Fixed-time Sliding Mode Observer-based Controller for a Class of Uncertain Nonlinear Double Integrator Systems

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Abstract—This paper investigates a fixed-time convergence issue using the sliding mode observer-based controller for a class of uncertain nonlinear double integrator systems. This observer-based controller is designed assuming that only the first state measurement is available and there is no information about disturbances. A new form of sliding mode observer in combination with a sliding mode controller is designed to estimate unmeasured state and unknown disturbances as well as provide the estimated data in the control law. A novel form of sliding surfaces for the robust observer-based controller is proposed for which fixed-time convergence is guaranteed to achieve trajectory tracking. In the proposed fixed-time scheme, the bound on the settling time is user-defined using design parameters regardless of the system's initial conditions. The control law and observer law are designed such that the chattering issue is alleviated in the control signal. The stability analysis of the closed-loop system using the observer-based controller is established via the Lyapunov theory. The validity of the controller design is tested by applying and simulating an example of a robot manipulator in Simulink/MATLAB. The superiority of the proposed method is demonstrated by comparing it with two other methods from the relevant literature.

Keywords: sliding mode controller, sliding mode observer, fixed-time stability, chattering-free, Lyapunov stability.

1. INTRODUCTION

Many practical high order systems are modelled using nonlinear differential equations. Furthermore, the true system may be affected by stochastic noise, uncertainties in the parameters and variations in the external environment which are unknown beforehand [1]. This makes the control of these systems a challenge and as a result, different nonlinear control methods including the nonlinear stability theory [2], the back stepping technique [3], Lyapunov function [4] and the sliding mode control [5, 6] have come into existence.

An effective approach found in the literature for dealing with parametric uncertainties in single or double integral system is adaptive control [7, 8] and notable adaptive design method has been proposed in [9-11] for the control of high order systems. On the other hand, finite-time stability is known for its fast transient performance achievement [12]. Hence, adaptive finite time control encompasses the advantage of both control techniques. It guarantees faster convergence rates, better disturbance rejection properties, and better robustness against uncertainties [13, 14]. The downside to adaptive finite time control is the complications involved in estimating the upper bound of uncertainties and external disturbances [15, 16].

Another approach found in the literature for dealing with parametric uncertainties is the sliding mode control (SMC) [17-22]. It is a robust control approach as it is unresponsive to external disturbances and parametric uncertainties or variations [23, 24]. However, exact information about the uncertainties and the disturbances are unknown when applying the SMC. This is a downside to this approach as it might result in chattering and an unstable closed loop system [25, 26]. Terminal sliding mode control was incorporated in [27] due to its robustness to obtain adaptive finite-time convergence, fast convergence, improved transient performance and higher precision for high order systems. However some singularities were present in the controller [28]. Nonsingular terminal sliding mode control was applied in [29] to solve the singularity problem however finite time convergence was not achieved and the convergence rate to the equilibrium was slow. Integration of adaptive control with nonsingular terminal sliding mode control was proposed in [30] to tackle the problem of unknown upper boundaries in adaptive control. The resulting control laws were however discontinuous across the terminal sliding mode surface when external disturbances were involved. An adaptive fuzzy hierarchical SMC scheme has been proposed in [31] to control uncertain under-actuated switched nonlinear systems with actuator faults where the singular issue of denominators has been solved utilizing a projection algorithm, and fuzzy logic systems has been used to estimate the unknown uncertain functions of the system. An adaptive neural finite-time hierarchical SMC scheme has been suggested in [32] for uncertain under-actuated nonlinear systems with backlash-like hysteresis where unknown functions of the system have been estimated using neural networks, and the singular issue of denominators has been solved using a projection algorithm. In [33], a sliding-mode surface (SMS)-based adaptive optimal control method has been proposed for switched nonlinear systems with average dwell time.

State and disturbance observers can be alternatively utilized to produce an estimate of the system states and disturbances, respectively. The system's output and input are used as the observer input [34]. The finite-time observers including the Kalman filter and the Luenberger observer have been given [35, 36] for linear systems. The extended Kalman filter proposed for nonlinear systems was not able to deal with the problem of the system parametric uncertainty [37]. Likewise, the nonlinear observers including adaptive estimators [38], backstepping method [39], Hamiltonian method [40], and sub-Lyapunov exponents [41] could only ensure the asymptotic convergence of the estimation errors [42]. Moreover, observer design for uncertain nonlinear systems has been challenging and slow because of the presence of the singular inputs in the nonlinear systems that makes them unobservable [43]. Finite-time sliding mode observers have been proposed to deal with the estimation issue of the unknown states and parametric uncertainties in a finite time and to provide robustness features [44]. An observer-based fuzzy feedback control method has been proposed in [45] for a type of discrete-time nonlinear systems. In [46], an adaptive sliding mode observer based controller has been incorporated fixed-time stability notion to control chaotic support structures for offshore wind turbines in the presence of unknown disturbances and parametric uncertainties.

In comparison with infinite-time control, finite/fixed-time control has better robustness. However, the estimated convergence time of the finite-time control approaches relies on the initial condition of the system trajectory, which would limit its practical applications because of likely unknown initial conditions of the system. To handle this problem, fixed-time convergence was initially suggested by [47], which can ensure that the settling time is globally bounded and unrelated to the initial condition of systems. In other words, unlike finite-time stability, fixed-time stability has nothing to do with the initial value and is only associated with the selected design parameters that is a great advantage over finite-time stability in practice. In [48], the use of fixed-time stability design methods has been proposed for line control systems. In [49], the fixed-time stability notion has been incorporated with disturbance and state observer based controller for a class of high-order nonlinear systems. Furthermore, an undesired chattering issue has been seen in most control signals of SMC including the fixed-time SMC [50], finite-time SMC [51, 52], and disturbance-observer based finite-time SMC [53] because of using signum function in the control laws of SMC. This inherent chattering phenomenon of conventional SMC requires ample energy for good efficiency, and it might damage the system physical parts [54]. In [55, 56], the signum function in the controllers has been estimated by saturation or sigmoid functions resulted in alleviating chattering. However, this method creates large steady-state errors because of the boundary layer around sliding surfaces.

Motivated by the above discussions, in this paper, a fixed-time sliding mode observer-based controller for a class of uncertain nonlinear double integrator systems is designed despite unknown disturbances and assuming the availability of the first state's measurement. The stability analysis of the closed-loop systems is investigated using Lyapunov stability theory. As the separation principle does not hold for nonlinear systems, one of the challenging parts of designing the combined observer-controller is establishing the stability analysis using the Lyapunov theory [49]. So, one of the advantages of this paper is that the stability proof of the closed-loop systems is obtained using only one novel Lyapunov candidate function. Also, another key feature of the proposed observer-based controller is that the chattering issue is alleviated from SMC laws by using the integral of signum function in the control law and observer laws. The effectiveness of the proposed schemes is revealed by applying on an example of a robot manipulator (given in [57, 58]) and comparing with two other control methods from the relevant literature. In fact, our proposed chattering-free observer-based fixed-time SMC (FFSMC) is compared with the similar fixed-time method (i.e., observer-based fixed-time SMC (OFSMC)) to demonstrate the efficacy of using FFSMC in alleviating chattering and improving tracking performance over OFSMC. Note that the sliding surface of OFSMC is defined using the conventional SMC methods given in our previous publication [49]. Also, FFSMC is compared to a similar finite-time method (i.e., chattering-free observer-based Terminal SMC (FTSMC)) to reveal the efficacy of using the fixed-time stability notion over finite-time stability notion in overall performance of the proposed observer-based controller method. Note that the idea behind the SMC law of FTSMC method is similar to the finite-time control method given in [59].

Reviewing the recent literature on the different fixed-time control approaches for various applications demonstrates that each has critical drawbacks and limitations. These limitations are given in Table 1, and the solutions to deal with them using the fixed-time observer-based controller proposed in this research are provided.

Table 1. Comparing the proposed approach and the different existing fixed-time approaches.

Fixed-time methods	Limitations	Proposed Fixed-time observer-based controller
Barrier Lyapunov function design method [60] Backstepping methods [60] Adaptive method [60]	Requires the knowledge of the system's initial conditions in advance that might be unknown in practice. It is needed once the barrier Lyapunov function design scheme is used.	No information is required about the system's initial conditions, even for determining the upper bound of convergence time.
Homogeneous methods [61] Sliding mode control [61], [62], [63], [64], [65] Gradient flows method [62]	Ignoring disturbances in the control design that is not true in practice.	Considering the model of external disturbances in the control design procedure.
Homogeneous methods [61], [66], [67]	Requiring full information of the state and the upper bounds of disturbances in advance that might not be	This sensorless controller does not require full-sate information and there is no requirement for the

Sliding mode control [61], [66], [62], [63], [64], [65], [68], [69], [47], [70], [71] Backstepping methods [66], [67], [72], [73], [74], [75], [76], [77] Gradient flows method [62] Adaptive method [68], [78]	available in many practical cases.	knowledge of the upper bounds of disturbances. Disturbance and state observers estimate them and provide the estimated data in the controller.
Barrier Lyapunov function design method [60] Backstepping methods [60], [79], [80] Adaptive method [60], [79], [68], [81], [78], [82] Sliding mode control [68], [81]	The upper bound of convergence time is not presented and it is unknown after designing the controller.	The upper bound of convergence time is presented without any information of the initial conditions of the system.
Homogeneous methods [61], [66], [67] Sliding mode control [61], [66] Backstepping methods [66], [67]	The homogeneous condition must be satisfied by the dynamic equations where homogeneous schemes are used.	No requirement of satisfying the homogeneous condition.
Homogeneous methods [66], [67] Backstepping methods [66], [67], [60], [79], [72], [80], [73], [74], [75], [76], [77], [83] Sliding mode control [66], [71] Barrier Lyapunov function design method [60] Adaptive method [60], [79]	Requiring a strict feedback form for the dynamic equations where the concept of backstepping control scheme is used.	The concept of SMC used that overcome this drawback of using backstepping control scheme.
Homogeneous methods [61], [67] Sliding mode control [61], [63], [64], [68], [69], [47] Backstepping methods [67], [72], [73] Adaptive method [68]	Only applicable for the linear systems.	The proposed fixed-time method is designed for a nonlinear system that is applicable for wide range of linear and nonlinear applications.

In summary, the main contributions of this paper are listed in the following:

1. A new form of combined fixed-time stability notion, sliding mode observer, and sliding mode controller is designed for a class of uncertain nonlinear double integrator systems despite unknown disturbances and assuming that the first state's measurement of the system is only available, the other one needs to be estimated. So, this sensorless controller with disturbance rejection does not require a full-state information and a priori knowledge of the upper bounds of the external disturbances and modelling uncertainties.
2. The chattering issue is eliminated from the control signal of the proposed observer-based controller by proposing a new solution where the integral of the signum function in the control law and the observer law is used.

3. The fixed-time stability proof of the closed-loop system is obtained by choosing a proper candidate Lyapunov function (considering the fact that the separation principle does not hold for nonlinear systems) as well as the upper bound of convergence time is obtained regardless of the system's initial conditions.

This paper is organized as follows. Mathematical preliminaries used throughout the paper are given in Section 2. The control problem in this research is mathematically defined in Section 3. The design of the observers-based controller and its stability proof is given in Section 4. Section 5 gives the simulation results for an example of a robot manipulator along with the results and discussion. The conclusions of this paper are given in Section 6.

2. MATHEMATICAL PRELIMINARIES

Definition 1. The signum function is given as,

$$sign(a) = \begin{cases} 1 & ; a > 0 \\ 0 & ; a = 0 \\ -1 & ; a < 0 \end{cases} \tag{1}$$

Note that the function $sig^b(.)$ is defined in [84] as $sig^b(a) = |a|^b sign(a)$. Moreover, the following relations are always true.

$$\begin{cases} sign(a) \times sign(a) = 1 \\ a \times sign(a) = |a| \\ a \times sig^b(a) = |a|^{b+1} \\ \frac{d|u|}{dt} = \dot{u} \times sign(u)\,; \dot{u} = \frac{du}{dt} \\ |a \times sign(b)| \le |a| \end{cases} \tag{2}$$

where $a, b \in \mathcal{R}$ and u is a differentiable function.

Definition 2. Consider a nonlinear system as follows,

$$\dot{x} = f(t, x) \tag{3}$$

where $x \in \mathbb{R}^n$ is the vector of the system states; $f : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function; t is considered on the interval $[t_0, \infty)$, where $t_0 \in \mathbb{R}_+ \cup \{0\}$. The system's initial conditions are $x(t_0) = x_0$.

The origin of the system (3) is globally finite-time stable if it is globally asymptotically stable and any solution $x(t, x_0)$ of (3) converges to the origin at some finite time moment for all x_0; i.e., $\forall t \ge T(x_0)$: $x(t, x_0) = 0$, where $T: \mathbb{R}^n \to \mathbb{R}_{\ge 0}, \forall\, x_0 \in \mathbb{R}^n$, is called settling time function, then the origin of (3) is globally finite-time stable [85, 86].

Definition 3. The origin of (3) is globally fixed-time stable if it is globally finite-time stable and the settling time function T is bounded; i.e., $\exists T > 0 : T \le T_{max}, \forall x_0 \in \mathbb{R}^n$. Therefore, the settling time is always bounded regardless of the system's initial conditions in fixed-time control methods [64, 87].

Lemma 1. Consider $a_1, a_2, \ldots, a_n \in \mathbb{R}$ and $0 < \gamma < 2$, then we have [88].

$$|a_1|^\gamma + |a_2|^\gamma + \cdots + |a_n|^\gamma \ge (a_1^2 + a_2^2 + \cdots + a_n^2)^{\frac{\gamma}{2}} \tag{4}$$

Lemma 2. Consider $a_1, a_2, \ldots, a_n \geq 0$, $0 < b \leq 1$ and $c > 1$, then we have [89].

$$\sum_{i=1}^{n} a_i^b \geq \left(\sum_{i=1}^{n} a_i\right)^b, \sum_{i=1}^{n} a_i^c \geq n^{1-c}\left(\sum_{i=1}^{n} a_i\right)^c \tag{5}$$

Lemma 3. Assume there exist four real numbers as $\rho_1, \rho_3 > 0$ and $0 < \rho_2 < 1$, $\rho_4 > 1$, and a continuously differentiable positive function $V(x): \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that: $V(x) = 0$ for $x(t) = 0$. If any solution $x(t)$ of Eq. (3) satisfies the inequality $\dot{V}(x) \leq -\rho_1 V^{\rho_2} - \rho_3 V^{\rho_4}$. Then, the origin is globally fixed-time stable for the system of Eq. (3) and the settling time function is as $T(x_0) \leq \frac{1}{\rho_1(1-\rho_2)} + \frac{1}{\rho_3(\rho_4-1)}$ [87, 90].

Lemma 4 ([50]). Consider a scalar system as follows,

$$\dot{y} = -\alpha y^{\frac{p}{q}} - \beta y^{\frac{m}{n}}, y(0) = 0 \tag{6}$$

where $\alpha, \beta > 0, q > p > 0, 0 < n < m < 2n$. Then the equilibrium of (6) is fixed-time stable and the settling time T is bounded by:

$$T \leq T_{max} = \frac{1}{\alpha}\frac{m}{m-n} + \frac{1}{\beta}\frac{q}{q-p} \tag{7}$$

3. PROBLEM FORMULATION

Consider a class of uncertain double integrator systems as,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(t,x) + g(t,x_1)u + d(t,x) \end{cases} \tag{8}$$

where $x = [x_1, x_2]^T$ are the system states; x_1 is available and measurable; $f(t,x)$ and $g(t,x_1)$ are smooth nonlinear functions; $g(t,x_1)$ is bounded and invertible; i.e., $g(t,x_1) \neq 0 \; \forall \; t, x$, and $g^{-1}(t,x_1)$ is available; u is the control input; $d(t,x)$ is a model of unknown external disturbances and modelling uncertainties (that is called disturbances throughout the paper).

To design the observer-based controller in this paper, it is assumed that there is no information about disturbances as well as the second state. Hence, the observer-based-controller is designed only with the measured value of x_1. The following assumptions are used in this paper:

Assumption 1. For $x_2(t)$, we have $|x_2(t)| \leq \eta$; η is a known positive constant.

This assumption (Assumption 1) ensures that the second state of the system (e.g., manipulator's rotary joints) is bounded which is true in most physical systems (such as manipulators).

Remark 1. In practice, a wide range of physical systems can be described by a set of independent double integrator subsystems (given by (8)) particularly robotic manipulators such as the three-link robotic manipulator [84, 91], the two-link robotic manipulator [92], the robot manipulator [57, 58], etc. For the case of the robotic manipulators (as an example of the system described by a set of independent double integrator subsystems), because the second state indicates velocities of joints, Assumption 1 seems to be necessary and rational for the industrial robot manipulators due to safety reasons and practical restrictions [93]. For industrial robot manipulators, the maximum velocity of each joint is usually provided in the data sheet.

Assumption 2. The below relations holds:

$$\begin{cases} |f(t,\hat{x}) - f(t,x)| \le \delta_1(\hat{x}_2) \\ \left|\dot{f}(t,\hat{x}) - \dot{f}(t,x)\right| \le \delta_2(\hat{x}_2) \end{cases} \tag{9}$$

where we have $x = [x_1, x_2]^T, \hat{x} = [x_1, \hat{x}_2]^T$; $\delta_i(\hat{x}_2)$, $i = 1, 2$, are positive functions (that can be obtained using η).

Remark 2. It should be noted that for designing our state observer-based controller only the availability of η (i.e., the upper bound of the second state) is necessary because the upper bound of $|f(t,\hat{x}) - f(t,x)|$ and $\left|\dot{f}(t,\hat{x}) - \dot{f}(t,x)\right|$ can be obtained using η. So, if Assumption 1 is true, then Assumption 2 will be ensured. An example of the calculation of the upper bounds of $|f(t,\hat{x}) - f(t,x)|$ and $\left|\dot{f}(t,\hat{x}) - \dot{f}(t,x)\right|$ using η is shown for the numerical example (given in section 5).

Remark 3. The successful applications of the assumptions considered in this research can be found in [84, 93-99].

The tracking errors are given as,

$$\begin{cases} e_i = x_i - x_{i_d} \\ \tilde{x}_i = \hat{x}_i - x_i \\ \hat{e}_i = \hat{x}_i - x_{i_d} \end{cases} \tag{10}$$

where x_{i_d} are known smooth trajectories that $x_{2_d} = \dot{x}_{1_d}$; hence, the tracking error dynamics is given as,

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f(t,x) + g(t,x_1)\,u + d(t,x) - \dot{x}_{2_d} \end{cases} \tag{11}$$

From now on, the objective is to design a sliding mode controller in combination with sliding mode state observer, sliding mode disturbance observer, and fixed-time stability notion.

4. STATE AND DISTURBANCE OBSERVER-BASED CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, a combined disturbance observer (DO), state observer (SO), and chattering-free fixed-time SMC (FFSMC) is developed and provides estimated data in the control laws. In order to design FFSMC, it is assumed that the disturbances are unknown and only x_1 (the first state measurement) is available, and x_2 (the second state) is not available and needs to be estimated. The block diagram of proposed method is illustrated in Fig. 1.

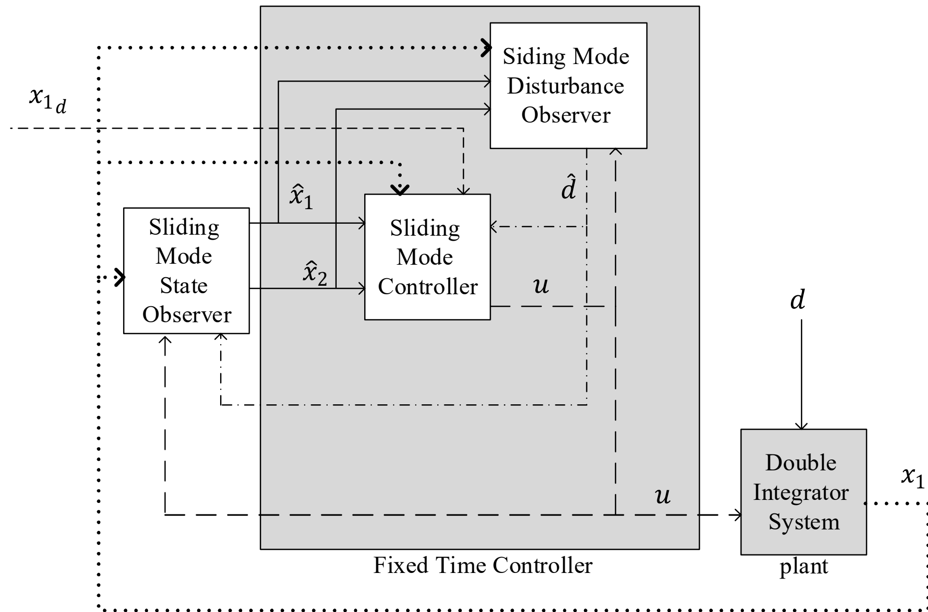

Fig. 1. The schematic block diagram of the proposed fixed-time sliding mode controller in combination with state and disturbance observers.

To proceed with the design, the estimation error of the disturbance observer (Z) is defined as $Z = h - q$; where $q = \dot{e}_1 - \int p dt$, $p = f(t,\hat{x}) + g(t,x_1)u - \dot{x}_{2_d}$, and $\dot{h} = \hat{d}(t,x)$. Note that the reason for defining q is that we assumed $d(t,x)$ is not available and needs to be estimated; hence, it is not correct to use the unknown terms (including $d(t,x)$ and even e_2) in the disturbance observer design. While we can use $f(t,\hat{x})$, $g(t,x_1)$, u, $\dot{x}_{2_d}$, and e_1 in the disturbance observer design.

Taking the time derivative of q we obtain: $\dot{q} = \dot{e}_2 - p$, so we have $\dot{q} = d(t,x)$. As a result, if $Z = 0$, then we have: $h = q \Rightarrow \dot{h} = \dot{q} \Rightarrow \dot{h} = d(t,x)$; where $\dot{h} = \hat{d}(t,x)$.

The sliding surfaces are given as,

$$\begin{cases} s_1 = \dot{e}_1 + A_1(e_1) + B_1(e_1) \\ s_2 = \dot{s}_1 + A_2(s_1) + B_2(s_1) \\ \sigma = \dot{\tilde{x}}_1 + A_3(\tilde{x}_1) + B_3(\tilde{x}_1) \\ \xi = Z + \int A_4(Z)dt + \int B_4(Z)dt \end{cases} \tag{12}$$

where we have $A_j(\varrho) = a_j \varrho^{\frac{p_j}{q_j}}$ and $B_j(\varrho) = b_j \varrho^{\frac{m_j}{n_j}}$; a_j, b_j are positive constants; $0 < p_j < q_j$ and $0 < n_j < m_j < 2n_j$. Also, p_j, q_j, m_j, and n_j must be odd numbers to avoid singularity problem. The control law is defined as,

$$\begin{cases} u = g^{-1}(t,x_1) \times \left(-f(t,\hat{x}) + \dot{x}_{2_d} - \dot{A}_1(e_1) - \dot{B}_1(e_1) - A_2(s_1) - B_2(s_1) - \hat{d}(t,x) + u_1\right) \\ \dot{u}_1 = \left(-\delta_2(\hat{x}_2) - \iota_1|s_2|^{\beta_1} - \iota_2|s_2|^{\beta_2}\right) sign(s_2) \end{cases} \tag{13}$$

where we have $0 < \beta_1 < 1$, and $\beta_2 > 1$; $\hat{d}(t,x)$ is the estimation of the disturbances; ι_i is positive constant. Note that the control law is designed such that the alleviation of the chattering issue is considered by defining $\dot{u}_1$. Then, the integral of signum function will be used in u (in the control signal) that makes the control signal smoother and alleviates chattering. The state observer is given as,

$$\begin{cases} \dot{\hat{x}}_1 = -A_3(\tilde{x}_1) - B_3(\tilde{x}_1) + \hat{x}_{11} \\ \dot{\hat{x}}_{11} = Q sign(\sigma) + f(t,\hat{x}) + g(t,x_1)\, u \\ Q = -2\left(\left|\hat{d}(t,x)\right| + \delta_1(\hat{x}_2)\right) - |c_1\hat{x}_2| + Q_1 \\ Q_1 = -c_2|\sigma|^{\beta_1} - c_3\left||\hat{x}_2| + \eta\right|^{\beta_1} - c_4|\sigma|^{\beta_2} - c_5\left||\hat{x}_2| + \eta\right|^{\beta_2} \\ \dot{\hat{x}}_2 = -c_1\hat{x}_2 + f(t,\hat{x}) + g(t,x_1)\, u \end{cases} \quad (14)$$

where $c_\nu, \nu = (1,2,3,4,5)$ are positive constants. Similarly, to alleviate the chattering issue in the state observer, the signal $\dot{\hat{x}}_{11}$ is defined to provide the integral of the signum function in the computation of $\dot{\hat{x}}_1$ and as such the state observer provides smoother estimates of the state and alleviates chattering. Note that the output of the state observer is used in the control law. That's why, it could create chattering in the control signal (if it's not chattering-free).

$$\begin{cases} \hat{d}(t,x) = \dot{h} = h_1 - A_4(Z) - B_4(Z) + \dot{q} \\ h_1 = \left(-r_1|\xi|^{\beta_1} - r_2|\xi|^{\beta_2}\right) sign(\xi) \end{cases} \quad (15)$$

where r_i is positive constant.

Theorem 3. Let system (8) satisfy Assumption 1 and 2. Consider the tracking error (11), sliding surfaces (12), control law (13), state observer (14), and disturbance observer (15). The trajectory tracking goal is fulfilled in fixed time by applying (13) to (11) and using (12), (14), and (15) where the effect of disturbances is fully rejected, and the estimated data of the states is provided in the controller.

Proof. Consider the candidate Lyapunov function as $V = |s_2| + |\sigma| + |\tilde{x}_2| + |\xi|$. Taking its time derivative, we obtain $\dot{V} = \dot{s}_2 sign(s_2) + \dot{\sigma} sign(\sigma) + \dot{\tilde{x}}_2 sign(\tilde{x}_2) + \dot{\xi} sign(\xi)$. Then, using (13), (14), and (15), yields,

$$\dot{V} = \left(\dot{f}(t,x) - \dot{f}(t,\hat{x}) + \dot{u}_1\right) sign(s_2) + \left(\dot{\hat{x}}_{11} - \dot{x}_2\right) sign(\sigma) + \left(\dot{\hat{x}}_2 - \dot{x}_2\right) sign(\tilde{x}_2) + (h_1) sign(\xi) \quad (16)$$

Then, we have,

$$\dot{V} \le \left|\dot{f}(t,\hat{x}) - \dot{f}(t,x)\right||s_2| - \delta_2|s_2| - \iota_1|s_2|^{\beta_1} - \iota_2|s_2|^{\beta_2} + \left(Q sign(\sigma) + f(t,\hat{x}) - f(t,x) - d(t,x)\right) sign(\sigma) + \left(-c_1\hat{x}_2 + f(t,\hat{x}) - f(t,x) - d(t,x)\right) sign(\tilde{x}_2) - r_1|\xi|^{\beta_1} - r_2|\xi|^{\beta_2} \quad (17)$$

Considering $\left|\dot{f}(t,\hat{x}) - \dot{f}(t,x)\right| \le \delta_2(\hat{x}_2)$, there is,

$$\dot{V} \le -\iota_1|s_2|^{\beta_1} - \iota_2|s_2|^{\beta_2} - r_1|\xi|^{\beta_1} - r_2|\xi|^{\beta_2} + 2|f(t,\hat{x}) - f(t,x)| + 2|d(t,x)| + |c_1\hat{x}_2| + Q \quad (18)$$

Considering $|f(t,\hat{x}) - f(t,x)| \le \delta_1(\hat{x}_2)$ and substituting Q into (18), one gets,

$$\dot{V} \le -\iota_1|s_2|^{\beta_1} - \iota_2|s_2|^{\beta_2} - r_1|\xi|^{\beta_1} - r_2|\xi|^{\beta_2} - c_2|\sigma|^{\beta_1} - c_3\big||\hat{x}_2| + \eta\big|^{\beta_1} - c_4|\sigma|^{\beta_2} - c_5\big||\hat{x}_2| + \eta\big|^{\beta_2} \quad (19)$$

Using $|\tilde{x}_2| \le \big||\hat{x}_2| + \eta\big|$, yields,

$$\dot{V} \le -\iota_1|s_2|^{\beta_1} - \iota_2|s_2|^{\beta_2} - r_1|\xi|^{\beta_1} - r_2|\xi|^{\beta_2} - c_2|\sigma|^{\beta_1} - c_3|\tilde{x}_2|^{\beta_1} - c_4|\sigma|^{\beta_2} - c_5|\tilde{x}_2|^{\beta_2} \quad (20)$$

Assuming $\omega_1 = \min(r_1, c_2, c_3, \iota_1)$, $\omega_2 = \min(r_2, c_4, c_5, \iota_2)$, we get,

$$\dot{V} \le -\omega_1\big(|s_2|^{\beta_1} + |\sigma|^{\beta_1} + |\tilde{x}_2|^{\beta_1} + |\xi|^{\beta_1}\big) - \omega_2\big(|s_2|^{\beta_2} + |\sigma|^{\beta_2} + |\tilde{x}_2|^{\beta_2} + |\xi|^{\beta_2}\big) \quad (21)$$

According to Lemmas 1 and 2, there is,

$$\dot{V} \le -\omega_1(|s_2| + |\sigma| + |\tilde{x}_2| + |\xi|)^{\beta_1} - (4^{1-\beta_2})\omega_2(|s_2| + |\sigma| + |\tilde{x}_2| + |\xi|)^{\beta_2} \Rightarrow$$

$$\dot{V} \le -\omega_1(V)^{\beta_1} - (4^{1-\beta_2})\omega_2(V)^{\beta_2} \quad (22)$$

Using $\rho_1 = \omega_1$, $\rho_2 = \beta_1$, $\rho_3 = (4^{1-\beta_2})\omega_2$, $\rho_4 = \beta_2$, we obtain $\dot{V} \le -\rho_1 V^{\rho_2} - \rho_3 V^{\rho_4}$. Then, according to Lemma 3, there comes $s_2 \to 0$, $\tilde{x}_2 \to 0$, $\sigma \to 0$, and $\xi \to 0$; where we have $T_1 \le \frac{1}{\rho_1(1-\rho_2)} + \frac{1}{\rho_3(\rho_4-1)}$. Subsequently, we obtain,

$$\begin{cases} \dot{s}_1 = -A_2(s_1) - B_2(s_1) \\ \dot{\tilde{x}}_1 = -A_3(\tilde{x}_1) - B_3(\tilde{x}_1) \\ Z = -\int A_4(Z)dt - \int B_4(Z)dt \end{cases} \Rightarrow \begin{cases} \dot{s}_1 = -A_2(s_1) - B_2(s_1) \\ \dot{\tilde{x}}_1 = -A_3(\tilde{x}_1) - B_3(\tilde{x}_1) \\ \dot{Z} = -A_4(Z) - B_4(Z) \end{cases} \quad (23)$$

According to Lemma 4, one obtains,

$$\begin{cases} s_1 \to 0 \\ \tilde{x}_1 \to 0 \\ \tilde{x}_2 \to 0 \\ Z \to 0 \end{cases} \Rightarrow \begin{cases} e_1 \to 0 \Rightarrow \dot{e}_1 = e_2 \to 0 \\ \hat{x}_1 \to x_1 \\ \hat{x}_2 \to x_2 \\ \hat{d} \to d \end{cases} \quad (24)$$

where we have $T_2 \le \sum_{j=1}^{4} \frac{1}{a_j}\frac{m_j}{m_j-n_j} + \frac{1}{b_j}\frac{q_j}{q_j-p_j}$. Finally, we have $T = T_1 + T_2$; where T is the total stability time. ■

The configuration of the above-mentioned FFSMC algorithm is represented in Fig. 2.

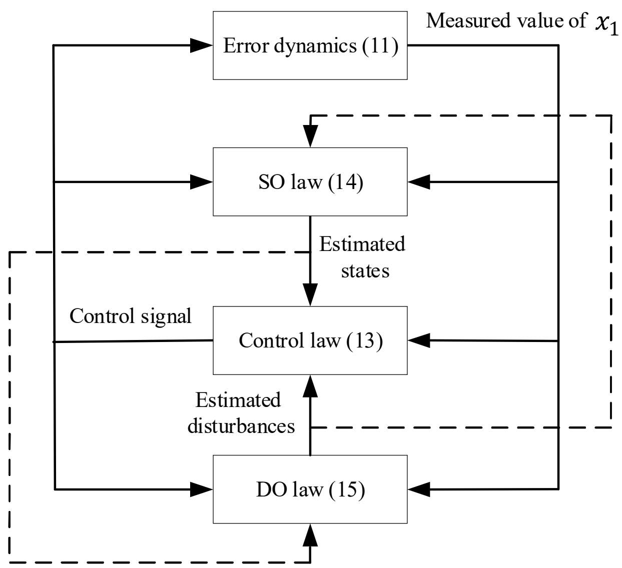

Fig. 2. Flowchart of the proposed FFSMC design.

Proposition 1. By using (25) to (28), the chattering-free finite-time sliding mode controller in combination with state and disturbance observers (FTSMC) can be designed. The sliding surfaces are defined as,

$$\begin{cases} s_1 = \dot{e}_1 + A_1(e_1) \\ s_2 = \dot{s}_1 + A_2(s_1) \\ \sigma = \dot{\tilde{x}}_1 + A_3(\tilde{x}_1) \\ \xi = Z + \int A_4(Z)dt \end{cases} \tag{25}$$

where $A_j(\varrho) = a_j \varrho^{\frac{p_j}{q_j}}$; a_j is positive constant and we have $0 < p_j < q_j$.

$$\begin{cases} u = g^{-1}(t, x_1) \times \left(-f(t, \hat{x}) + \dot{x}_{2_d} - \dot{A}_1(e_1) - A_2(s_1) - \hat{d}(t, x) + u_1\right) \\ \dot{u}_1 = \left(-\delta_2(\hat{x}_2) - \iota_1 |s_2|^{\beta_1}\right) sign(s_2) \end{cases} \tag{26}$$

where $0 < \beta_1 < 1$; $\hat{d}(t, x)$ is the estimation of the disturbances; ι_1 is a positive constant. Note that our proposed approach to alleviate chattering is used for designing the control law as well as the state observer.

$$\begin{cases} \dot{\hat{x}}_1 = -A_3(\tilde{x}_1) + \hat{x}_{11} \\ \dot{\hat{x}}_{11} = Q sign(\sigma) + f(t, \hat{x}) + g(t, x_1)\, u \\ Q = -2\left(\left|\hat{d}(t, x)\right| + \delta_1(\hat{x}_2)\right) - |c_1 \hat{x}_2| + Q_1 \\ Q_1 = -c_2 |\sigma|^{\beta_1} - c_3 \left||\hat{x}_2| + \eta\right|^{\beta_1} \\ \dot{\hat{x}}_2 = -c_1 \hat{x}_2 + f(t, \hat{x}) + g(t, x_1)\, u \end{cases} \tag{27}$$

where $c_\nu, \nu = (1,2,3)$ are positive constants.

$$\begin{cases} \hat{d}(t,x) = \dot{h} = h_1 - A_4(Z) + \dot{q} \\ h_1 = \left(-r_1|\xi|^{\beta_1}\right) sign(\xi) \end{cases} \quad (28)$$

where r are positive constants and we have $Z = h - q$, $q = \dot{e}_1 - \int p dt$, $p = f(t,\hat{x}) + g(t,x_1)\, u - \dot{x}_{2_d}$, and $\dot{h} = \hat{d}(t,x)$.

Remark 4. It should be noted that the idea behind the design of the SMC law of FTSMC method (that is given in Proposition 1) is similar to the finite-time control method given in [59] that is incorporated with our proposed observer.

Remark 5. Alternatively, the observer-based controller can be designed by modifying the sliding surfaces where $A_j(\varrho) = a_j sig^{\frac{p_j}{q_j}}(\varrho)$ and $B_j(\varrho) = b_j sig^{\frac{m_j}{n_j}}(\varrho)$; a_j, b_j are positive constants and $0 < p_j < q_j$ and $0 < n_j < m_j$. By using this modified sliding surface, a similar fixed-time sliding mode controller in combination with state and disturbance observer (OFSMC) can be designed. However, it would be likely to create unwanted chattering issue in the control signal due to utilizing the function of $sig(.)$ (that is given in Definition 1 as $sig^a(x) = |x|^a sign(x)$). Similar concept of OFSMC has been considered in our previous publication [49], while the proposed FFSMC in this research is upgraded and can alleviate the chattering issue from the control signal.

Remark 6. In practice, the direct measurement of velocity using physical sensors might be costly and the measurements tend to be contaminated by noise. Also, there might be no information about disturbances in practice. Thus, the proposed observer-based controller in this research will be beneficial to apply for a wide range of practical applications described by a set of independent double integrator subsystems (similar to (8)) including the three-link robotic manipulator [84, 91], ship course system [90], two-link robotic manipulator [92], etc.

5. NUMERICAL SIMULATION

In this section, the applicability and validity of the observer-based controller design is tested based on a simulation of a robot manipulator (given in [57, 58]) that is in a form of double integrator system given by (8). The proposed FFSMC in this research is also compared with two other similar control methods (OFSMC and FTSMC) from the relevant literature. In fact, FFSMC is compared with another fixed-time observer-based controller (OFSMC) to demonstrate its efficacy in alleviating the chattering phenomenon and providing a better general tracking performance than OFSMC. Also, FFSMC is compared with a finite-time observer-based controller (FTSMC) to reveal its superiority in terms of different performance criteria over FTSMC. The simulation has been done in Simulink/MATLAB by utilizing ode4 (as the numerical method) and the step-size of 0.001. Also, the convergence time is adjusted to be almost equal by a proper selection of the design parameters for all control methods (FFSMC, OFSMC, and FTSMC) to make a reasonable comparison among the simulation results. The following performance criteria (given in [100, 101]) are used to provide a numerical comparison among the simulation results of FFSMC, OFSMC, and FTSMC.

I. Integral of the absolute value of the error (IAE)

$$IAE_{e_i} = \int_0^{t_f} |e_i|\, dt \quad (29)$$

II. Integral of the square value (ISV) of the control input

$$ISV_u = \int_0^{t_f} u^2\, dt \tag{30}$$

where t_f is the total running time. The IAE provides the numerical measures of tracking performance for a whole error curve. The energy consumption can be compared using ISV criterion.

5.1.Application to a robot manipulator

In this section, the following robot manipulator given in [57, 58] is considered.

$$J\ddot{q} + B\dot{q} + MGL\sin(q) + d(t,x) = u(t) \tag{31}$$

where q and $\dot{q}$ represent the angle (or position) and angular velocity (or velocity) of the rigid link, respectively; J is the rotation inertia of the servo motor (kgm^2); B is the damping coefficient (Ns/m); L is the length from the axis of joint to the mass center (m); M is the mass of the link (kg); and G is the gravitational acceleration (m/s^2); $u(t)$ is the control input (or torque) (Nm). The control objective is to make all the system states, angle q (that is denoted by x_1) and angular velocity $\dot{q}$ (that is denoted by x_2), synchronize to sinusoidal prescribed motion trajectories. The system parameters are given in [57, 58] as,

$$J = 1\ (kgm^2),\ MGL = 10\ (kgm^2/s^2),\ B = 2\ (Ns/m),$$

$$x_1 = q\ (rad),\ x_2 = \dot{q}\ (rad/s),\ \theta_1 = -\frac{B}{J},\ \theta_2 = -\frac{MGL}{J} \tag{32}$$

Then, we have,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(t,x) + g(t,x_1)u + d(t,x) \end{cases} \tag{33}$$

where $f(t,x) = \theta_1 x_2 + \theta_2 \sin(x_1)$, $f(t,\hat{x}) = \theta_1 \hat{x}_2 + \theta_2 \sin(x_1)$, and $g(t,x_1) = \frac{1}{J}$. The simulation conditions are considered the same for the three methods given in this research. The system's initial conditions are considered as,

$$x_1(0) = 2\,,\ x_2(0) = -1\,,\ \hat{x}_1(0) = 0\,,\ \hat{x}_2(0) = 0\,,\ h(0) = 0 \tag{34}$$

The desired trajectories (i.e., sinusoidal prescribed motion trajectories) and the model of the external disturbances and modelling uncertainties are given as,

$$x_{1_d} = \cos(t),\ x_{2_d} = -\sin(t),\ d(t,x) = 0.1\|x\| + 0.1\sin(10t) \tag{35}$$

where $\|x\| = \sqrt{x_1^2 + x_2^2}$. The δ_i is calculated as follows and the considered design parameters are given in Table 1.

$$\begin{cases} |f(t,\hat{x}) - f(t,x)| \le |\theta_1 \hat{x}_2 + \theta_2 \sin(x_1) - \theta_1 x_2 - \theta_2 \sin(x_1)| \le \delta_1(\hat{x}_2) = |\theta_1|(|\hat{x}_2| + \eta) \\ |\dot{f}(t,\hat{x}) - \dot{f}(t,x)| \le |\theta_1 \dot{\hat{x}}_2 - \theta_1 \dot{x}_2| \le \\ \qquad \le |\theta_1(-c_1\hat{x}_2 + f(t,\hat{x}) + g(t,x_1)u) - \theta_1(f(t,x) + g(t,x_1)u + d(t,x))| \le \\ \qquad \le |\theta_1(-c_1\hat{x}_2 + |f(t,\hat{x}) - f(t,x)| + |d(t,x)|)| \le \\ \qquad \le \delta_2(\hat{x}_2) = |\theta_1|(|c_1\hat{x}_2| + \delta_1 + |\hat{d}(t,x)|) \end{cases} \tag{36}$$

Table 1. The design parameters considered for the simulation.

	FFSMC	OFSMC	FTSMC
a_1, b_1	7	35	14
a_2, b_2	7	35	14
a_3, b_3	30	30	30
a_4, b_4	10	10	10
c_v	0.0001	0.0001	0.0001
r_i , ι_i	0.1	1	0.1
$\frac{p_j}{q_j}$	$\frac{101}{103}$	$\frac{101}{103}$	$\frac{101}{103}$
$\frac{m_j}{n_j}$	$\frac{105}{103}$	$\frac{105}{103}$	$\frac{105}{103}$
β_1	0.5	0.5	0.5
β_2	1.1	1.1	1.1
η	1	1	1

Remark 7. The arbitrary design parameters given in Table 1 are selected by the designer. Hence, the fixed settling time and control effort of the system can be adjusted by properly choosing them. More importantly, the convergence time is adjustable and can be determined a priori utilizing control parameters regardless of the system's initial conditions (thanks to the fixed-time stability notion) which is a great advantage over the finite-time method. Also, it should be noted that p_j, q_j, m_j, and n_j must be selected from odd numbers to avoid the singularity issue.

Figs. 3 to 9 show the simulation results of FFSMC, OFSMC, and FTSMC methods, simultaneously. Figs. 3 and 4 show the convergence of the estimated data ($\hat{x}_1$ and $\hat{x}_2$) to the actual data (x_1 and x_2) and then to the desired states (x_{1_d} and x_{2_d}) using the three methods. Fig. 5 shows the error of state estimations ($\tilde{x}_1$ and $\tilde{x}_2$) using the three schemes. Fig. 6 shows the tracking error (e_1 and e_2) using the three methods. From Figs. 3 and 5, it can be seen that the estimated data ($\hat{x}_1$) reaches the actual data (x_1) as follows:

- $\hat{x}_1 \to x_1$ within $t \approx 0.1(s)$ using FFSMC.
- $\hat{x}_1 \to x_1$ within $t \approx 0.4(s)$ using OFSMC.
- $\hat{x}_1 \to x_1$ within $t \approx 0.2(s)$ using FTSMC.

As a results, the FFSMC provides a faster estimation of the angle of the link (i.e., $x_1 \to \hat{x}_1$) with a smoother tracking response compared with the other two methods. Note that the undesirable undershoot can be obviously seen in Figs. 3 and 5 (especially in Fig. 5) for tracking response of the angle's estimation of OFSMC.

From Figs. 4 and 5, it can be also observed that the estimated data ($\hat{x}_2$) convergences to the actual data (x_2) of the angular velocity of the link, as follows:

- $\hat{x}_2 \to x_2$ within $t \approx 0.3(s)$ using FFSMC.
- $\hat{x}_2 \to x_2$ within $t \approx 0.35(s)$ using OFSMC.
- $\hat{x}_2 \to x_2$ within $t \approx 0.3(s)$ using FTSMC.

Although the convergence time of the estimated data to the actual data for the angular velocity of the link is almost equal for the three methods, the FFSMC provide a better tracking accuracy after $t \approx 0.3(s)$ compared to the other methods.

From Figs. 3 and 6, it can be seen that after converging the estimated data ($\hat{x}_1$) to the actual data (x_1) of the angle of the link, it takes a short time to fulfil the trajectory tracking goal (i.e., $e_1 = 0$ or $\hat{x}_1 \rightarrow\ x_1 \rightarrow x_{1_d}$); i.e., we have:

- $\hat{x}_1 \rightarrow\ x_1 \rightarrow x_{1_d}$ within $t \approx 0.5(s)$ using FFSMC.
- $\hat{x}_1 \rightarrow\ x_1 \rightarrow x_{1_d}$ within $t \approx 0.8(s)$ using OFSMC.
- $\hat{x}_1 \rightarrow\ x_1 \rightarrow x_{1_d}$ within $t \approx 0.5(s)$ using FTSMC.

It can be observed that the fastest response of tracking the reference of the angle of the link is provided by FFSMC and FTSMC with an almost similar tracking performance. However, the FFSMC provides a better tracking performance over the OFSMC in terms of convergence time and preciseness of the tracking response of the angle of the link (see Figs. 3 and 6).

From Figs. 4 and 6, it can be observed that after converging the estimated data ($\hat{x}_2$) to the actual data (x_2) of the angular velocity of the link, it takes a short time to reach the desired trajectory (i.e., $e_2 = 0$ or $\hat{x}_2 \rightarrow\ x_2 \rightarrow x_{2_d}$); i.e., we have:

- $\hat{x}_2 \rightarrow\ x_2 \rightarrow x_{2_d}$ within $t \approx 0.4(s)$ using FFSMC.
- $\hat{x}_2 \rightarrow\ x_2 \rightarrow x_{2_d}$ within $t \approx 0.6(s)$ using OFSMC.
- $\hat{x}_2 \rightarrow\ x_2 \rightarrow x_{2_d}$ within $t \approx 0.4(s)$ using FTSMC.

It is clear that FFSMC and FTSMC provides a faster convergence time compared to OFSMC. Also, an unwanted undershoot can be observed in Figs. 4 and 6 (especially in Fig. 6) for the tracking response of the angular velocity of the link of OFSMC.

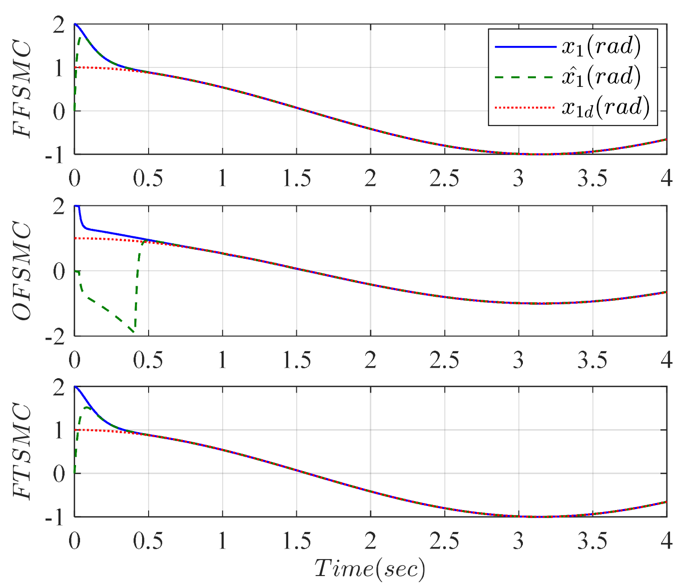

Fig. 3. The tracking performance of the angle, x_1, to its references, x_{1_d}, as well as the estimation performance of the angle using FFSMC, OFSMC, and FTSMC schemes.

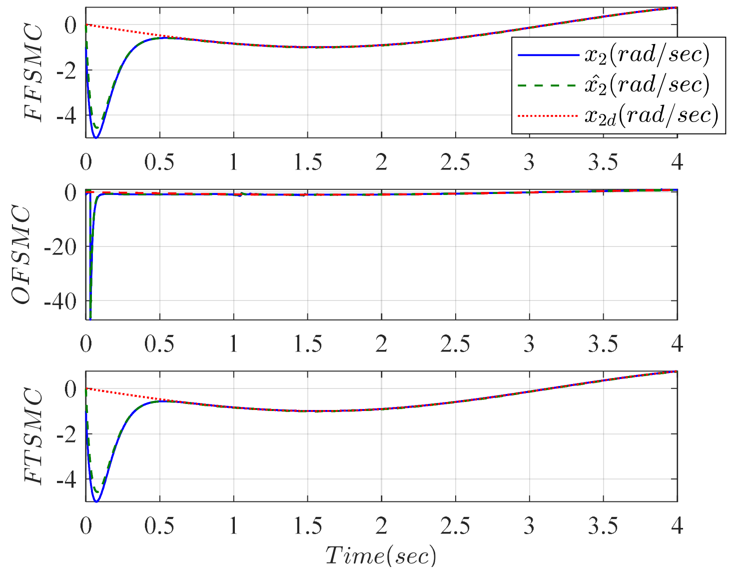

Fig. 4. The tracking performance of the angular velocity, x_2, to its references, x_{2_d}, as well as the estimation performance of the angular velocity using FFSMC, OFSMC, and FTSMC schemes.

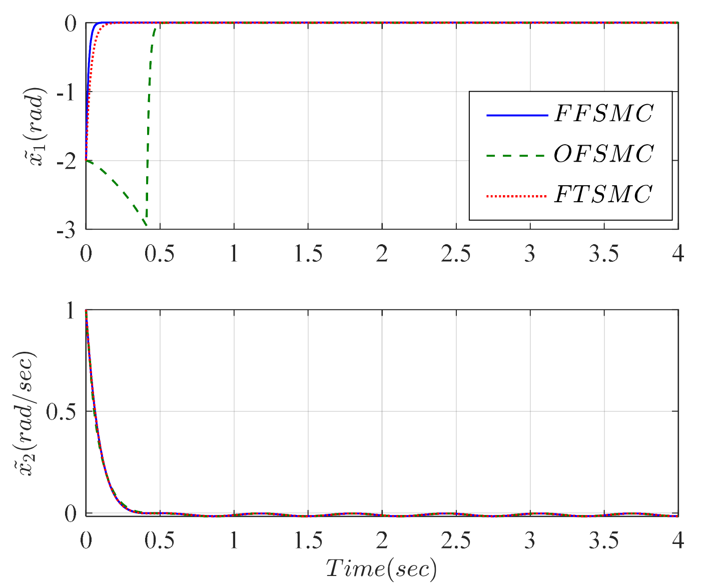

Fig. 5. The estimation errors of the angle, $\tilde{x}_1$, and the angular velocity, $\tilde{x}_2$, using FFSMC, OFSMC, and FTSMC schemes.

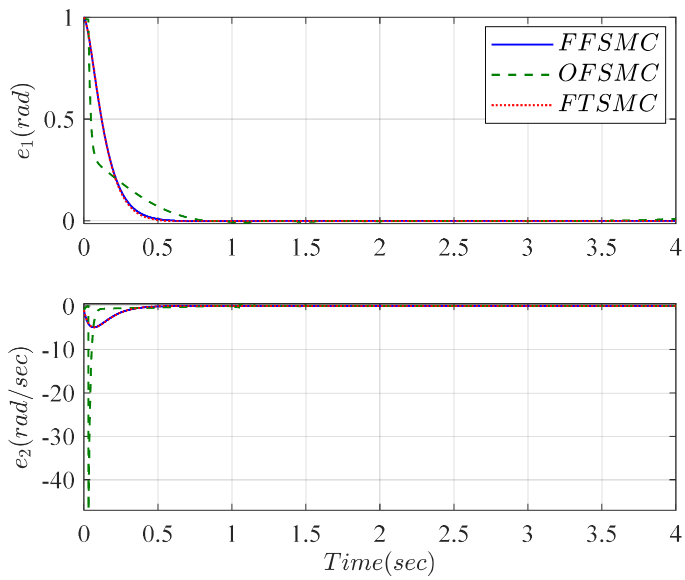

Fig. 6. The tracking errors of the angle, e_1, and the angular velocity, e_2, using FFSMC, OFSMC, and FTSMC schemes.

Fig. 7 shows the time response of the estimation error (Z) of the disturbances $(d(t,x))$ using the three schemes. Fig. 8 shows a zoomed view of the convergence of the estimated date $(\hat{d}(t,x))$ to the actual data $(d(t,x))$ for the disturbances of the robot manipulator using the three methods.

It can be observed that the estimated data reaches the actual data as follows:

- $\hat{d}(t,x) \rightarrow d(t,x)$ $(i.e., Z = 0)$ within $t \approx 0.8(s)$ using FFSMC.
- $\hat{d}(t,x) \rightarrow d(t,x)$ within $t \approx 0.9(s)$ using OFSMC.
- $\hat{d}(t,x) \rightarrow d(t,x)$ within $t \approx 0.9(s)$ using FTSMC.

Hence, the FFSMC provides the fastest response for estimating d compared to the other two methods. Its time response gives a lowest amount of undershoot and overshoot compared with OFSMC and FTSMC (see Fig. 7). Most importantly, the chattering issue can be observed in the time response of DO using OFSMC (as expected, see Remark 5), while this undesired phenomenon does not exist in the time response of DO using FFSMC and FTSMC (see Fig. 8).

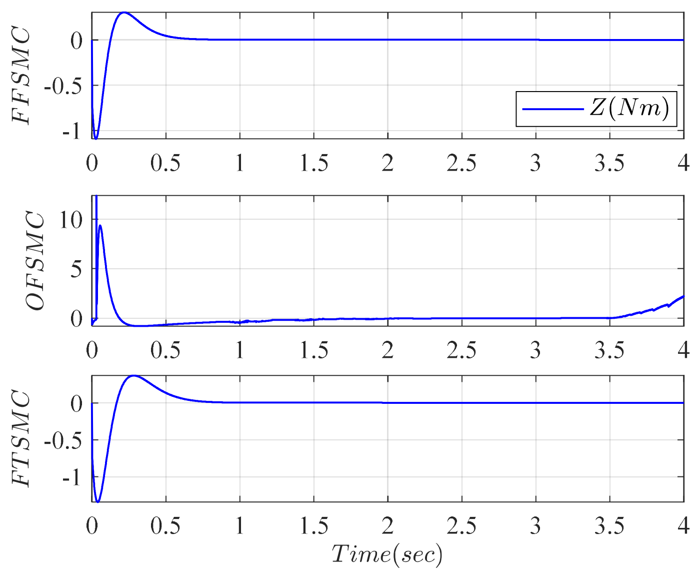

Fig. 7. The estimation errors of the modelling uncertainties and external disturbances, Z, using FFSMC, OFSMC, and FTSMC schemes.

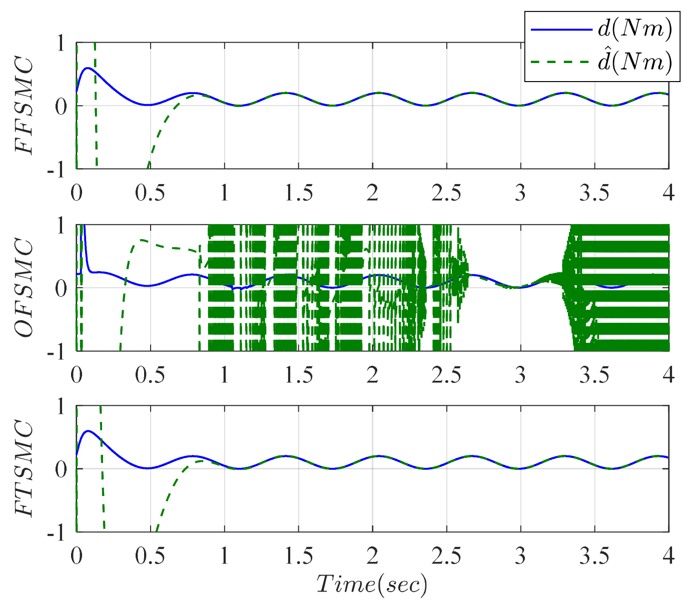

Fig. 8. The tracking performance of the estimated data, $\hat{d}(t,x)$, to the actual data, $d(t,x)$, of the modelling uncertainties and external disturbances using FFSMC, OFSMC, and FTSMC schemes.

Fig. 9 shows the control signals of the three methods, FFSMC, OFSMC, and FTSMC. The chattering phenomenon can be seen in the control signal of FFSMC (as expected, see Remark 5), while this unwanted phenomenon does not exist in the control signal of FFSMC and FTSMC (see the zoomed view in Fig. 9). Also, the amplitude of the control signal of OFSMC is much higher than the other two methods that is not desirable.

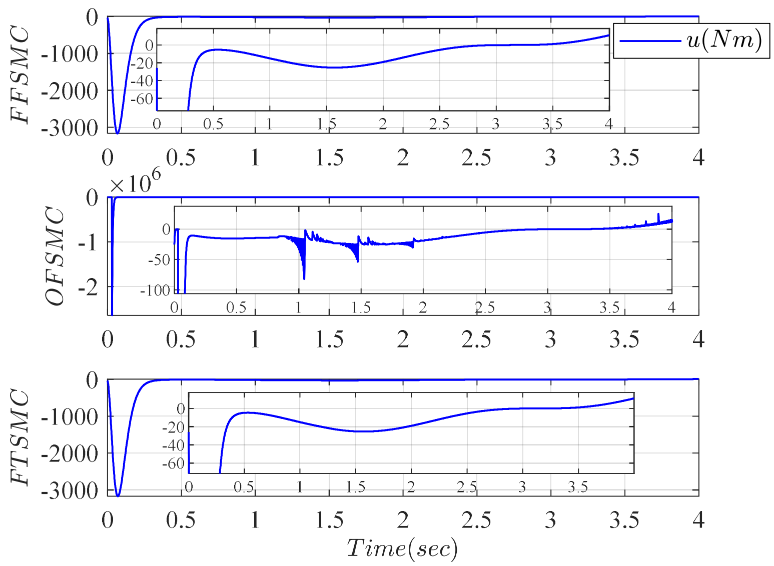

Fig. 9. Time responses of the control signal (u) using FFSMC, OFSMC, and FTSMC schemes.

Table 2. The comparison of the performance indexes of FFSMC, OFSMC, and FTSMC.

	FFSMC	OFSMC	FTSMC
$ISV_u \times 10^6$	3.1075	1.8411×10^5	3.1782
IAE_{e_1}	0.5550	0.5256	0.5628
IAE_{e_2}	3.8595	3.9949	3.8667
$IAE_{\hat{e}_1}$	0.3837	3.4218	0.4746
$IAE_{\hat{e}_2}$	3.5932	3.8411	3.5965
$IAE_{\tilde{x}_1}$	0.1296	3.8621	0.2568
$IAE_{\tilde{x}_2}$	0.3750	0.3754	0.3758
IAE_Z	0.5956	4.9613	0.9282

Table 2 provides a comparison of the performance indexes of the three methods, FFSMC, OFSMC, and FTSMC. From Table 2, it can be seen that our proposed FFSMC gives lower numerical values (in most cases) for ISV_u and IAE_{e_i}. Consequently, the FFSMC outperforms the other two methods in terms of these two performance criteria, ISV and IAE.

6. CONCLUSION

In this article, the issue of disturbance and state observer-based chattering-free fixed-time sliding mode controller design is investigated for a class of uncertain nonlinear double integrator systems. The observer-based controller is designed while there is no information about disturbances as well as the second state of the system. The stability proof is obtained for the closed-loop nonlinear system of the proposed observer-based controller by defining a novel Lyapunov function and considering the fact that the separation principle does not hold for nonlinear systems. The proposed observer-based controller is applied and simulated for a robot manipulator that demonstrates the applicability and efficacy of our method. The simulation results proved that the observer and controller are able to operate simultaneously to achieve the trajectory tracking goal for the robot manipulator. Comparison results reveal that the FFSMC outperforms the OFSMC in terms of alleviating chattering, tracking performance, and the performance criteria, ISV and IAE. Also, FFSMC is better than FTSMC because of using the notion of fixed-time stability in FFSMC as well as comparing the simulation results and the performance criteria. For the future works, a combined chattering-free robust SMC with deep learning and reinforcement learning is aimed to be developed to provide an incorporation of a conventional controller and intelligent controller.

7. COMPETING INTERESTS

The authors declare that they have no known competing financial interests or personal relationships which have, or could be perceived to have, influenced the work reported in this article.

8. DATA AVAILABILITY STATEMENTS

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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