' includes one start component (s_2'), nine data components ($d_{2,1}'$, $d_{2,2}'$, $d_{2,3}'$, $d_{2,4}'$, $d_{2,5}'$, $d_{2,6}'$, $d_{2,7}'$, $d_{2,8}'$, $d_{2,9}'$), one end component (e_2'), depicted in Table 4, as well as six function components ($h_{2,1}'$, $h_{2,2}'$, $h_{2,3}'$, $h_{2,4}'$, $h_{2,5}'$, $h_{2,6}'$).

- $h_{2,1}'$: Select the neighbouring threats between which there would be an interrelationship that may lead to a new secondary conflict or even potential collision. If the distance and time at CPA of both threats meet the dimension criteria ($d_{2,1}'$) and time threshold ($d_{2,2}'$), the relationship is declared to be adjacent.
- $h_{2,2}'$: Summary the resolution waypoints. This function component consolidates the multi-threat waypoints at the beginning of RA and records the pilot responses to advisories for the successive research of domino threat. The aircraft involved in nearby threats which may initiate domino effects are summarized in $d_{2,4}'$.
- $h_{2,3}'$: Screen the approaching aircraft. To improve the computational efficiency, only the approaching aircraft which are resolving their separate primary threat should be screened out for the secondary threat detection.
- $h_{2,4}'$: Update the flight state. Operating this function component one time indicates that the aircraft fly to the next waypoints, until all threats are resolved or a new secondary threat is detected based on $h_{2,5}'$.
- $h_{2,5}'$: Detect the domino threat. This function component is similar to $h_{1,2}'$, while the difference is that the involved aircraft have been in the process of resolving their own primary threat.
- $h_{2,6}'$: Estimate the collision/conflict situation. This function component determines whether the new secondary threat could be resolved or not. The pilots have to resolve the emergent threat, and if the D_{cl} and H_{cl} volumes of both aircraft overlap, an induced collision would emerge as the consequence of previous decisions.

In summary, the nested function component “Domino threat computation” as the sub-model is ingeniously designed to narrow the expanded state space based on those function components such as $h_{2,1}'$ and $h_{2,3}'$, which can play an important role in dealing with the complex multi-threat scenarios (e.g., flocks).

Table 4
Start, data and end components specification of the sub-model “Domino threat computation”.

Num.	Components	Description
s_2'	TCAS processor information and potential manoeuvre	input the states of involved aircraft and the pilot's potential manoeuvre
$d_{2,1}'$	Distance criteria	provide the dimension criteria for neighbouring threats
$d_{2,2}'$	Time threshold	provide the time threshold for neighbouring threats
$d_{2,3}'$	Neighbouring threat	indicate the neighbouring threat
$d_{2,4}'$	Amended RA waypoints	preserve the states of aircraft involved in nearby threats
$d_{2,5}'$	Approaching aircraft	represent the approaching aircraft whose distance are getting smaller
$d_{2,6}'$	Time control	take the time control cooperating with $h_{2,4}'$ to realize its control function
$d_{2,7}'$	Sequence control	take the sequence control cooperating with $h_{2,4}'$ to realize its control function
$d_{2,8}'$	Domino threat aircraft	keep the aircraft states involved in a domino threat
$d_{2,9}'$	Judgement criteria	provide the values of the collision volume (D_{cl} and H_{cl})
e_2'	Domino state	output the predictive domino threat states

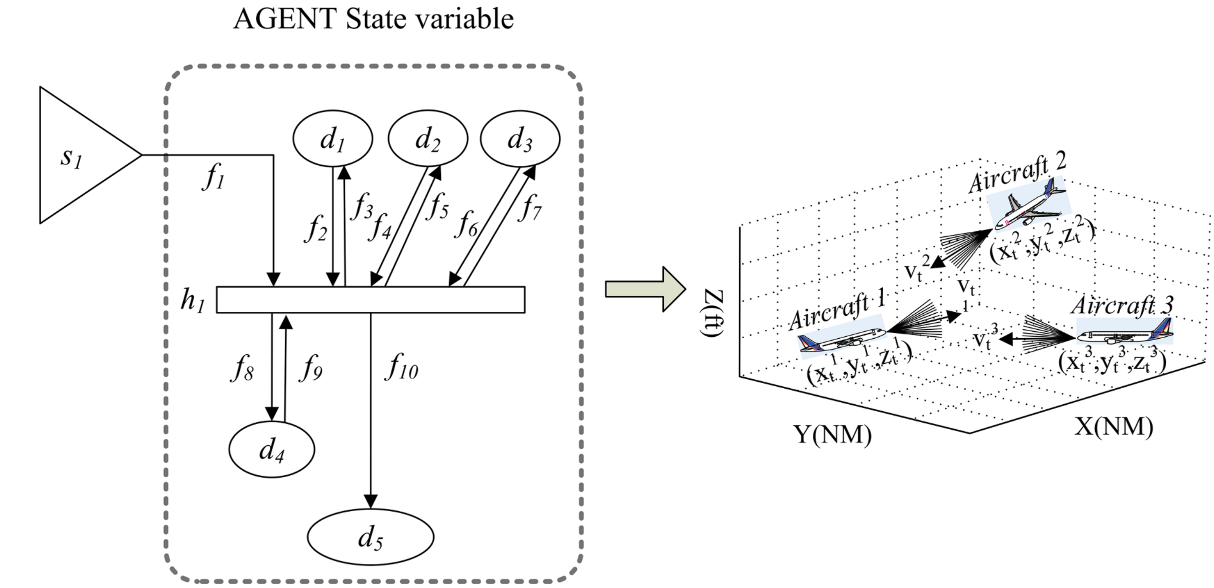

Fig. 5. Variable speed generation for each involved aircraft.

4.4. Event explanation example

This research has been developed with an assumption of 4D trajectories, sequence of 3D position of an aircraft together with its time stamp, TCAS II-equipped aircraft and en-route traffic. When accounting for the vertical, lateral and longitudinal bearings, the speed of *Aircraft i (i = 1, 2,…, n)* also have 3D components $v_t^i = (v_{t,x}^i, v_{t,y}^i, v_{t,z}^i)$. TCAS, independent of any ground inputs, performs surveillance of nearby aircraft to provide information on the position and altitude of these aircraft so that the collision avoidance algorithms can perform their function (DoT, 2011). However, because of the wind influence, pilot behaviours or aircraft performance errors, the speed vectors of aircraft may be variable in a certain range during the execution phase, illustrated in Fig. 5; thus it leads to different possible future situations in which several potential collision scenarios could be induced.

Agent State variable containing the function component h_1 is proposed to simulate the initial feasible speed vector generation, and data components d_1, d_2, d_3 provide several options for random selection in corresponding x, y, z axis. Considering the initial speed in the x axis as an example, the changes in the initial speed could be various, from several negative to several positive options, such as $(-v_x^n, \dots,-v_x^r, \dots,-v_x^2, -v_x^1, 0, v_x^1, v_x^2, \dots,v_x^r, \dots,v_x^n)$, $v_x^1 < v_x^2 < \dots < v_x^r < \dots < v_x^n$. Similarly, the variable range in the y and z axis could respectively be $(-v_y^n, \dots,-v_y^r, \dots,-v_y^2, -v_y^1, 0, v_y^1, v_y^2, \dots,v_y^r, \dots,v_y^n)$, $v_y^1 < v_y^2 < \dots < v_y^r < \dots < v_y^n$ and $(-v_z^n, \dots,-v_z^r, \dots,-v_z^2, -v_z^1, 0, v_z^1, v_z^2, \dots,v_z^r, \dots,v_z^n)$, $v_z^1 < v_z^2 < \dots < v_z^r < \dots < v_z^n$. The extremum $v_x^n/v_y^n/v_z^n$ in each axis is primarily based on the simulated aircraft performance. After firing h_1, the initial state $s_0^i = (p_t^i, v_t^i) = [(x_0^i, y_0^i, z_0^i), (v_{0,x}^i, v_{0,y}^i, v_{0,z}^i)]$ of *Aircraft i* in the initial situation would add a random variable of velocity in each axis, and the more authentic kinematic state should be $s_{0,\Delta}^i = (p_t^i, v_{t,\Delta}^i) = [(x_0^i, y_0^i, z_0^i), (v_{0,x}^i + v_x^r, v_{0,y}^i + v_x^r, v_{0,z}^i + v_x^r)]$, $r \in \{0, 1, \dots,n\}$. The execution times of the function component h_1 is equal to the number of multiple aircraft and the sequence control is stored in the data component d_4.

In summary, the proposed GMAS-based causal encounter model in this paper not only analyses the paths of involve aircraft, but also focuses on reproducing the TCAS operations which could be strengthened in the multi-aircraft scenarios. Certainly, other potential methods could be applied in the analysis and improvement of TCAS scenarios. The GMAS as the enhanced version of TIMSPAT, designed and implemented by our research group, is employed in this research has the following advantages: First, it provides the graphical modelling interface faced industrial systems. The GMAS model components (start component, data component, function component, nested function component, link component, and end component), are convenient for the model construction and the representation of the dynamic TCAS behaviours based on the transformation of state identification. Second, it has a state space analysis plug-in, and also contains efficient search algorithms which could scale up to industrial-sized problems. Events (i.e., the state changes) can be depicted by a graph-based notation with several nodes and the relations between those events are represented using the links. Thus, a series of discrete events that form the model record the dynamics of a system to perform the state changes, and the links define the relations between events. The recorded state changes constitute the state space, providing a global view of the scenario dynamics and a better understanding of the induced collision occurrence, as well as the improved advisory for the pilots to make an optimal option.

5. Results

Table 5 provides the relevant parameter used in the different experiments to validate the feasibility of the GMAS model.

The diameter and height of the collision cylinders are twice as long as the horizontal and vertical sizes of the aircraft, respectively

Table 5
Parameter values for the scenarios.

TCAS Equipment	Kind of reaction	Detection range (NM)	RA acceleration (g)	Subsequent RA acceleration (g)
TCAS II 7.1	Pilot react	40	0.25/−0.25	0.35/−0.35
Initial primary pilot delay (s)	Subsequent pilot delay (s)	Horizontal size (m)	Vertical size (m)	SL
5	2.5	40.74	11.95	6

(i.e., D_{cl} = 0.044 NM and H_{cl} = 78.44 ft) (EuroControl, 2001). The computer used for this simulation is a T450 laptop with a 2.6 GHz Intel i7 processor and 8 GB of RAM, which is enough for the memory requirements of the algorithmic operations and simulation.

5.1. Potential collision representation

To illustrate the possibility of induced collisions between multiple aircraft, let us consider the 5-aircraft traffic scenario (*Aircraft 1*, *Aircraft 2*, *Aircraft 3*, *Aircraft 4* and *Aircraft 5*) illustrated in Fig. 6. In this scenario, five fully equipped aircraft are considered with two predicted encounters (*threat 1* between *Aircraft 1* and *Aircraft 2*, and the other one is *threat 2* between *Aircraft 3* and *Aircraft 4*). Variable $t_{TA}^{i}(i = 1, 2, 3, 4, 5)$ is used for the TA emergence time, and variable $t_{RA}^{i}(i = 1, 2, 3, 4, 5)$ indicates the RA. In normal flight, *Aircraft 1* is cruising at FL160 and *Aircraft 2* is cruising at FL180 on an opposing route. When *Aircraft 2* starts a descending operation and flies into the range of *Aircraft 1*, a TA is issued by TCAS to warn the crew of *Aircraft 1* that a collision is predicted to occur within t_{TA}^{1}. An RA is issued at t_{RA}^{1} to ask the crew to take responsibility for achieving safe separation. Once the threat is detected, *Aircraft 1* performs a descend operation while *Aircraft 2* climbs to provide the greatest vertical separation at CPA. Normally, the RA strength selects the ALIM as the smallest safe separation that requires a minimal speed change. Meanwhile, a similar TA and RA process is initiated between *Aircraft 3* and *Aircraft 4*. When *Aircraft 4* comes within the range of *Aircraft 3* and a collision is predicted to occur, a TA is issued at t_{TA}^{3} and an RA is issued at t_{RA}^{3}. The crew in *Aircraft 3* respond to the RA by attempting to descend, while *Aircraft 4* climbs with the strength of ALIM. Unfortunately, despite the RA's resolution of both encounters, a new secondary threat is initiated between *Aircraft 4* and *Aircraft 1* as a consequence of previous decisions. This is detected by the TCAS at 7:22:15. The crew has to address the emergent encounter. However, there is not enough time left for the pilot reaction, and a collision would occur. Besides, the descending *Aircraft 3* would encounter the cruising *Aircraft 5*, and successfully the new emerging threat could be normally resolved based on TCAS coordination.

This typical compositive scenario includes the following characteristics: (1) It is a fairly complex scenario with multiple aircraft. These five fully equipped aircraft are considered with two predicted encounters (*threat 1* between *Aircraft 1* and *Aircraft 2*, and the other one is *threat 2* between *Aircraft 3* and *Aircraft 4*). And the neighbouring *Aircraft 5* further increases the complexity of the scenario. (2) It involves the interrelationship (also called domino effects) between neighbouring threats. For instance, a new secondary threat between *Aircraft 3* and *Aircraft 5* may emerge in the process of resolving the previous threat.

The compositive scenario has been represented and validated by the Interactive Collision Avoidance Simulator (InCAS) (ICAO, 2006), which is designed for the replay of a real or a synthetic TCAS scenario in which multiple aircraft are involved. It would be mainly used to provide a relatively exact reconstruction of reality for the evaluation, study, demonstrating the TCAS operations.

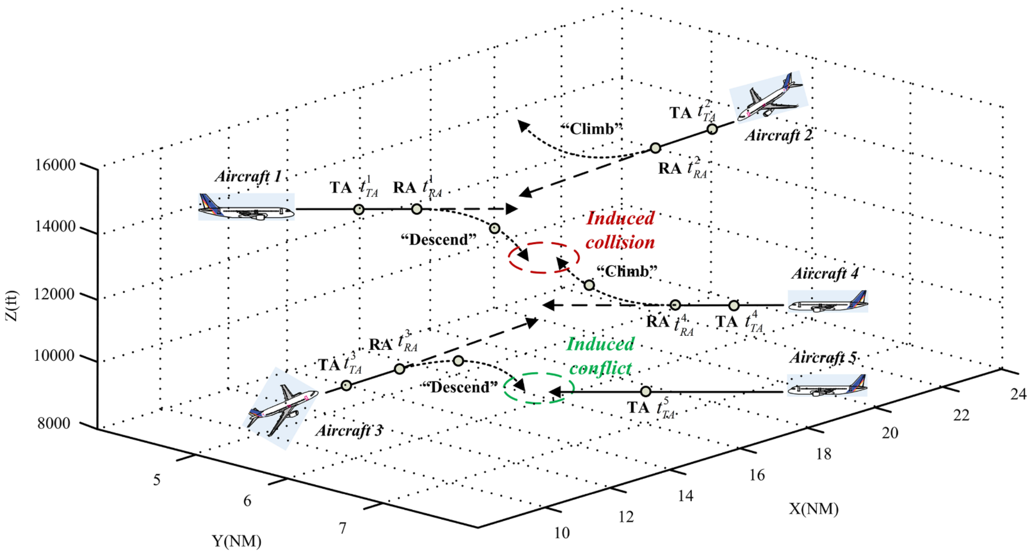

Fig. 6. Five-aircraft collision scenario.

Table 6
Waypoints of partial trajectory.

Time	Aircraft	X (Nm)	Y (Nm)	Z (ft)	Time	Aircraft	X (Nm)	Y (Nm)	Z (ft)
7:22:11	*Aircraft 1*	15.36	6.60	15620.00	7:22:15	*Aircraft 1*	15.76	6.92	15566.65
7:22:11	*Aircraft 2*	22.87	6.59	15900.00	7:22:15	*Aircraft 2*	22.27	6.91	15893.33
7:22:11	*Aircraft 3*	3.36	4.48	15453.00	7:22:15	*Aircraft 3*	3.44	4.48	15465.00
7:22:11	*Aircraft 4*	17.84	7.88	15420.00	7:22:15	*Aircraft 4*	16.84	7.48	15420.00
7:22:11	*Aircraft 5*	4.51	4.83	15350.00	7:22:15	*Aircraft 5*	4.43	4.63	15350.00
7:22:12	*Aircraft 1*	15.46	6.68	15606.65	7:22:16	*Aircraft 1*	15.86	7.00	15553.31
7:22:12	*Aircraft 2*	22.72	6.67	15898.33	7:22:16	*Aircraft 2*	22.12	6.99	15891.67
7:22:12	*Aircraft 3*	3.38	4.48	15456.00	7:22:16	*Aircraft 3*	3.46	4.48	15452.00
7:22:12	*Aircraft 4*	17.59	7.78	15420.00	7:22:16	*Aircraft 4*	16.59	7.38	15432.00
7:22:12	*Aircraft 5*	4.49	4.78	15350.00	7:22:16	*Aircraft 5*	4.41	4.58	15350.00
7:22:13	*Aircraft 1*	15.56	6.76	15593.31	7:22:17	*Aircraft 1*	15.96	7.08	15539.98
7:22:13	*Aircraft 2*	22.57	6.75	15896.67	7:22:17	*Aircraft 2*	21.97	7.07	15890.00
7:22:13	*Aircraft 3*	3.40	4.48	15459.00	7:22:17	*Aircraft 3*	3.48	4.48	15438.00
7:22:13	*Aircraft 4*	17.34	7.68	15420.00	7:22:17	*Aircraft 4*	16.34	7.28	15445.00
7:22:13	*Aircraft 5*	4.47	4.73	15350.00	7:22:17	*Aircraft 5*	4.39	4.53	15350.00
7:22:14	*Aircraft 1*	15.66	6.84	15579.98	7:22:18	***Aircraft 1***	**16.06**	**7.16**	**15526.65**
7:22:14	*Aircraft 2*	22.42	6.83	15895.00	7:22:18	*Aircraft 2*	21.82	7.15	15888.33
7:22:14	*Aircraft 3*	3.42	4.48	15462.00	7:22:18	*Aircraft 3*	3.50	4.48	15421.00
7:22:14	*Aircraft 4*	17.09	7.58	15420.00	7:22:18	***Aircraft 4***	**16.09**	**7.18**	**15465.00**
7:22:14	*Aircraft 5*	4.45	4.68	15350.00	7:22:18	*Aircraft 5*	4.37	4.48	15350.00

Specifically, in the InCAS the induced collision between *Aircraft 1* and *Aircraft 4*, and the induced conflict between *Aircraft 3* and *Aircraft 5* could emerge from the initial set states.

Note that this GMAS encounter model is not only built for this five-aircraft scenario, but for all complex scenarios in which multiple aircraft are involved. It consists of three kinds of agents and two nested function components: Agent State variable is used to improve robustness based on the uncertainty of motion state; Agent Pilot response tries to generate all possible pilot reactions for the TCAS advisories; Agent Horizontal resolution advisory aims to take the horizontal manoeuvring into consideration when a domino conflict could not be resolved; the nested function component h_1' contains the process of carrying out the TCAS operations and h_2' detects the domino threat of the possible future situations.

For *threat 1*, when *Aircraft 1* and *Aircraft 2* fly within range of one another, a TA is issued to fire a warning at 7:21:51. The RA emerges to ask the crew to act upon this encounter at 7:22:06, as the situation is getting worse. For *threat 2*, the TA is issued at 7:21:55, and the RA appears at 7:22:10. Table 6 illustrates the waypoints of a partial trajectory of each aircraft before the collision occurs. At 7:22:17, a new threat is detected between *Aircraft 3* and *Aircraft 5*, and the pilots react 2.5 s later to resolve it. At 7:22:18, *Aircraft 1* and *Aircraft 4* burst into each other's collision safety volume, their horizontal distance is $\sqrt{(16.06-16.09)^2 + (7.16-7.18)^2} = 0.036NM < 0.044NM$, and the altitude interval is $15526.65-15465.00 = 61.65ft < 78.44ft$. The key process of this five-aircraft scenario evolution is illustrated in Fig. 7.

5.2. State space with horizontal resolution

This section presents the state space of this five-aircraft scenario that is generated by our causal GMAS model, displayed in Fig. 8. The causal model begins to operate when both the adjacent and interactional TAs are declared. For *Aircraft 1* and *Aircraft 2* which are involved in *conflict 1*, based on the current TCAS logic *Aircraft 1* descends and *Aircraft 2* climbs in general when the RA is issued. If the horizontal resolution is introduced to improve the RA ability, their response combination could be (Descend, Climb), (Left, Right) or (Right, Left). As shown in level 1 of this state space, *state b*, *state c* and *state d* is generated. Immediately afterwards, *Aircraft 3* and *Aircraft 4* have to resolve the neighbouring *conflict 2*. Analogously, their possible reaction also has three options (Descend, Climb), (Left, Right) or (Right, Left). In view of the overall situation, there would be $3 \times 3 = 9$ states in Level 2 for the five-aircraft response combination, i.e., *state e*, *state f*, *state g*, *state h*, *state i*, *state j*, *state k*, *state l* and *state m.*

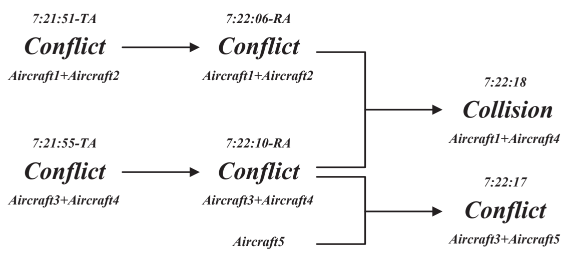

Fig. 7. Key process of this scenario evolution.

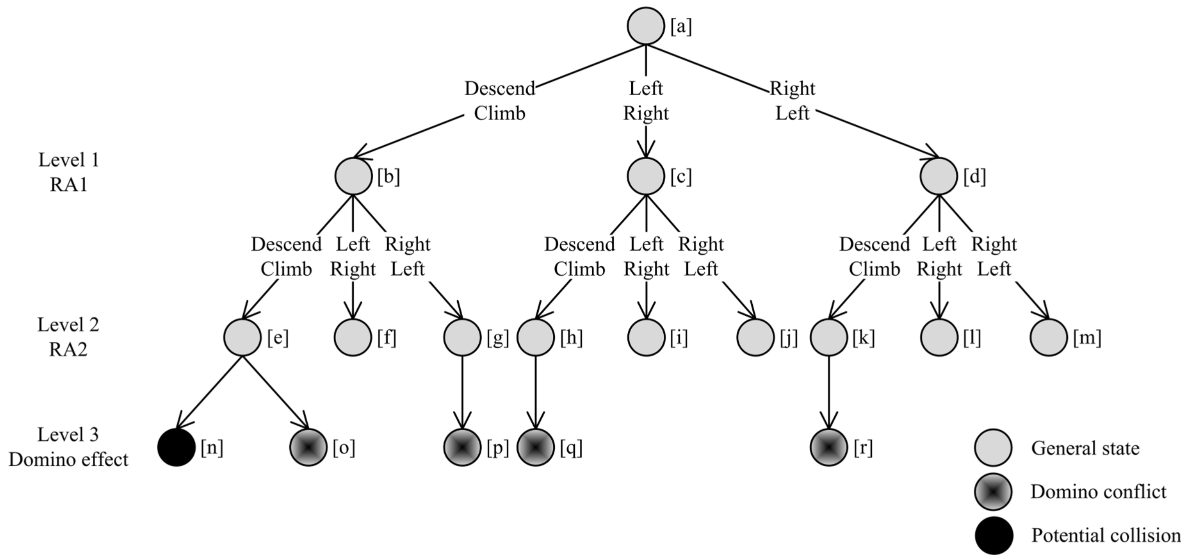

Fig. 8. The state space of this five-aircraft scenario.

The two nearby threats may produce domino effects that would initiate a new secondary encounter or even a potential collision. In addition, the amending aircraft in the RA process also may encounter neighbouring vehicle, e.g., *Aircraft 5* in this scenario. The tests of domino effect are implemented on the involved aircraft, and four response combinations containing five domino effects (*state n*, *state o*, *state p*, *state q*, and *state r*) have been detected. Besides, the conflict would deteriorate into a collision when the response combination (Descend, Climb, Descend, Climb) is utilized, and it has been validated and represented in the above section. Thus, *state f*, *state i*, *state j*, *state l*, and *state m* without any domino effect are preferred. Considering the fact that the horizontal resolution advisory is used as an auxiliary strategy to ensure the traffic safety, the vertical means in altitude is still the primary choice and therefore *state f* corresponding to the response combination (Descend, Climb, Left, Right) is the optimal advisory for this multi-threat scenario based on the state prediction.

5.3. Further investigations

5.3.1. The impact of pilot's response delay

The Agent pilot response can randomly generate the pilot reaction time of the TCAS advisories, and the response delays are set based on the experimental objective. TCAS acts as the last resort when there is a failure in ATCo-provided separation services, thus the increment in the pilot response time may initiate more potential accidents. We changed the initial pilot delay 5 s to 1 s in the previous five-aircraft collision scenario to check the state evolution.

Fig. 9 depicts the relative range and altitude between *Aircraft 1* and *Aircraft 4* with the different pilot response times of 1 s and 5 s. When the pilot delay is 5 s, *Aircraft 1* turns downward to avoid *threat 1* from 7:22:11 and *Aircraft 4* turns upward to avoid *threat 2* from 7:22:15, the generated trajectories correspond to the pattern based on the TCAS logic. From the red line for relative altitude and the yellow line for relative range, it shows that *Aircraft 1* and *Aircraft 4* invade the minimum separation region of each other at 7:22:18. Going by the standard RA instructions, the data flow of relative range (yellow line) is the same irrespective of the pilot's response time. If the aircraft are assumed to react as soon as an advisory emerges and the pilot response time is set to 1 s (blue line records the relative altitude between *Aircraft 1* and *Aircraft 4*), there would be no secondary threat that appears as a downstream effect of previous decisions. Though *Aircraft 1* descends while *Aircraft 4* climbs, as they are approaching, they never come within the safety volume of each other and no threat is activated. At the time they were on the same flight level (at the relative altitude 0 ft), their relative range is nearly 1.20 NM which is out of the TA region. Besides, at 7:22:18, when the two aircraft are nearest in the range, their relative altitude is 120.00 ft which is larger than the height of the collision cylinder (78.44 ft), and they keep away from each other in different flight levels. In addition, though the descending *Aircraft 3* would still encounter the cruising *Aircraft 5*, because of the shorter pilot's response delay there would be more time left for *Aircraft 3* to implement the reactive manoeuvres of subsequent RA, as well as *Aircraft 5* for the emerging threat.

Thus, by reducing the pilot delay, the risk rate of TCAS logic failures is expected to reduce while providing extra capacity to mitigate the TCAS induced collisions. Delay to respond TCAS advisories during flight can degrade safety, and the pilots in the involved aircraft should react their respective TCAS advisories as soon as possible.

5.3.2. Computational efficiency analysis

This research presents the development and application of modified TCAS operations with horizontal RA improvement when using the GMAS formalism, and it creatively generates the state space of the involved aircraft in a scenario over a period of time.

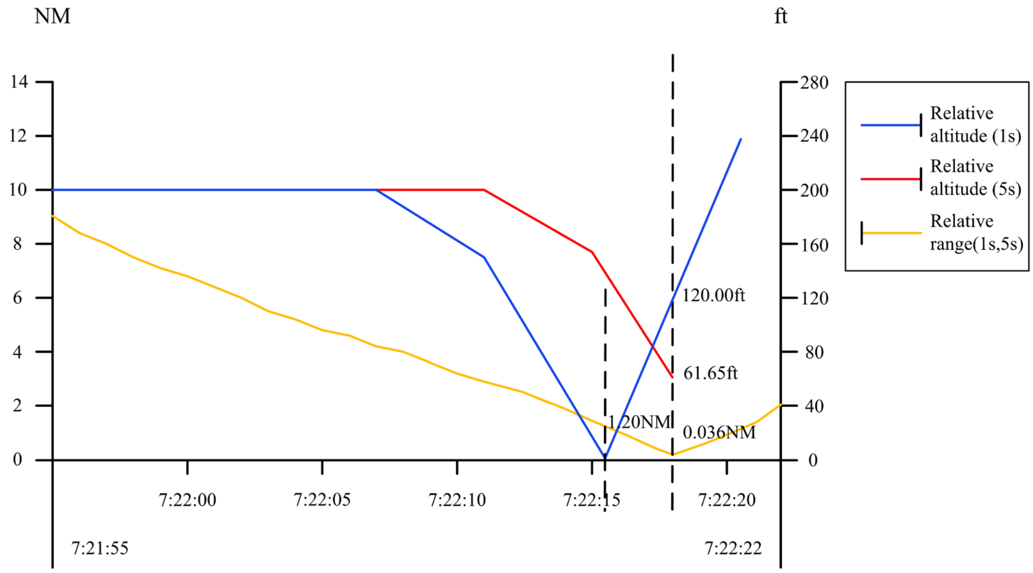

Fig. 9. Relative range and altitude between *Aircraft 1* and *Aircraft 4* with the pilot response time of 1 s and 5 s.

Table 7
Average computation time of the causal model with/without $h'_{2,1}$ and $h'_{2,3}$.*

Amount	Time taken (abridged causal model without $h'_{2,1}$ and $h'_{2,3}$)	Time taken (full causal model)
3	3.56 s	1.37 s
4	14.79 s	3.11 s
5	1 min20.15 s	8.68 s
6	9 min57.72 s	17.97 s
7	1 h40 min34.81 s	41.82 s

* Test case of different initial states (Averaged over 25 Runs).

TCAS is used at a tactical and operational level to alert about those scenarios in which TCAS could fail and provide optimal coordinated advisories, thus the computational efficiency is a prerequisite to ensure the flight safety.

Evidently it is challenging for the present causal model to handle a large number of aircraft in the entire airspace, however the discussion scope focuses on the local particular scenario of hot spots in which several aircraft are involved. Based on the radar data from the FAA and Department of Defence sites throughout the United States, the distribution over the number of aircraft involved in a multi-threat encounter is illustrated (Billingsley et al., 2009). Over 95% involve three aircraft, but only one involves seven aircraft in the total identified 3803 such multi-threat encounters. The proposed causal model is competent for safety assessment and advisory optimization of different numbers of aircraft in a hot spot, and the average computation times are recorded in Table 7.

Analysing the computation time of this model with the increasing aircraft number is almost an exponential growing problem that the proposed causal model could not take effect in the TCAS advisory optimization. Innovatively, some function components (e.g., $h'_{2,1}$ and $h'_{2,3}$) which are similar to filters to screen out the valuable states for the subsequent calculation, play an important role in effectively narrowing the expanded state space. Based on the simulation results summarized in Table 7, it is illustrated that the average computation time is clearly reduced compared to the abridged causal model without $h'_{2,1}$ and $h'_{2,3}$.

6. Conclusions

TCAS constitute a last-resort means, which is accepted worldwide, of effective and significant reducing the collision probability between aircraft. It executes independently of ground-based systems and relies fully on relevant surveillance equipment on-board the aircraft. TCAS equipped in aircraft does not control the vehicle directly; it just issues advisories to pilots on how to manoeuvre vertically to prevent collision. However, the increased airspace usage can induce a secondary threat (negative domino effect) as a result of manoeuvre advisory issued by TCAS, which may initiate an improper manoeuvre that would induce a collision. This paper contributes to the study for a better understanding of the induced effects of resolution advisories aroused by TCAS in an over-extending airspace, and reasonably provides a optimal advisory when a potential collision would occur. The main contributions of this paper are listed below:

- Specify the mathematical model for TCAS II algorithm with the horizontal RA capability. TCAS issue TAs to provide proximity warning of nearby traffic to assist the pilot in the visual acquisition of intruder aircraft, and RAs to provide recommended escape manoeuvres in the vertical dimension to increase the existing separation between aircraft. The horizontal improvement is designed to handle multi-threat encounters in which a potential collision is predicted. This process serves as the theoretical basis for construction of the encounter model.
- Describe the powerful graphical and mathematical modelling tool GMAS. It overcomes the defect of TIMSPAT that fails to provide the graphic modelling interface and CPN Tools which is still difficult to integrate complex operations. The state space concept is introduced to represent the system evolution process, i.e., a global perspective on the scenario dynamics and a better understanding of the induced collision occurrence for risk assessment.
- Propose the causal encounter model absolutely in view of the improved TCAS-based avoidance operations. Based on the detailed model specification and explanation, the model logic process is explicit; the implemented model provides a downstream trace of the different effects of potential RAs issued to avoid a collision through the generation of state space. Besides, several techniques, e.g., eliminating the situations that the aircraft are not approaching, are utilized to improve the computational efficiency, inventively resolved the problem of fleetly expansive state exploration.
- Summarize and analyse the simulation results of the typical induced collision scenarios. For safety assessment and advisory optimization, quantitative measurement experiments are conducted to validate the feasibility and effectiveness of the encounter model with horizontal resolution.

Actually, there are various sources of uncertainty which make influence on the deterministic TCAS logic, and these should be considered in the flight status evolution when we built the causal encounter model. The weather conditions (particularly the wind), the aircraft control systems (both the pilot and aircraft performance) and the positioning/tracking precision (even considering the more precise navigation systems), that affect the aircraft while they are flying their trajectories in a precise and straight way. Due to these interference factors, the speed vectors of the involved aircraft, i.e., the major affected parameter, may be variable in a certain range during the execution phase; thus this leads to different possible future situations (state space) in which several potential collision scenarios could be induced. In the proposed causal encounter model, the Agent State variable aims to improve robustness through considering the uncertainty of motion state. Several typical disturbances should be introduced in simulations to test the robustness of the amended trajectories suggested by TCAS advisories under conditions of uncertainties in operational level. Specially, the speed variation owing to wind instability discussed in this model is identified as the most common factor affecting the en-route trajectory predictions. Besides, the Agent Pilot response is used to provide probabilistic pilot reaction for the TCAS advisories, which can play a considerable role in the process of collision avoidance. Different reaction time and sense choices of pilots as two kinds of the uncertainties also could induce different future scenario states.

As the next research step, it could be designed for the replay of a real or a synthetic event in which multiple aircraft are involved. It would be mainly used as an interactive system for the evaluation, study, demonstrating the potential TCAS-induced collisions, and to simulate incidents that provide a relatively exact reconstruction of reality to support the safety assessment and advisory optimization.

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