COMMENTS ON MULTI-CARRIER AND SINGLE-CARRIER DIGITAL MODULATION IN A MULTIPATH RADIO CHANNEL

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Abstract In this paper, an analytical comparison of single-carrier (SC) and multi-carrier (MC) transmission is presented for binary signalling on slow fading mobile radio channels defined by Rayleigh, Rician, and one-sided normal distributions. For the same rate of transmission and signal bandwidth, it is demonstrated that the SC transmission system is superior to the MC system on the slow fading multipath channel.

I. Introduction

One of the main issues in the application of digital modulation to radio communications is the choice of multiplexing/multiple access techniques. Debates on the benefits of single carrier (SC) transmission over multi-carrier (MC) transmission have led to the choice of the SC system in several applications. On the other hand, there has also been a renewed interest in multi-carrier modulation (MCM) [1], a form of FDM. It is indicated in [1] that MCM out-performs single carrier modulation in the case of flat fast fading, since the fading signal is integrated over a longer symbol interval.

In this paper, a comparison of SC and MC transmission on multipath slow fading radio channels is presented. For given transmitted power, rate of transmission and channel bandwidth, it is demonstrated that on such channels the SC transmission system is superior to the MC system. For the sake of simplicity, the comparison is carried out for binary signalling, although the conclusions can be extended to M-ary signalling as well.

The slowly varying quasi-stationary multipath radio channel is modeled by a complex-valued Gaussian random process, with independent quadrature components characterized, generally, by different mean values, m_x and m_y, and different variances σ_x^2 and σ_y^2. This generalized Gaussian channel model (GGM) leads to a four-parameter probability density function of the received signal envelope [2]. Various combinations of the values of the parameters m_x, m_y, σ_x^2, and σ_y^2 generate well-known channel models (Rayleigh, Rician, one-sided normal, etc.) as particular cases of the four-parameter distribution.

In addition to the channel distortion resulting from the time-variant multipath, the signal on each path is corrupted by an additive noise which is modeled as a zero mean, white Gaussian random process with two-sided power spectral density $N_0/2$. The additive noises on the L paths are assumed to be mutually statistically independent.

The error performance of the MC system with N carriers in a multipath environment is evaluated for the limiting case, when the channel symbol duration T_N is much greater than the multipath spread, $T_N >> T_m$.

The error performance of the SC system in the multipath environment is evaluated for the case, when the channel symbol duration T_1 (T_N/N) satisfies the condition $T_1 < \Delta\tau_{min}$, where $\Delta\tau_{min}$ is the minimum differential delay between the paths.

Coherent symbol-by-symbol detection with an observation interval $T_0 = (1 + D)T_1$, where D is the decision delay (equal to the channel memory), is employed, and the decision feedback equalizer eliminates the intersymbol interference from previously detected symbols.

The performances of the MC system and the SC system are compared as a ratio of SNRs required to maintain the same average error probability in the two systems. The channel fading is assumed to be sufficiently slow in order to obtain perfect channel state information (CSI), including the estimate of the phase of the received signal, and the delays of the paths.

Sections II and III of this paper contain the performance evaluation of the MC system and the SC system, respectively. The performance comparison of MC and SC systems is presented in Section IV, and Section V contains a discussion.

II. Performance Evaluation of a Multi-Carrier Transmission System

Each propagation path of the multipath channel (with L paths) is characterized by a time-variant complex-valued transfer function

$$H_\ell(f;t) = x_\ell(f;t) + j\, y_\ell(f;t), \quad \ell = 1,2,\ldots,L \qquad (1)$$

where the quadrature components $x_\ell(f;t)$ and $y(f;t)$ are Gaussian random processes in the t variable, statistically independent, with mean values $m_{x\ell}$, $m_{y\ell}$ and variances $\sigma_{x\ell}^2$, $\sigma_{y\ell}^2$, respectively.

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$$P_{eav} = \frac{1}{\pi} \int_0^\infty \frac{\dfrac{m_{x\Sigma}^2 \lambda\gamma_b(1+t^2)}{2[1+\lambda\gamma_b L\sigma_x^2(1+t^2)]} - \dfrac{m_{y\Sigma}^2 \lambda\gamma_b(1+t^2)}{2[1+\lambda\gamma_b L\sigma_y^2(1+t^2)]}}{\{[1+(1+t^2)\lambda\gamma_b L\sigma_x^2][1+(1+t^2)\lambda\gamma_b L\sigma_y^2]\}^{1/2}} dt \quad (6)$$

When the symbol duration T_N in a MC system (with N carriers) is much greater than the multipath spread T_m of the channel, $T_N >> T_m$, it implies that the intersymbol interference (ISI) is negligible, and the channel is frequency-nonselective. All L component waves overlap almost entirely, and the observation interval T_o required for optimum symbol-by-symbol detection is equal to the symbol duration, $T_o = T_N$.

Under this condition, the error probability for binary transmission over a deterministic multipath channel with AWGN and optimum detection is given by

$$P_e = Q\left[\sqrt{\lambda \gamma_b (x_\Sigma^2 + y_\Sigma^2)}\right] \quad (2)$$

where $\gamma_b = \frac{S_{tr} T_N}{N_0}$ is the SNR, S_{tr} - the average transmitted signal power, $\lambda = 0.5$, 1, and 2 for binary ASK, FSK, and PSK, respectively, and

$$x_\Sigma = \sum_{\ell=1}^{L} x_\ell, \quad y_\Sigma = \sum_{\ell=1}^{L} y_\ell \quad (3)$$

are the quadrature components of the equivalent "single-path" channel transfer function.

In the case of slow fading the random process (1) can be considered constant for each signalling interval, i.e., x_ℓ, y_ℓ are random variables. In accordance with (3) we have

$$m_{x_\Sigma} = \sum_{\ell=1}^{L} m_{x_\ell}, \quad \sigma_{x\Sigma}^2 = \sum_{\ell=1}^{L} \sigma_{x_\ell}^2,$$

$$m_{y_\Sigma} = \sum_{\ell=1}^{L} m_{y_\ell}, \quad \sigma_{y\Sigma}^2 = \sum_{\ell=1}^{L} \sigma_{y_\ell}^2 \quad (4)$$

In order to calculate the error performance on a slowly fading channel we use the integral representation of the probability function [3] in (2)

$$Q(z) = \frac{1}{\pi} \int_0^\infty \frac{1}{1+t^2} \exp\left[-\frac{z^2}{2}(1+t^2)\right] dt \quad (5)$$

and assume that the variances of the paths are the same, $\sigma_{x_\ell} = \sigma_x$, and $\sigma_{y_\ell} = \sigma_y$.

By substituting (5) in (2), and averaging over the joint pdf of x_Σ and y_Σ, the average error probability is obtained [2] as given in (6).

From (6), the average error probability can be easily evaluated for a number of practical channels.

In the range of parameter values satisfying the conditions

$$\gamma_b \sigma_x^2 >> 1, \quad \gamma_b \sigma_y^2 >> 1 \quad (7)$$

(6) can be approximated by

$$P_{eav} \simeq \frac{1}{4\lambda\gamma_b L\sigma_x\sigma_y} \exp\left[-\frac{m_{x\Sigma}^2}{2L\sigma_x^2} - \frac{m_{y\Sigma}^2}{2L\sigma_x^2}\right] \quad (8)$$

For the Rician channel ($m_{x\Sigma}^2 + m_{y\Sigma}^2 \neq 0$, $\sigma_x^2 = \sigma_y^2 = \sigma^2$), it follows from (8) that

$$P_{eav} \simeq \frac{1}{4\lambda\gamma_b L\sigma^2} \exp\left[-\frac{m_{x\Sigma}^2 + m_{y\Sigma}^2}{2L\sigma^2}\right] \quad (9)$$

and for the Rayleigh channel ($m_{x\Sigma} = m_{y\Sigma} = 0$)

$$P_{eav} \simeq \frac{1}{4\lambda\gamma_b L\sigma^2} \quad (10)$$

For the one-sided normal channel ($m_{x\Sigma} = m_{y\Sigma} = 0$, $\sigma_x^2 = 0$, $\sigma_y^2 \neq 0$), which represents the most severe fading conditions, it follows from (6) when (7) is satisfied, that

$$P_{eav} \simeq \frac{1}{\pi\sqrt{\lambda\gamma_b L\sigma_y^2}} \quad (11)$$

When the fluctuations in the paths are small ($\sigma_x^2 << 1$, $\sigma_y^2 << 1$), the average error probability is determined by (2) if x_Σ and y_Σ are replaced by $m_{x\Sigma}$ and $m_{y\Sigma}$, respectively

$$P_{eav} \simeq Q\left(\sqrt{\lambda\gamma_b(m_{x\Sigma}^2 + m_{y\Sigma}^2)}\right) \quad (12)$$

III. Performance Evaluation of a Single-Carrier Transmission System

If we assume $T_1 < \Delta\tau_{min}$, i.e. the symbol duration is less than the minimum differential delay between paths, the lower bound on the error probability for the deterministic multipath channel in the case of optimum symbol-by-symbol detection with perfect CSI is given as

$$Pe \geq Q\left(\sqrt{\lambda \gamma_b \left[\sum_{\ell=1}^{L} \left(x_\ell^2 + y_\ell^2\right) \right]}\right) \quad (13)$$

In order to find the error performance in the slowly fading multipath channel it is necessary to average (13) with respect to x_ℓ and y_ℓ. Using (5) and the joint pdf for iid random variables $\{x_\ell, y_\ell\}$, we find

$$Pe_{av} \geq \frac{1}{\pi}\int_0^{\infty} \frac{1}{(1+t^2)} \prod_{\ell=1}^{L} \frac{\exp\left\{ -\frac{m_{x_\ell}^2 \lambda\gamma_b(1+t^2)}{2[1+\lambda\gamma_b\sigma_{x_\ell}^2(1+t^2)]} - \frac{m_{y_\ell}^2 \lambda\gamma_b(1+t^2)}{2[1+\lambda\gamma_b\sigma_{y_\ell}^2(1+t^2)]} \right\}}{\{[1+(1+t^2)\lambda\gamma_b\sigma_x^2][1+(1+t^2)\lambda\gamma_b\sigma_y^2]\}^{1/2}} dt \quad (14)$$

In the case of large SNR satisfying the condition (7), and $\sigma_{x\ell}^2 = \sigma_x^2$, $\sigma_{y\ell}^2 = \sigma_y^2$

$$Pe_{av} \geq \frac{(2L-1)!}{(2\lambda\bar{\gamma}_b)^L!\,(L-1)!} \prod_{\ell=1}^{L} \exp\left[-\frac{m_{x,\ell}^2}{2\sigma_x^2} - \frac{m_{y,\ell}^2}{2\sigma_y^2} \right] \quad (15)$$

where $\bar{\gamma}_b = 2\gamma_b\,\sigma_x\,\sigma_y$.

For the Rician channel it follows from (15) that

$$Pe_{av} \geq \frac{(2L-1)!}{(2\lambda\bar{\gamma}_b)^L\,(L-1)!} \prod_{\ell=1}^{L} \exp\left[-\frac{m_{x\ell}^2 + m_{y\ell}^2}{2\sigma^2} \right] \quad (16)$$

and for the Rayleigh channel

$$Pe_{av} \geq \frac{1}{(2\lambda\bar{\gamma}_b)^L} \frac{(2L-1)!}{L!(L-1)!} \quad (17)$$

For the one-sided normal distribution ($m_x = m_y = \sigma_x^2 = 0$) with $\gamma_b\,\sigma_y^2 >> 1$ it follows from (14) that

$$Pe_{av} \geq \frac{\Gamma\left(\frac{L+1}{2}\right)}{L\,\Gamma\left(\frac{L}{2}\right)\sqrt{\pi}\,(\lambda\,\gamma_b\,\sigma_y^2)^{L/2}} \quad (18)$$

where $\Gamma(.)$ is the gamma-function.

Under the conditions $\sigma_x^2 << 1$, $\sigma_y^2 << 1$, the average error probability can be found from (13) by substituting $m_{x_\ell}^2$ and $m_{y_\ell}^2$ for x_ℓ^2 and y_ℓ^2, respectively.

IV. Performance Comparison over Slowly Fading Multipath Channels

Employing the formulas (8) - (12) and (15) - (18), we compare the error performance of the MC and the SC transmission systems on a slowly fading multipath channel under the conditions outlined above.
Assuming the same bit rate, type of modulation, and the same channel, we compare the transmitted powers required in the two systems for the same error probability.

$$\eta = \frac{\gamma_{bN}}{\gamma_{b1}} = \frac{\exp\left[-\frac{m_{x\Sigma}^2}{2L\,\sigma_x^2} - \frac{m_{y\Sigma}^2}{2L\,\sigma_y^2} \right]}{L\,p^{\frac{L-1}{L}}} \left\{ \frac{\frac{L!(L-1)!}{(2L-1)!}}{n \prod_{\ell=1}^{L} \exp\left[-\frac{m_{x\ell}^2}{2\sigma_x^2} - \frac{m_{y\ell}^2}{2\sigma_y^2} \right]} \right\}^{1/2} \quad (19)$$

Comparing (8) and (15), the ratio of the required powers is obtained as (19) above, where p is the fixed average error probability, and γ_{bN}, γ_{b1} are the required SNR's for the MC and SC systems, respectively.

The power loss (gain) on the Rayleigh and on the Rician channels is easily obtained from (19); e.g., for the Rayleigh channel

$$\eta = \frac{1}{L\,p^{\frac{L-1}{2}}} \left[\frac{L!(L-1)!}{(2\,L-1)!}\right]^{1/L} \quad (20)$$

Comparing (11) and (18), the power ratio for the one-sided normal channel model is obtained as

$$\eta = \frac{\gamma_{bN}}{\gamma_{b_1}} = \frac{1}{\pi L\, p^{\frac{2(L-1)}{L}}} \left[\frac{L\,\Gamma\left(\frac{L}{2}\right)\sqrt{\pi}}{\Gamma\left(\frac{L+1}{2}\right)} \right]^{2/L} \qquad (21)$$

From (19) - (21), obtained under the condition (7), it follows that on a single-path (L=1) slow fading channel $\eta = 1$, and there is no difference in the performance of the two systems being compared. For $L \geq 2$, on the Rayleigh and one-sided normal fading channels η is always > 1, and the SC system requires less power than the MC system. Numerical results (power gain in dB for the SC system) for these two channel models are presented in Table 1. The power gain increases for smaller error probabilities, as well as with an increasing number of paths (except for the case $p=10^{-2}$).

$$\eta^* = \exp\left[- \frac{\left(\sum_{\ell=1}^{L} m_{x\ell}\right)^2 + \left(\sum_{\ell=1}^{L} m_{y\ell}\right)^2 - \sum_{\ell=1}^{L} \left(m_{x\ell}^2 + m_{y\ell}^2\right)}{2L\,\sigma^2} \right] \leq 1 \qquad (22)$$

As far as the Rician channel is concerned, it can be verified by comparing (21) with (20) that the additional gain (loss) (with respect to the Rayleigh channel) associated with non-zero mean values of the quadrature components is given by (22) above.

In the most likely case, when only one path has non-zero mean values, $\eta^* = 1$, i.e., the power ratio (19) is the same as on the Rayleigh channel.

Finally, in the case of small fluctuations in the channel, it follows from (12) and (13) that

$$\eta = \frac{\sum_{\ell=1}^{L} \left(m_{x\ell}^2 + m_{y\ell}^2\right)}{\left(\sum_{\ell=1}^{L} m_{x\ell}\right)^2 + \left(\sum_{\ell=1}^{L} m_{y\ell}\right)^2} \qquad (23)$$

where η may be either less or more than one; for a channel with $L = 1$, $\eta = 1$.

V. Discussion

An evaluation of the relative performance of the MC and SC digital signalling over a slowly fading multipath channel has been presented. For a given channel, the symbol duration T_N was assumed to be much greater than the multipath spread T_m ($T_N >> T_m$) in the case of the MC system, and the symbol duration T_1 was assumed to be smaller than the minimum differential delay $\Delta\tau_{min}$ between the paths in the case of the SC system. Such an assumption is justified since with the same bit rate, channel bandwidth, and type of modulation, $T_N = N.T_1$, where N is the number of carriers in the MC system.

Under these assumptions, there is negligible ISI in the case of the MC system, and an optimum symbol-by-symbol detector with an observation interval $T_o = T_N$, is utilized. In the case of the SC system, it is assumed that perfect CSI is available and the signal shapes at the receiver are known. The ISI is dealt with by using an optimum symbol-by-symbol detector with an observation interval $T_o = (1+ D)\, T_1$ and a DFE scheme.

On a multipath channel ($L \geq 2$), the SC system significantly outperforms the MC system when the fading is modeled by the Rayleigh or one-sided normal pdf's. In the case of a Rician channel, the advantage of the SC system is the same (in the most practical case) as the Rayleigh channel.

Generally, the deeper the fading the greater is the advantage of the SC system.

Table I: The power ratio η (in dB) required for the same Pe_{av} with MC and SC binary transmission systems on the slowly fading multipath channel (L - number of paths)

	Rayleigh fading channe model, η(dB)				One-sided normal fading channel model, η(dB)			
Pe_{av} \ L	2	3	4	5	2	3	4	5
10^{-2}	4.6	5.2	5.1	4.8	13.7	16.4	17.7	18.1
10^{-3}	9.6	11.9	12.6	12.8	23.1	29.8	32.7	34.2
10^{-4}	14.6	18.6	20.1	20.8	33.1	43.1	47.7	50.2

References

1. J. A. C. Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come", IEEE Communications Magazine, May 1990, pp. 5-14.
2. D. Klovsky and B. Nikolaev, "Sequential Transmission of Digital Information in the Presence of Intersymbol Interference", Mir Publishers, Moscow 1978, p. 216.
3. I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series and Products, Academic Press, N.Y., 1965.