# ACD++: a Domain Specific Language for Cell-DEVS Modelling 

Chong Jiao and Baohong Liu


#### Abstract

This paper introduces a library ACD++ for modeling and simulation of cellular models based on Cell-DEVS formalism. The goal is to allow the modeling of cellular models more flexible and adaptive. ACD++ is implemented in Ruby programming language, providing an internal Domain Specific Language (DSL) to simplify the construction of cellular models. Ruby's meta-programming characteristics and plentiful syntactic sugar enables the easy expression of complex logics behind the models. The Cell-DEVS formalism proved consistent with the DEVS hierarchy, improving the description of complex systems. Another strength lies in the extensibility of the DSL, allowing the modelers to introduce their domain specific vocabulary to facilitate the definition of specific models. The use of this library has allowed the development more flexible and adaptive, significantly reducing development time.

Index Terms-DSL; Cell-DEVS; cellular model; modeling and simulation


## I. INTRODUCTION

In recent years, the flexibility and adaptability of modeling and simulation has become the main concern especially in the area of decision-making. For instance, in the simulation of complex systems for decision making, the decision makers often expect the simulation to be adaptive to the dynamic changing decision environment [1]. Many technologies have been proposed to enable the flexibility of modeling from the perspective of design pattern, aspect-oriented programming, or modeling language. Our work should be placed at the latter one, which aims to provide an internal Domain Specific Language (DSL) to improve the description of complex systems which can be represented by cell spaces. Cellular Automata formalism (CA) has been widely used to describe those systems which can be represented as cell spaces [2][3]. CA is defined as $n$-dimensional lattices composed of cells with discrete and finite states. It can be viewed as a dynamic system, which evolves at separate points in time. However, the discrete time paradigm poses constraints on the precision and efficiency of the simulated models [4]. In addition, most cells would not update their states in each iteration, which further reduce efficiency. To solve it, the discrete event cellular models are proposed in paper [5] and are specified as Cell-DEVS, which is an extension of Discrete EVent System specification (DEVS). DEVS is proposed by professor Zeigler in 1976 and used to specify formally discrete event systems using a modular description. The discrete event paradigm and formal specification of CellDEVS formalism provide great advantage of being efficient,

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Chong Jiao and Baohong Liu are with College of Information System and Management, National University of Defense Technology, Changsha, 410073, P. R. China (jch_email@qq.com; liubh@nudt.edu.cn)
accurate and easy to verify. Many simulation tools have provided support for the Cell-DEVS formalism, such as James-II, CD++, and ADEVS [6]. However, these tools are either limited to the implemented programming language, or constrained by the modeling methodology. Thus they cannot provide enough flexibility and adaptability. To overcome it, we expect the simulation tools can provide mechanism for domain experts to create and modify the models according to domain knowledge. DSL is competent for it. A DomainSpecific Language (DSL) is a programming or description language tied to a specific application domain [7]. Compared with general programming language, DSL is more expressive, easily verified, and provides a good way to enable the flexibility of modeling. Ruby is a programming language invented in 1993, whose meta-programming characteristics and plentiful syntactic sugar provide strong support to grow a DSL. In this paper, we propose a modeling and simulation library, ACD++, based on Cell-DEVS specification. It is implemented in Ruby programming language and is devote to providing an internal DSL to improve the description of complex systems which can be represented by cell spaces.

## II. BACKGROUND

## A. DEVS and Cell-DEVS formalism

DEVS formalism originates from system theory, providing a framework for the construction of hierarchical models in a modular manner [8]. In DEVS, basic models are specified as black boxes with input and output ports. Several models can be integrated together to form a hierarchical model. The integrated model is either atomic (behavioral) model or coupled (structural) model. The former models autonomous behavior and is specified as:

$$
A M=<X, Y, S, \delta_{i n t}, \delta_{e x t}, \lambda, t a>
$$

Where
$X$ is the set of inputs
$Y$ is the set of outputs
$S$ is the set of states
$\delta_{\text {int }}: S \rightarrow S$ is the internal transition function
$\delta_{\text {ext }}: Q \times X \rightarrow S$ is the external transition function, where $Q=\{(s, e) \mid s \in S, 0 \leq t a(s)\}$ is the total state set and $e$ is the elapsed time since last transition
$\lambda: S \rightarrow Y$ is the output function
$t a: S \rightarrow R_{0}^{+}$is the time advance function
On reception of external event, $\delta_{\text {ext }}(s, e, x)$ is invoked using input value $x$, elapsed time $e$, and current state $s$. In the absence of external events, an atomic model will remain in state $s$ until ta(s) expires. Then the output function $\lambda(s)$ is called and the model will transform into the new state
$\delta_{\text {int }}(s)$. The coupled model is composed of child models, each of them being atomic or coupled. Formally, the classical coupled models are specified as:
$N=<X, Y, D,\left\{M_{d} \mid d \in D\right\}, E I C, E O C, I C, S E L E C T>$
where
$X$ is the input set of the coupled model
$Y$ is the output set of the coupled model
$D$ is the set of component references
$\left\{M_{d} \mid d \in D\right\}$ is the set of child models
EIC is the external input coupling set which connect inputs to component inputs
$E O C$ is the external output coupling set which connect component outputs to outputs of $N$.
$I C$ is the internal coupling set which connect component outputs to component inputs
Select: $2^{Z}-\phi \rightarrow Z$ is the tie-breaking function
Therefore, a coupled model defines the components and their interactions. Detailed description can be found in [8].

Cell-DEVS is an extension of DEVS formalism, which defines each cell as an atomic model, and the cell space as a coupled model. Each cell holds state variables and some rules which are used to update its state using its present state and neighborhoods. The formal specification of Cell-DEVS atomic model can be found in [9]-[10]. It can be specified as:

$$
T D C=<X, Y, S, N, d, \delta_{i n t}, \delta_{e x t}, \tau, \lambda, t a>
$$

Where
$X, Y$, and $S$ represent input set, output set and state set respectively. $N$ represents the number of inputs of a cell. The delay function $d$ is associated with each cell, after which, the new state value is sent out. $\tau$ represents the local rules which are responsible to compute the future state using its current state and neighbourhoods' state. A complete cell space model can be defined as:
$G C C=<X_{\text {list }}, Y_{\text {list }}, X, Y, n,\left\{t_{1}, t_{2}, \cdots, t_{n}\right\}, N, C, B, Z>$
Where
$X_{\text {list }}$ and $Y_{\text {list }}$ represent input coupling list and output coupling list, which are used to define the coupling relation among external DEVS models. $n$ defines the dimension of the cell space. $\left\{t_{1}, t_{2}, \cdots, t_{n}\right\}$ is the number of cells in each dimension. $N$ defines the number of inputs of each cell, which is the same as the definition of TDC. $C$ is an array of atomic cells. $B$ represents the border cells, which may have different behaviours from the rest cells. $Z$ represents the transition rules.

Both of DEVS and Cell-DEVS provide a formal specification that can reduce the cost of development. Thus they provide a good starting point to discuss flexibility of modelling and simulation.

## B. Domain Specified Language

A Domain-Specific Language (DSL) is a programming or description language tied to a specific application domain.

Unlike general-purpose language, it is designed for a particular kind of problem.

A DSL can either be external or internal. The former exists independently from other languages. The latter rely on the hosting language, which can be viewed as an enhancement of the hosting language [11]. In this paper, we provide an internal DSL implemented in Ruby. Ruby's dynamic binding mechanism and closures (a function of block can capture its referencing environment) makes you can execute the closure with the context of arbitrary objects, which provides strong support for the internal DSL.

Many simulation tools have provided support for the Cell-DEVS formalism, such as ADEVS [12], CD++ [13][14] and James-II [15]. Except for ADEVS that provides API for modelers to create Cell-DEVS models, CD++ and James-II both provide a DSL. However, both of them are based on an external DSL, which poses constraints on the extensibility and expressive power. Our work proposes an internal DSL implemented in Ruby for the modeling of CellDEVS models. Modellers can benefit from the convenience brought by DSL, while defining complex logics using Ruby. Another strength lies in the extensibility, allowing it to be extended to the specified domain.

## III. DESIGN AND IMPLEMENTATION

ACD++ has been implemented using Ruby. The ACD++ library itself focuses on the improvement of modelling flexibility for cellular models. Our work is an extension of DEVS-Ruby, which is a DEVS modelling and simulation library implemented in Ruby proposed by paper [11].

In this section, we will first propose our design of DSL. Then, we discuss our modelling architecture and implementation.

## A. Design of DSL

To improve the flexibility of modelling, we propose some requirements that an ideal DSL should possesses. (1) Be consistent with the Cell-DEVS specification; (2) Support the Cell-DEVS vocabulary, and be more accessible to the non-experts; (3) The created models can be easily integrated into the DEVS architecture; (4) Offer a way to extend the DSL.

As ACD++ implements Cell-DEVS theory, our first concern is to remain consistent with the specifications. The vocabulary of our DSL includes all the elements in the specification, such as neighbourhood, state, dimension.

Table I shows the specification of 'Game of Life' [16]. From the specification, we can learn that the dimension of the cell space is $20 \times 20$. Each cell owns a state variable whose domain is [DEAD, ALIVE]. The state of cells whose coordinator is beyond the dimension is regarded as DEAD.

Moreover, the behaviour of cells is defined as a set of rules with the form \{ACTION, DELAY, CONDITION $\}$. These indicate that when the CONDITION is satisfied, the cell would take the ACTION after the DELAYed time expires. This is similar to the definition in CD++, which is described

## TABLE I

THE SPECIFICATION OF GAME OF LIFE

```
size 20,20
states :state=> [:DEAD,:ALIVE]
border :constant, :state=>:DEAD
neighbor_type :moore, 1
init_with_value state: :ALIVE
init_with_maps [0,0]=>{:state => :DEAD},
        [0,1] => {:state => :DEAD }
```

TABLE II
THE RULES IN GAME OF LIFE

```
rule action{state(:DEAD)}, delay{1},
    condition{state == :ALIVE and
        count_range(2..3, :state => :DEAD)}
rule action{state(:ALIVE)}, delay{1},
    condition{state == :DEAD and
    count(:state=> :ALIVE) == 3}
```

in paper [13]-[14]. However, unlike CD++ which limits that each cell can only hold one state variable, ACD++ allows a cell having multi-variables. Thus, the ACTION can update several state variables at the same time. The rules in 'Game of Life‘ are specified in Table II. A cell will transform into dead if its 2 or 3 neighbourhoods are dead. If the cell is dead, it will transform into alive when 3 of its neighbourhoods are alive.

In fact, the rules specified in Table II are purely Ruby code. The vocabularies such as rule, action and condition are just functions in Ruby. Those in the braces are block parameter of Ruby. This means modellers can define any logic conformed to Ruby's syntax, which provides convenience for models with complex logics. Each cell space can be coupled with other DEVS models. 1 presents the default structure of cell space informally. The default cell space model has an input port in and output port out. On reception of external event from input port in, the received messages will be broadcast to each cell. In this way, we can communicate with the cells on the fly, providing a flexible way to influence the behaviour of cells. Besides, each cell will send its current state when its state changes. Fig.

Table III gives an example in which a cell space model is coupled with a DEVS model, which receives messages from the port out and then prints the states of the cell space. First, we add the cell space model and the observer model into the DEVS hierarchy. Then, we couple the output out of the cell


Fig. 1. The default structure of cell space
TABLE III
COUPLE THE CELL SPACE MODEL WITH OTHER DEVS MODELS

```
DEVS.simulate do
    duration 300
    add_cellspace(name: :cellspace) do
    size 20, 20
# other specification of cell space
    end
# another DEVS model
    add_model type: CellObserver, name::observer,
            with_args: [[20,20]]
        # coupled the two mode
    plug cellspace@out, with: observer@input
end
```

space with the input port in of the observer by calling the function plug and finish our specification.

In addition, to further improve the flexibility and adaptation, ACD++ provides several hooks to customize the behaviours of cell space. These hooks are listed is Table IV. Modellers can customize the behaviours of cell space like any other DEVS models.

Despite that we simplify the definition of Cell-DEVS models, we encourage the modeler to extend our DSL to introduce their own specific vocabulary. In this way, the end user can focus on the specified domains. To extend our DSL,

TABLE IV
PROVIDED HOOKS TO CUSTOMIZE THE BEHAVIOURS OF CELLS

| hooks | description |
| :--- | :--- |
| cell_control(\&block) | To customize the behavior of each cell. The block <br> will be executed when a cell is instantiated. You can <br> customize the cells behavior in the block the same as <br> other DEVS models. |
| To customize the structure of cell space model. The |  |
| execute_before_cells- | block will be executed before all cells established. You <br> can add sub-models, ports or coupling relations in the <br> block for the space model. |
| _be_established-(\&block) |  |
| execute_after_cells- | To customize the structure of the cell space model. The <br> block will be executed after all cells established. |

the modellers should define the specified vocabulary based on our provided vocabulary, and then ACD++ provides a way to integrate them into our modelling architecture.

## B. Implementation

Fig. 2 shows the modelling architecture of ACD++. The CellModel class represents the class of cells, and it is an atomic model. The CellSpace class represents the class of cell space and it is a coupled model.


Fig. 2. Modelling architecture
Each cell is associated with a delay. According to the type of delay, the cell model can be divided into two kinds: transport and inertial, which is represented by TDCellModel and IDCellmodel in Fig. 2. Fig. 3 presents the informal specification of a cell with transport delay and inertial delay. When received an external event, the local rules are activated. The result of this computation will be delayed during delay time units. To do so, an internal event is scheduled. For a cell with transport delay in Fig 3 (a), a queue is used to preserve the result of computation. During the time of delay, new external event can arrive. But for a cell with inertial delay shown in Fig 3 (b), the result of computation is saved in $f$. External event during the delay may change the value of $f$.

Paper [9] proved these two kinds of models are both consistent with DEVS specification.

To build a simulation, we propose an internal DSL aiming to make the procedure of modelling and simulation more convenient and flexible. Our implementation is on the basis of DEVS-Ruby, which provides a DSL for DEVS modelling


Fig. 3. Informal specification of a cell with transport delay and inertial delay. (a) transport delay (b) inertial delay
and simulation, whereas our work mainly focus on the modelling of Cell-DEVS models.

As seen in Table III, the function simulate, defined under the DEVS namespace, serves as the entry point into our DSL. It will do two things. First, it instantiates a SimulationBuilder, which is responsible to instantiate the root coordinator and root model. Then, it will execute the given block within the context of the builder.

The entry point of our DSL proposed for Cell-DEVS models is the function add_cellspace. Similarly, this will do three things. First, it will instantiate a CellBuilder, which is responsible to instantiate the cell space model and its associated processor. Then it will execute the given block within the context of the builder to complete the specification. Finally, it instantiates the cells and their associated processor according to the specification.

In addition, to improve the flexibility and adaptability of modelling, we adopt several hooks which has been listed in Table IV. The detailed procedures when establishing a CellDEVS model are depicted in Table V.

The relationship among builders is shown in Fig. 4. The AtomicBuilder and CoupledBuilder are responsible for the building of atomic model and coupled model. The SimulationBuilder is inherited from CoupledBuilder, and it defines some vocabulary relevant to simulation and is responsible for the building of root model. The CellBuilder is inherited from CoupledBuilder, which is responsible for the building of Cell-DEVS models. Through the Mix-In mechanism of Ruby, the function add_cellspace can be mixed into CoupledBuilder, thus each coupled model can add CellDEVS model as its child model.

One of our strengths of our proposed DSL lies in its extensibility. We encourage the modellers to extend our DSL to introduce their own specific vocabulary. To extend our DSL, the modeller should define a builder inherited from our CellBuilder, which is responsible to instantiate the model and its associated processor. Then, the modeller defines the

TABLE V
PROCEDURES OF ESTABLISHING A CELL-DEVS MODEL

```
1. the function add_cellspace is called with
    a block parameter;
2. Instantiate the CellBuilder, and invoke
    its constructor function using the given
    block parameter;
    2.1 Instantiate a coupled model as the
        cell space as well as its processor;
    2.2 Execute the block within the context
        of CellBuider and complete the spec-
        ification;
    2.3 Invoke the before_establish hook to
        customize the structure of cell space
        model, which is described in hook
        execute_before_cells_be_established;
    2.4 Instantiate the cell model as the
        child model of cell space according
        to specification;
    2.5 Establish the coupling relation among
        the child models;
    2.6 Invoke the after_established hook to
        customize the structure of cell space
        model.
3. Finish the specification.
```



Fig. 4. The relationship among builders

1. Declare the TerrainBuilder inherited from CellBuilder, and introduce the specified vocabulary as the instance methods, for example, side_length.
2. Define the entry point of DSL; module Terrain
def add_terrain_model(name, \&block)
TerrainBuilder. new(name: name, \&block)
end
end
3. Mix the function add_terrain_model into the
CoupledBuilder.
DEVS: : CoupledBuilder.send :include Terrain
specified vocabulary as the instance methods of this builder. Third, the modeller should define an entry point similar to the function add_cellspace, which is responsible to instantiate the CellBuilder. Similar to the provided function add_cellspace, the entry point should also be mixed into CoupledBuilder so that each coupled model can add the specified model as its child model. An example which is used to extend out DSL to describe the terrain model is illustrated in Table VI.

On benefit of the abstract simulation mechanism of DEVS, we just utilize the simulation algorithms implemented in DEVS-Ruby [11], so that we can focus on the modelling of Cell-DEVS models.

## IV. Use Case Study

One of the strength of ACD++ lies in that it provides an internal DSL implemented in Ruby. Modellers can benefit from the convenience brought by DSL, while defining complex logics using Ruby. Another strength lies in the extensibility, allowing it to be extended to the specified domain. This section will illustrate a well-known model for fire propagation of forest fires [13], [17]. This model uses environmental and vegetation conditions to infer the fire spread ratio. To simplify the model, we only take the wind into account here. The state variable of each cell is a continuous variable which represent the time when the cell begin to burn. The fuel model, the speed and direction of the wind, terrain of topology and dimensions of cell are used to get the spread ratio in each direction. For instance, if the current is not burning (ignite-time $==0$ ), and the northeast neighbourhood $(1,-1)$ has started to burn (whose ignitetime $>0$ ), the state variable ignite-time will be set to the result of the ignite-time of $(1,-1)$ plus distance of the two cells

## TABLE VII

FIRE SPREADING BUILDER
TABLE VIII
SPECIFICATION OF THE FIRE PROPAGATION SIMULATION

```
class FireSpreadBuilder < CellBuilder
    #east spreading ratio
    def east_speed(speed=nil)
        @east_speed=speed unless speed.nil?
        @east_speed
    end
    #other directions of spreading ratio are
        omitted out
    def fire_behavior
        #east_spread
        rule(
            action{igniteTime(cell_igniteTime_at(
            [1,0])+ side_length_of_cell/
            east_speed)},
        delay {side_length_of_cell/east_speed},
        condition{ igniteTime==0 and
            cell_igniteTime_at([1,0])>0}
            )
        # north_east_spread
        rule(
        action{igniteTime(cell_igniteTime_at(
            [1,-1]) +Math.sqrt(2) *side_length-
            _of_cell/north_east_speed)},
        delay {Math.sqrt(2)*side_length_of_cell/
            north_east_speed},
        condition{ igniteTime==0 and cell_ignite-
            Time_at ([1,-1])>0}
                )
        #north, north_west, west, south_west,
        # south, and south_east
    end
end
```

divided by north-east spreading ratio. To demonstrate the extensibility of our DSL, we will extend our DSL for this fire propagation simulation. As mentioned in section III, we first define a builder inherited from CellBuilder. As seen in the Table VII, we declare a builder FireSpreadBuilder which defines the specified vocabulary as its instance methods. For instance, the function fire_behavior defines the rules which are used to update their state. For instance, the first rule defined in the function fire_behavior indicates that when the cell is not burning or burned (ignite-time $==0$ ) and the east neighbourhood ( 1,0 ) is burning (ignite-time $>0$ ), it will become burning after the delayed time (using distance divided by east spread ratio). Due to the limitation of space, we did not give all the instance methods.

Last, we should define the entry point, function $a d$ d_fire_spread_model, which is responsible to instantiate the FireSpreadBuilder, and mix it into the CoupledBuilder. Thus, we can add this fire propagation model as the child model of any coupled model. Finally, we finish specification of our simulation using the extended DSL in Table VIII.

```
DEVS.simulate do
    duration 30
    add_fire_spread_model(name: :fire_model) do
        size 20,20
        states :igniteTime=> [0,inf]
        neighbor_type :moore, 1
        border :constant, igniteTime: 0
        init_with_value igniteTime: 0
        init_with_maps [9,9]=>{igniteTime: 1}
        north_speed 5.106976
        north_west_speed 17.967136
        west_speed 5.106976
        south_west_speed 1.872060
        south_speed 1.146091
        south_east_speed 0.987474
        east_speed 1.146091
        north_east_speed 1.872060
        side_length_of_cell 15.24
fire_behavior
    end
# add the observer
    add_model type: CellObserver, name::observer
# couple the two models
    plug "fire_model@out", with:"observer@input"
end
```

As is shown is Table VIII, the length of simulation is set to 30 . The first cell to start burning is at $(9,9)$ at time 1 , and Fig. 5 gives the 4 snapshots during the simulation.

Compared with CD++, one of strengths of our proposed DSL lies in its extensibility, which providing a good way to distinguish the role of modeller from the end-user. Thus the end-user does not have to know a lot about the library but focus on the domain knowledge.

## V. CONCLUSION

This paper proposes a modelling and simulation library, ACD++, based on Cell-DEVS specification. The start point of our work is to allow the modelling of cellular models more flexible and adaptive. To achieve this, we propose an internal DSL allowing to easily express the modelling specification of Cell-DEVS. By using the Cell-DEVS specification, the cell model can be easily coupled with other DEVS models, providing a flexible way to influence the behaviour of cells. In addition, the internal DSL implemented in Ruby determines that modeller can easily define complex logics using Ruby.


Fig. 5. snapshots during the simulation

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