Performance Analysis of Distance-based Inter-cell Interference Coordination in Small Cell Networks

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***Abstract*—In this paper we propose a distance-based Inter-Cell Interference Coordination (ICIC) scheme for Small Cell Networks (SCNs). While most of the previous works focus on a typical user, not an edge user, and do not consider the spatial distribution of the edge users, we carefully consider the spatial distribution of edge users and then apply a distance-based ICIC scheme only to the edge users, where small cell base stations within a certain radius from each edge user cooperate to improve the coverage probability of edge users. With the help of the stochastic geometry theory we analyze the performance of the proposed ICIC scheme and derive the optimal ICIC cooperation radius.**

I. INTRODUCTION

Despite many advantages of Small Cell Networks (SCNs), co-channel interference among base stations due to the full frequency reuse still limits the attainable performance of SCNs. Cross-tier interference between small cells and macro cells can be avoided with split-spectrum assignment [2], but a more technical interference management scheme is required to avoid co-tier interference among Small cell Base Stations (SBSs).

The inter-cell interference has been recognized as the main bottleneck since 3GPP release 8 and there have been several approaches to manage inter-cell interference. Inter-Cell Interference Coordination (ICIC) is one of them [1]. ICIC can be divided into two types. One is cell-centric ICIC and the other is user-centric ICIC. Under cell-centric ICIC, the resource management is done by a pre-designed fixed frequency reuse pattern. On the other hand, in user-centric ICIC the resource management is coordinated based on the actual locations of users through multi-cell cooperation [3]. This is more complicated and needs more signaling overhead than cell-centric ICIC, but it is suitable for SCN environment because it can cope with the dynamics of user locations well and there are various efforts to overcome overhead in 5G networks [4].

Recently, since a cellular network model based on stochastic geometry has been proposed [5], more and more cellular network analyses have been based on the locations of randomly distributed Base Stations (BSs) and users [6]. The Homogeneous Poisson Point Process (HPPP) model plausibly captures the irregular network topology of practical networks, especially SCNs, and gives mathematically tractable results. ICIC has been also analyzed using stochastic geometry. In [7], the authors analyzed fractional frequency reuse schemes which is a type of cell-centric ICIC. In [3], the authors proposed and analyzed a user-centric ICIC that uses the signal strength order from the viewpoint of an ICIC-scheduled user. In [2], the authors proposed and analyzed a user-centric ICIC in an SCN where SBSs that send interfering signals with strength over a certain threshold to an ICIC-scheduled user stop using the allocated resource.

Most of the previous works consider a typical user, not an edge user, and analyze the coverage probability improvement of the typical user by ICIC. Noting that the main purpose of ICIC is to improve not a typical user but an edge user who experiences performance degradation by interference from neighboring SBSs, we carefully define edge users in an SCN based on their coverage probabilities and apply our distance-based ICIC scheme only to the edge users. We then analyze and investigate the coverage probability improvement. Taking into account fundamental trade-off between the proposed ICIC scheme and resource efficiency, we derive the optimal ICIC cooperation radius.

The rest of the paper is organized as follows: We explain our ICIC scheme in Section II. Section III presents the network model. We derive the coverage probability of an edge user with and without ICIC and the resource efficiency under ICIC in Section IV. We validate our model and investigate the performance improvement through numerical and simulation study in Section V.

II. THE INTERFERENCE COORDINATION SCHEME

We propose a user-centric ICIC scheme using distance-based cooperation to enhance the edge user performance in a network consisting of a number of small cells. Each small cell has one SBS and SBSs in the network are connected via a backhaul to exchange the information for interference coordination.

A. Edge User in SCN

In SCNs, since the activation of an SBS changes dynamically depending on whether it has a user to server or not, the fact that a user is located near a small cell boundary does not always mean that the user experiences performance degradation due to interfering neighbor SBSs. Therefore, an edge user in an SCN need to be defined by considering whether neighbor SBSs are acutally active or not. From this perspective, we define an edge user as follows.

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978-1-5386-2723-5/17/$31.00 ©2017 IEEE

We consider a user in an SCN and its associated SBS that is the nearest SBS to the user. We now define the degree of edge for the user as the ratio of the distance from the user to its associated SBS to the distance from the user to the nearest active neighbor SBS to the user.

We next consider a threshold to classify the users into edge users and interior users based on the degree of edge. That is, a threshold η is given such that the user is called an edge user if the user has the degree of edge greater than or equal to η. So η is called the edge threshold from now on.

We will later explain how to determine the edge threshold η based on a given coverage probability criterion.

B. Description of the interference coordination scheme

In the proposed distance-based ICIC scheme we assume the following.

1) Cross-tier interference between SBSs and their macro base stations is avoided with split-spectrum assignment. Therefore, our concern is co-tier interference coordination among SBSs.
2) Each user is associated with its nearest SBS, called the associated SBS, and thus small cells are constructed as the voronoi diagram generated by SBSs.
3) If there are no users in a small cell, the SBS of the small cell is called an inactive SBS. Otherwise, the SBS is called an active SBS. Note that each inactive SBS does not transmit any signal because there are no users to serve.
4) Intra-cell Time Division Multiple Access (TDMA) is adopted. Each active SBS randomly selects one user in its cell to allocate a Resource Block (RB) at each time slot. We tag an arbitrary time slot and focus on an RB allocated at the tagged time slot, called the tagged RB. A user selected by its associated SBS to allocate the tagged RB, is called a served user.
5) Each SBS has location information about its neighbor SBSs and can receive activation information from its neighbor SBSs through the backhaul. In addition, each active SBS can estimate the location information of its served user.
6) It is assumed that the proposed ICIC scheme is employed at the tagged time slot to enhance the coverage probability of edge users. In addition, we assume that the network information necessary for ICIC is exchanged prior to the tagged time slot. The information exchange procedure is out of the scope of this paper.
7) The edge threshold η is assumed to be given. The detailed explanation on how to determine η will be explained later.
8) The cooperation radius d for ICIC is assumed to be given. The optimal cooperation radius d will be analyzed and determined later.

Now the proposed distance-based ICIC scheme is performed as follows.

1) Each SBS exchanges its activation information with the neighbor SBSs through the backhaul prior to the tagged time slot.
2) Each active SBS uses its own location information and activation information to determine if its served user is an edge user.
3) Any active SBS with its served user being an edge user sends an ICIC cooperation request to its neighbor SBSs within the cooperation radius d from its served user via the backhaul.
4) If an active SBS with its served user being an interior user receives the ICIC cooperation request, the active SBS does not transmit any signal to its served user at the tagged time slot.

III. SYSTEM MODEL

A. Spatial Model

The locations of the SBSs are modeled by a HPPP Φ^b in $\mathbb{R}^2$ with intensity λ^b and small cells are constructed by the Voronoi diagram of the SBSs. The location of the users are modeled by an independent HPPP Φ^u in $\mathbb{R}^2$ with intensity λ^u. Without loss of generality, we consider a typical user denoted by $\mathbf{u}_o \in \Phi^u$ and located at the Cartesian origin o by Slivnyak's theorem [8]. We assume that user $\mathbf{u}_o$ is served by its associated SBS.

Let $\mathbf{x}_1$ denote the location of the nearest SBS and hence the associated SBS of $\mathbf{u}_o$ and $\mathbf{r}_1$ be the distance from user $\mathbf{u}_o$ to $\mathbf{x}_1$. Let $\mathbf{x}_{na}$ denote the location of the nearest active neighbor SBS of $\mathbf{u}_o$ and $\mathbf{r}_{na}$ be the distance from user $\mathbf{u}_o$ to $\mathbf{x}_{na}$.

B. Channel Model

The path loss exponent is given by $\alpha > 2$. It is assumed that the network is interference limited and hence the thermal noise is ignored in our analysis. For each $\mathbf{x} \in \Phi^b$, we add independent marks $\mathbf{h}_\mathbf{x} \in \mathbb{R}$ to denote the small-scale fading on the link from $\mathbf{x} \in \Phi^b$ to $\mathbf{u}_o$ at the tagged RB. $\mathbf{h}_\mathbf{x}$ is assumed to be exponentially distributed with unit mean (which corresponds to Rayleigh fading). Although the above channel assumption simplifies the analytical analysis, the presented analysis can be extended to a more general channel model including log-normal shadowing as in [5].

C. Signal to Interference Ratio

SBSs are classified into active SBSs and inactive SBSs. Since an inactive SBS does not transmit any signal, all users experience inter-cell interference only from active SBSs. So the interference without ICIC at user $\mathbf{u}_o$ is derived as follows.

For $\mathbf{x} \in \Phi^b$, define $\mathbf{A}_\mathbf{x}$ as

$$\mathbf{A}_\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ is an active SBS,} \\ 0 & \text{if } \mathbf{x} \text{ is an inactive SBS.} \end{cases} \tag{1}$$

Then the interference without ICIC at user $\mathbf{u}_o$, denoted by $\mathbf{I}_{wo}$, is given by $\mathbf{I}_{wo} = \sum\limits_{\mathbf{x}\in\Phi^b\setminus\{\mathbf{x}_1\}} \mathbf{h}_\mathbf{x}\|\mathbf{x}\|^{-\alpha}\mathbf{A}_\mathbf{x}$.

The Signal to Interference Ratio (SIR) at user $\mathbf{u}_o$ without ICIC is given by $\mathbf{SIR}_{wo} = \frac{\mathbf{h}_{\mathbf{x}_1}\mathbf{r}_1^{-\alpha}}{\mathbf{I}_{wo}}$.

When the distance-based ICIC scheme is applied, any active SBS receiving an ICIC cooperation request does not transmit any signal to its served user if its served user is not an edge user. So, when $\mathbf{u}_o$ is an edge user, the interference with ICIC, denoted by $\mathbf{I}_w$, is derived as follows.

For $\mathbf{x} \in \Phi^b$, define $\mathbf{E}_\mathbf{x}$ as

$$\mathbf{E}_\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ serves an edge user,} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

and define $\mathbf{N}_\mathbf{x}$ as

$$\mathbf{N}_\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ does not receive an ICIC cooperation request,} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Then

$$\begin{aligned} \mathbf{I}_w = & \sum_{\mathbf{x} \in \Phi^b \cap B_d^c \setminus \{\mathbf{x}_1\}} \mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x} (1 - \mathbf{E}_\mathbf{x}) \mathbf{N}_\mathbf{x} \\ & + \sum_{\mathbf{x} \in \Phi^b \setminus \{\mathbf{x}_1\}} \mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x} \mathbf{E}_\mathbf{x}, \end{aligned} \tag{4}$$

where $B_d := \{x \in \mathbb{R}^2 : \|x\| \leq d\}$. The SIR at edge user $\mathbf{u}_o$ with ICIC is given by $\mathbf{SIR}_w = \frac{\mathbf{h}_{\mathbf{x}_1} \mathbf{r}_1^{-\alpha}}{\mathbf{I}_w}$.

D. Coverage Probability

For a given coverage threshold θ,

$$\begin{aligned} & cp_{wo}(\theta \mid \cdot) := \mathbb{P}\left[\mathbf{SIR}_{wo} > \theta \mid \cdot\right] \text{ and} \\ & cp_w(\theta, d \mid \cdot) := \mathbb{P}\left[\mathbf{SIR}_w > \theta \mid \cdot\right] \end{aligned}$$

are the coverage probabilities of user $\mathbf{u}_o$ without and with ICIC given some condition $\cdot$, respectively.

IV. PERFORMANCE ANALYSIS

In this section, our goal is to analyze the coverage probabilities of edge user $\mathbf{u}_o$ without and with ICIC.

A. Coverage Probability without ICIC

For a tractable analysis which takes into account the random positioning of active SBSs, suppose that $\mathbf{A}_\mathbf{x}$ in (1) for $\mathbf{x} \in \Phi^b$ is an independent Bernoulli random variable with success probability $p^{b,a}$ where $p^{b,a}$ is the probability that a typical SBS is active. It has been shown that $p^{b,a}$ is a function of the ratio $\frac{\lambda^b}{\lambda^u}$ of two intensities of SBSs and users, given by [9]

$$p^{b,a} = 1 - \left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}. \tag{5}$$

Let $\Phi^{b,a} := \{\mathbf{x} \in \Phi^b : \mathbf{A}_\mathbf{x} = 1\}$, which is approximated by an independently thinning of Φ^b with thinning probability $p^{b,a}$. Then the intensity of active SBSs $\Phi^{b,a}$, denoted by $\lambda^{b,a}$, satisfies $\lambda^{b,a} = p^{b,a}\lambda^b$. Using (5) we have

$$\lambda^{b,a} = \lambda^b \left(1 - \left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}\right). \tag{6}$$

In what follows, we provide the analysis of the coverage probability of user $\mathbf{u}_o$ without ICIC, of which proofs are omitted here due to page limitation.

LEMMA 1: The joint probability density function (p.d.f.) of $\mathbf{r}_1$ and $\mathbf{r}_{na}$ is given by, for $r_1 < r_{na}$

$$\begin{aligned} & f_{\mathbf{r}_1, \mathbf{r}_{na}}(r_1, r_{na}) \\ & = p^{b,a} (2\pi\lambda^b)^2 r_1 r_{na} e^{-p^{b,a}\lambda^b \pi r_{na}^2} e^{-(1-p^{b,a})\lambda^b \pi r_1^2}. \end{aligned}$$

Using Lemma 1, we obtain the following theorem.

THEOREM 1: The probability p_{int}^u that user $\mathbf{u}_o$ is an interior user and the probability p_{edg}^u that user $\mathbf{u}_o$ is an edge user, are explicitly given by

$$p_{int}^u = \frac{\eta^2}{1 - (1 - \eta^2)\left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}}, \tag{7}$$

$$p_{edg}^u = \frac{(1 - \eta^2)\left(1 - \left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}\right)}{1 - (1 - \eta^2)\left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}}, \tag{8}$$

respectively.

For later use we define a function $H(x, y)$ by

$$\begin{aligned} H(x, y) &= 2 \int_{\frac{x}{y}}^{\infty} \frac{\theta v}{v^\alpha + \theta} \, dv \\ &= \frac{\theta^{\frac{2}{\alpha}}}{\operatorname{sinc}\left(\frac{2}{\alpha}\right)} - \left(\frac{x}{y}\right)^2 {}_2F_1\left(1, \frac{2}{\alpha}; 1 + \frac{2}{\alpha}; -\frac{1}{\theta}\left(\frac{x}{y}\right)^\alpha\right) \end{aligned} \tag{9}$$

where ${}_2F_1(a, b; c; z)$ is a hypergeometric function and $\operatorname{sinc}(x)$ is a sinc function.

The next theorem provides the coverage probability of edge user $\mathbf{u}_o$ without ICIC.

THEOREM 2:

$$\begin{aligned} & cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) \\ & = (2\pi\lambda^b)^2 \frac{p^{b,a}}{p_{edg}^u} \int_0^\infty \int_{r_1}^{\frac{r_1}{\eta}} \frac{r_1 r_{na}}{1 + \theta\left(\frac{r_1}{r_{na}}\right)^\alpha} \\ & \quad \times e^{-\pi\lambda^b\left(\left(p^{b,a}(H(r_{na}, r_1) - 1) + 1\right) r_1^2 + p^{b,a} r_{na}^2\right)} \, dr_{na} \, dr_1 \end{aligned}$$

where $p^{b,a}$, p_{edg}^u and $H(x, y)$ are given in (5), (8) and (9), respectively.

To discuss how to determine the edge threshold η, we here propose to use a coverage probability criterion ϕ that is assumed to be given *a priori*. The coverage probability criterion means an allowable coverage probability for a user without ICIC, i.e., if the coverage probability of a user is smaller than ϕ, then the user experiences a severe difficulty in communication. We now consider the coverage probability of an edge user without ICIC, i.e, $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right)$. Note first that this coverage probability is decreasing in η. Next, recall that our ICIC scheme is applied to improve the coverage probability of an edge user. So it is reasonable to define an edge user as a user whose coverage probability is severely small and needs to be improved. From this viewpoint, for a given coverage probability criterion ϕ we determine η that satisfies

$$\phi = cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right).$$

B. Some Thinning Probabilities

Recall that each active SBS randomly selects one user in its cell to allocate the tagged RB. To approximate the locations of served users, we need to compute the probability that a user is selected by its associated SBS, denoted by $p^{u,s}$, and hence actually served at the tagged RB. Observing that each active SBS selects one user in its cell, $p^{u,s}$ is given by

$$p^{u,s} = \lim_{W \uparrow \mathbb{R}^2} \mathbb{E}\left[\frac{\Phi^{b,a}(W)}{\Phi^{u}(W)}\right],$$

where W denotes a square of area w^2 centered at the origin. The following theorem shows that $p^{u,s}$ is expressed in terms of the intensities of active SBSs and users, which is also derived in [9]. As before, we omit the proofs of the following theorems due to page limitation.

THEOREM 3:

$$p^{u,s} = \frac{\lambda^{b,a}}{\lambda^{u}}. \tag{10}$$

Next, for later use we derive the probability that a typical SBS does not receive any ICIC cooperation request, denoted by p_{on}. To calculate p_{on} in a tractable way, the locations of served users who are edge users, denoted by $\Phi^{u,s}_{edg}$, are approximated by an independently thinning of Φ^u with thinning probability $p^u_{edg} p^{u,s}$. It then follows that the intensity of $\Phi^{u,s}_{edg}$, denoted by $\lambda^{u,s}_{edg}$, is given by $\lambda^{u,s}_{edg} = p^u_{edg} p^{u,s} \lambda^u$. Using (6), (8), and (10) we have

$$\lambda^{u,s}_{edg} = \frac{\lambda^b (1-\eta^2)\left(1 - \left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}\right)^2}{1 - (1-\eta^2)\left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}}. \tag{11}$$

Since a typical SBS receives an ICIC cooperation request if there exist edge users served by their associated SBSs within a distance d from itself, the probability p_{on} is given by

$$p_{on} = \mathbb{P}_o\left[\Phi^{u,s}_{edg}(B_d \setminus V) = 0\right]$$

where $\mathbb{P}_o[\cdot]$ denotes the Palm probability [8] for a typical SBS Φ^b and V is the voronoi cell of the typical SBS. The reason why the voronoi cell of the typical SBS is removed in the probability is that the typical SBS obviously receives ICIC cooperation request by an edge user served by another SBS in another cell, not its own cell.

Because of random cell shape, we approximate the probability p_{on} using the largest disk included in V as follows. Let $\mathbf{r}_m$ be the radius of the largest disk included in V. The tail distribution of $\mathbf{r}_m$ is given in [10] by $\mathbb{P}[\mathbf{r}_m > r] = e^{-4\pi\lambda^b r^2}$.

Using p.d.f. of $\mathbf{r}_m$ denoted by $f_{\mathbf{r}_m}(r)$ and given by

$$f_{\mathbf{r}_m}(r) = 8\pi\lambda^b r e^{-4\pi\lambda^b r^2}, \tag{2}$$

the following result is obtained.

THEOREM 4: A lower bound of p_{on}, denoted by $\underline{p_{on}}$, is given by

$$\begin{aligned} \underline{p_{on}} = & e^{-\lambda^{u,s}_{edg}\pi d^2} \frac{4\lambda^b}{4\lambda^b - \lambda^{u,s}_{edg}} \left(1 - e^{-\left(4\lambda^b - \lambda^{u,s}_{edg}\right)\pi d^2}\right) \\ & + e^{-4\lambda^b \pi d^2} \end{aligned} \tag{12}$$

where $\lambda^{u,d}_{edg}$ is given in (11).

In Section V, we will verify that the simulation results and the analytical results using $\underline{p_{on}}$ instead of p_{on} are well matched.

C. The Coverage Probability with ICIC

From now on we assume that user $\mathbf{u}_o$ is selected by its SBS, but becomes an edge user. In this case we analyze the coverage probability of edge user $\mathbf{u}_o$ with ICIC. The interference of edge user $\mathbf{u}_o$ with ICIC is given in (4).

When a typical SBS is active, i.e., $\mathbf{A}_\mathbf{x} = 1$, it randomly selects a user in its cell and a user is an edge user with probability p^u_{edg}. So, given that $\mathbf{A}_\mathbf{x} = 1$, $\mathbf{E}_\mathbf{x}$, defined in (2), is a Bernoulli random variable with success probability p^u_{edg}. For analysis, $\mathbf{N}_\mathbf{x}$, defined in (3), is approximated by a Bernoulli random variable with success probability p_{on}, independent of the others, given that $\mathbf{A}_\mathbf{x} = 1$ and $\mathbf{E}_\mathbf{x} = 0$, i.e., the SBS is active and serves an interior user.

Let $\Phi^{b,a}_{edg} := \left\{\mathbf{x} \in \Phi^b : \mathbf{A}_\mathbf{x}\mathbf{E}_\mathbf{x} = 1\right\}$ and $\Phi^{b,a}_{int,on} := \left\{\mathbf{x} \in \Phi^b : \mathbf{A}_\mathbf{x}(1-\mathbf{E}_\mathbf{x})\mathbf{N}_\mathbf{x} = 1\right\}$. By the above assumption, $\Phi^{b,a}_{edg}$ is approximated by an independent thinning of $\Phi^{b,a}$ with thinning probability p^u_{edg} and $\Phi^{b,a}_{int,on}$ is approximated by an independent thinning of $\Phi^{b,a}$ with thinning probability $p^u_{int} p_{on}$. It then follows that the intensity of $\Phi^{b,a}_{edg}$, denoted by $\lambda^{b,a}_{edg}$, satisfies that $\lambda^{b,a}_{edg} = p^u_{edg}\lambda^{b,a}$. Similarly, the intensity of $\Phi^{b,a}_{int,on}$, denoted by $\lambda^{b,a}_{int,on}$, satisfies that $\lambda^{b,a}_{int,on} = p^u_{int} p_{on} \lambda^{b,a}$.

From the above assumption, the coverage probability of edge user $\mathbf{u}_o$ with ICIC is derived as follows:

THEOREM 5: $cp_w(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta)$ is equal to (14).

D. The Optimal Cooperation Radius

In this section we consider the resource efficiency of our ICIC scheme which is defined by

$$R_{eff}(d) := \lim_{W \uparrow \mathbb{R}^2} \mathbb{E}\left[\frac{\Phi^{b,a}_{edg}(W) + \Phi^{b,a}_{int,on}(W)}{\Phi^{b,a}(W)}\right]$$

where W denotes a square of area w^2 centered at the origin.

In what follows we derive a closed form expression of the resource efficiency.

THEOREM 6:

$$R_{eff}(d) = 1 - p^u_{int}\left(1 - p_{on}\right)$$

where p^u_{int} and p_{on} are given in (7) and (12), respectively.

COROLLARY 1: $\lim_{d \to \infty} R_{eff}(d) = p^u_{edg}$.

It is obvious that the coverage probability of an edge user increases as the cooperation radius increases while the resource efficiency decreases as the cooperation radius increases. This is a trade-off between the coverage probability and the resource efficiency. From this viewpoint, to determine the optimal cooperation radius in our ICIC scheme we have to consider both the coverage probability and the resource efficiency, which results in the following optimization problem on the cooperation radius.

$$d^* = \operatorname*{arg\,max}_{d>0} cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) \times R_{eff}(d). \tag{13}$$

Since it is difficult to solve the optimization problem directly, we cannot derive a closed-form expression of d^*, but we can determine d^* numerically as shown in the following section.

V. NUMERICAL EVALUATION AND SIMULATIONS

In this section, we provide numerical results based on our analysis and simulation results for the proposed ICIC scheme using MATLAB. Let $W \subset \mathbb{R}^2$ be a finite square observation window whose edge length is 4000 m. For each realization, the SBSs are distributed on W as an HPPP with intensity λ^b and users are distributed as an HPPP with intensity λ^u. 10^5 realizations are averaged to obtain the simulation results. The approximations and numerical evaluation of the integrals in our analysis are validated by simulations.

A. Coverage Probability without and with ICIC

From now on, the path loss exponent, the intensity of SBSs and the intensity of users are fixed to be $\alpha = 4$, $\lambda^b = 10^{-3}$, and $\lambda^u = 10^{-2}$, respectively.

We consider the coverage probability without ICIC of an edge user and plot our analytical and simulation results in Fig. 1. The figure shows that the simulation results coincide with the analytical results. From the figure we also see that, as η increases, the coverage probability decreases and the decrease becomes more significant for high values of the coverage threshold θ.

We next consider the coverage probability of an edge user when our ICIC scheme is applied. The analytical and simulation results in this case are plotted in Fig. 2 with $\eta = 0.9$. From the figure we see that both results coincide. Moreover, we see that, as the cooperation radius d increases, the coverage probability of an edge user increases as we can easily expect.

B. Resource efficiency of ICIC

We consider the resource efficiency of our ICIC scheme analyzed in section IV-D. The analytical and simulation results are plotted in Fig. 3. While Fig. 2 show that the coverage probability increases as the cooperation radius d increases, Fig. 3 shows that the resource efficiency decreases as the cooperation radius d increases. This shows a trade-off between the coverage probability and the resource efficiency.

C. Optimal Cooperation Radius and Performance Improvement

In this section, based on our observation on the trade-off between the cooperation radius and the resource efficiency we consider the optimization problem to determine the optimal cooperation radius given in section IV-D. In numerical and simulation experiment we set the coverage probability criterion ϕ to be 0.2 for $\theta = 0$ dB. In this case, the edge threshold η is set to 0.9 based on the coverage probability criterion and the intensity of users. Fig. 4 plots the objective function given in (13) when $\theta = 0$ dB. As can be seen in the figure, the optimal cooperation radius d^* is about 31 m. We finally investigate the benefit of our ICIC scheme. In Fig. 1 and Fig. 2, we see that when the coverage threshold $\theta = 0$ dB, the edge threshold $\eta = 0.9$ and the cooperation radius $d = 31$ m, the coverage probability of an edge user without our ICIC scheme is about 0.2, while the coverage probability of an edge user with our ICIC scheme is about 0.5, which shows that our ICIC scheme significantly improves the coverage probability of an edge user.

VI. CONCLUSIONS

We proposed a distance-based ICIC scheme in a small cell network to improve the coverage probability of an edge user.

$$\begin{aligned}
&cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = (2\pi\lambda^b)^2 \frac{p^{b,a}}{p^u_{edg}} \\
&\times \left(\int_{\eta d}^{d} \int_{d}^{\frac{r_1}{\eta}} \left(1 + (1 - p_{on}) p^u_{int} \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}\right) \frac{r_1 r_{na}}{1 + \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}} e^{-\pi\lambda^b\left(\left(p^{b,a}\left((p^u_{edg} + p^u_{int} p_{on}) H(r_{na}, r_1) - 1\right) + 1\right) r_1^2 + p^{b,a} r_{na}^2\right)} \, dr_{na} \, dr_1 \right. \\
&+ \int_{d}^{\infty} \int_{r_1}^{\frac{r_1}{\eta}} \left(1 + (1 - p_{on}) p^u_{int} \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}\right) \frac{r_1 r_{na}}{1 + \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}} e^{-\pi\lambda^b\left(\left(p^{b,a}\left((p^u_{edg} + p^u_{int} p_{on}) H(r_{na}, r_1) - 1\right) + 1\right) r_1^2 + p^{b,a} r_{na}^2\right)} \, dr_{na} \, dr_1 \\
&+ \int_{0}^{\eta d} \int_{r_1}^{\frac{r_1}{\eta}} \left(1 + p^u_{int} \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}\right) \frac{r_1 r_{na}}{1 + \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}} e^{-\pi\lambda^b\left(\left(p^{b,a}\left(p^u_{edg} H(r_{na}, r_1) + p^u_{int} p_{on} H(d, r_1) - 1\right) + 1\right) r_1^2 + p^{b,a} r_{na}^2\right)} \, dr_{na} \, dr_1 \\
&\left. + \int_{\eta d}^{d} \int_{r_1}^{d} \left(1 + p^u_{int} \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}\right) \frac{r_1 r_{na}}{1 + \theta \left(\frac{r_1}{r_{na}}\right)^{\alpha}} e^{-\pi\lambda^b\left(\left(p^{b,a}\left(p^u_{edg} H(r_{na}, r_1) + p^u_{int} p_{on} H(d, r_1) - 1\right) + 1\right) r_1^2 + p^{b,a} r_{na}^2\right)} \, dr_{na} \, dr_1 \right)
\end{aligned}$$

where $p^{b,a}$, p^u_{int}, p^u_{edg}, p_{on}, and $H(x, y)$ are given in (5), (7), (8), (12), and (9), respectively.

(14)

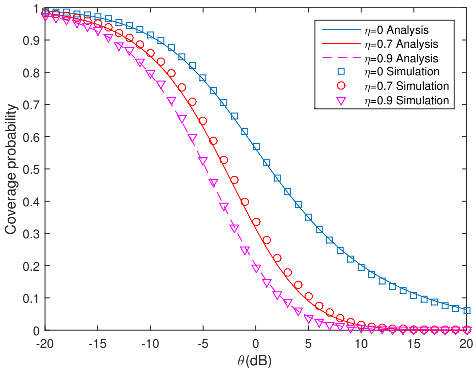

Fig. 1. The validation of the coverage probability without ICIC

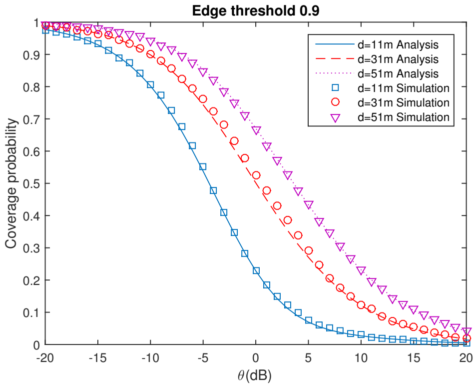

Fig. 2. The validation of the coverage probability with ICIC

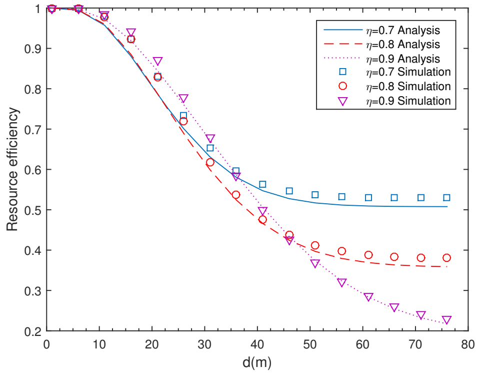

Fig. 3. The validation of the resource efficiency of ICIC

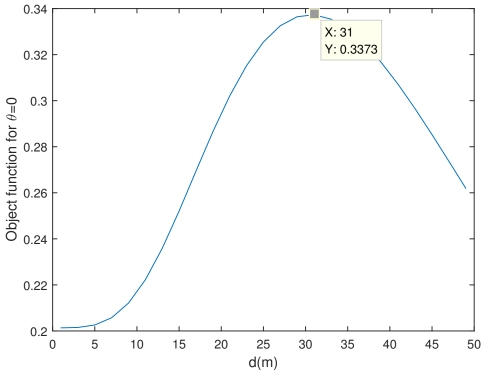

Fig. 4. The objective function when $\theta = 0$

We also developed an analytical framework to analyze the performance of the proposed ICIC scheme. In our analytical framework we defined edge users based on their coverage probabilities and considered the spatial distribution of edge users. Using the analytical framework, we derived semi-closed expressions of the coverage probabilities for an edge user without and with the proposed ICIC scheme. Considering the trade-off between the ICIC cooperation radius and the resource efficiency we formulated an optimization problem and obtained the optimized ICIC cooperation radius. Our analysis was validated through numerical and simulation study.

ACKNOWLEDGMENT

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(NRF-2017R1A2B4008581).

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