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MULTIPLE MODEL CONTROL IMPROVEMENTS: HYPOTHESIS TESTING AND MODIFIED MODEL ARRANGEMENT

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Abstract

This work demonstrates the fusion of two concepts in switching systems, namely, hypothesis testing and multiple model adaptive control. A hypothesis test switching method is defined to detect parameter jumps in a stochastic environment and select new models. The control of discrete-time systems with rapidly time-varying parameters is simulated. Hypothesis test switching is compared to the most frequently researched performance index switching method. The proposed method is found to be unique because it achieves lower control error and operates without user adjustment or *a priori* knowledge of parameter behaviour and model placement. In addition, a modification to the way multiple models are arranged is proposed. Using the modified arrangement, performance increases are demonstrated, stability is proven more easily, previously required assumptions can be relaxed, and new switching methods can be applied.

Key Words

Stochastic control, discrete-time control systems, intelligent control, digital control

1. Introduction

Multiple model adaptive control becomes necessary when plant parameters move too quickly to be tracked by a single estimation model. With proper model placement and switching, improved control and stability can be achieved during initialization, subsystem failures, and operating environment changes. Typically, a finite number of models are evaluated by a performance index and, at any instant, the most suitable model's controller is used to control the plant [1–7]. The most researched model arrangement, using $N-2$ fixed models and two adaptive models [1–6], is referred to as the *classical model arrangement* throughout this paper. Each of the N models in the classical model arrangement has an associated matched controller. Switching to a fixed model that well represents the plant state, and then performing adaptation from that point, yields the greatest benefit of the multiple model approach (Section 3) [1–6].

Logic-based switching schemes have incorporated a dwell-time [8] or hysteresis [9, 10] to prevent excessive switching. Hysteresis switching is a popular area of research and in some cases switching can cease within a finite time [7], as necessary for stability. The scheme has yet to be formally proposed for discrete-time stochastic systems. In future work, such a scheme will be proposed so that the hypothesis test method's stability and performance can be compared.

The problem of sudden changes in parameters is also well researched in the statistics community. Hypothesis testing is demonstrated useful for detecting parameter jumps when performing system identification by means of a Kalman filter [11, 12]. The Kalman residual is normalized by its standard deviation and used as a Gaussian test statistic [11]. In the event of a parameter jump, the resulting change in mean or variance is detected by a hypothesis test, allowing strategic re-initialization of the filter for faster convergence to new values.

This paper demonstrates the fusion of two concepts in switching systems: hypothesis testing and multiple model adaptive control. A hypothesis test switching method is defined to detect parameter jumps in a stochastic environment and perform model selection (Section 4). Hypothesis test switching is compared to performance index switching, the most researched and popular switching method. The stability analysis of the proposed method requires similar assumptions (Section 6). Simulations demonstrate that the hypothesis test method is unique because it operates optimally without user adjustment (i.e. tuning) or *a priori* knowledge of the time-varying conditions and model placement (Section 7).

In addition, a modification to the classical model arrangement is proposed (Section 5). Motivation for this modified arrangement comes from the inherent instability of the classical arrangement. The proposed method excludes all fixed controllers but still benefits from the existence of fixed models. Analysis and simulation demonstrate that assumptions can be relaxed (Section 6), stability is improved, and performance is increased (Section 7).

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2. Mathematical Preliminaries

A discrete-time plant with input $u(k)$, output $y(k)$, and disturbance $w(k)$ is described by the following equations:

$$y(k+1) = \theta^{\mathrm{T}}(k)\phi_0(k) + w(k+1) \tag{1}$$

$$\theta^{\mathrm{T}}(k) = [a_1(k), \ldots, a_{n_A}(k), b_1(k), \ldots, b_{n_B}(k), c_1(k), \ldots, c_{n_C}(k)] \tag{2}$$

$$\phi_0(k)^{\mathrm{T}} = [-y(k), \ldots, -y(k-n_A), u(k), \ldots, u(k-n_B+1), w(k), \ldots, w(k-n_C+1)] \tag{3}$$

The disturbance is zero mean Gaussian with variance σ^2. The plant is required to be of unity delay and have a minimum phase transfer function. Adaptive control is performed using ELS parameter estimation. The plant's order is known (i.e. n_A, n_B, and n_C) and the estimate of (2) is expressed:

$$\hat{\theta}^{\mathrm{T}}(k) = [\hat{a}_1(k), \ldots, \hat{a}_{n_A}(k), \hat{b}_1(k), \ldots, \hat{b}_{n_B}(k), \hat{c}_1(k), \ldots, \hat{c}_{n_C}(k)] \tag{4}$$

The disturbance statistics, $w(k), \ldots, w(k-n_C+1)$, in (3) cannot be measured and are represented instead by the *a posteriori* estimation errors, $e(k), \ldots, e(k-n_C+1)$ (defined below). The ELS regression vector is then defined:

$$\phi(k)^{\mathrm{T}} = [-y(k), \ldots, -y(k-n_A), u(k), \ldots, u(k-n_B+1), e(k), \ldots, e(k-n_C+1)] \tag{5}$$

The *a priori* estimate of $y(k+1)$ is defined $\hat{y}^0(k+1) = \hat{\theta}^{\mathrm{T}}(k)\phi(k)$ and the *a posteriori* estimate is defined $\hat{y}(k+1) = \hat{\theta}^{\mathrm{T}}(k+1)\phi(k)$. Using the *a posteriori* error from time-step k, $e(k) = y(k) - \hat{y}(k)$, the parameter estimation vector is updated as:

$$\hat{\theta}^{\mathrm{T}}(k+1) = \hat{\theta}^{\mathrm{T}}(k) + P(k+1)\phi(k)e(k) \tag{6}$$

The initial state, $\hat{\theta}^{\mathrm{T}}(0)$, is known and the covariance matrix, $P(k+1)$, is calculated as:

$$P(k+1) = \frac{1}{\lambda}\left(P(k) - \frac{P(k)\phi(k)\phi^{\mathrm{T}}(k)P(k)}{\lambda + \phi^{\mathrm{T}}(k)P(k)\phi(k)}\right) \tag{7}$$

where the user-defined forgetting factor, $\lambda \in (0,1)$, specifies how quickly the algorithm discounts past sample information. Assuming that $u(k)$ and $w(k)$ are persistently exciting, $\hat{\theta}^{\mathrm{T}}(k)$ will converge asymptotically to $\theta^{\mathrm{T}}(k)$.

The *a posteriori* estimate is equated with the desired plant output, $y^*(k+1)$, and the control error equation, $e_c(k+1) = y(k+1) - y^*(k+1)$, is used as follows:

$$\begin{aligned} y(k+1) &= \hat{y}(k+1) + e_c(k+1) \\ &= \hat{\theta}^{\mathrm{T}}(k+1)\phi(k) + e_c(k+1) \\ &= \hat{b}_1(k+1)u(k) + \hat{\theta}^{\mathrm{T}}(k+1)\tilde{\phi}(k) + e_c(k+1) \end{aligned} \tag{8}$$

where $\tilde{\phi}(k)^{\mathrm{T}}$, is the same as $\phi(k)^{\mathrm{T}}$ from (5), except that $u(k)$ is replaced by a zero. With the certainly equivalence principle, the control error is equal to the *a posteriori* estimation error. The control input is calculated by rearranging (8) to be:

$$u(k) = \frac{y^*(k+1) - \hat{\theta}^{\mathrm{T}}(k+1)\tilde{\phi}(k)}{\hat{b}_1(k+1)} \tag{9}$$

To ensure that $u(k)$ is bounded, it is required that $\hat{b}_1$ has a positive, known lower bound, $b_{min} > 0$. A more detailed definition of the system and control law can be found in [3].

3. Multiple Model Adaptive Control of Stochastic Systems

The multiple model method in [1–7] is well researched. The method's performance index is described in Section 3.1 and its classical model arrangement is described in Section 3.2.

3.1 Performance Index Switching

Model M_i, where $i \in \{1, \ldots, N\}$, is selected at the instant it has the smallest associated performance index, $J_i(k)$. If the disturbance statistics, plant parameters, and models are time-invariant, then a sum-of-error is a suitable performance index [2]. The performance index analyzed here, (10), is designed for improved switching in time-varying environments and is the most popular switching method [2–7]. Model M_i has the plant estimate output $\hat{y}_i(k)$, which is compared to $y(k)$ to produce the model's identification error, $e_i(k) = y(k) - \hat{y}_i(k)$. The performance index for M_i is calculated as:

$$J_i(k) = \frac{d_1}{k}e_i^2(k) + \frac{d_2}{k}\sum_{\tau=1}^{k} d_3^{k-\tau} e_i^2(\tau) \tag{10}$$

where the weights d_1, d_2, and d_3 are heuristically adjusting to achieve the desired switching behaviour. The compromise of switching "too often" and reacting to parameter changes "too slowly," is made by adjusting the ratio d_1:d_2. It is found that the ratio 1:2 yields suitable switching performance in most situations [5]. Weight $d_3 \in (0,1)$ is an adjustable forgetting factor that determines the rate at which past modelling errors are attenuated. When the time-varying properties of the plant parameters are unknown, there is no way of knowing what value for d_3 will achieve minimum control error and, as a result, heuristic adjustment of d_3 (i.e. tuning) is required. This process of tuning d_3 is demonstrated in Section 7.1. The proof of stability for performance index (10), which is discussed in Section 6.1, requires a low disturbance level or $d_3 \approx 1$ (undesirable assumptions).

3.2 Classical Model Arrangement

The classical model arrangement is used in [1–7]. Two adaptive models and $N-2$ fixed models are used to identify the plant. Each of the N models has a corresponding

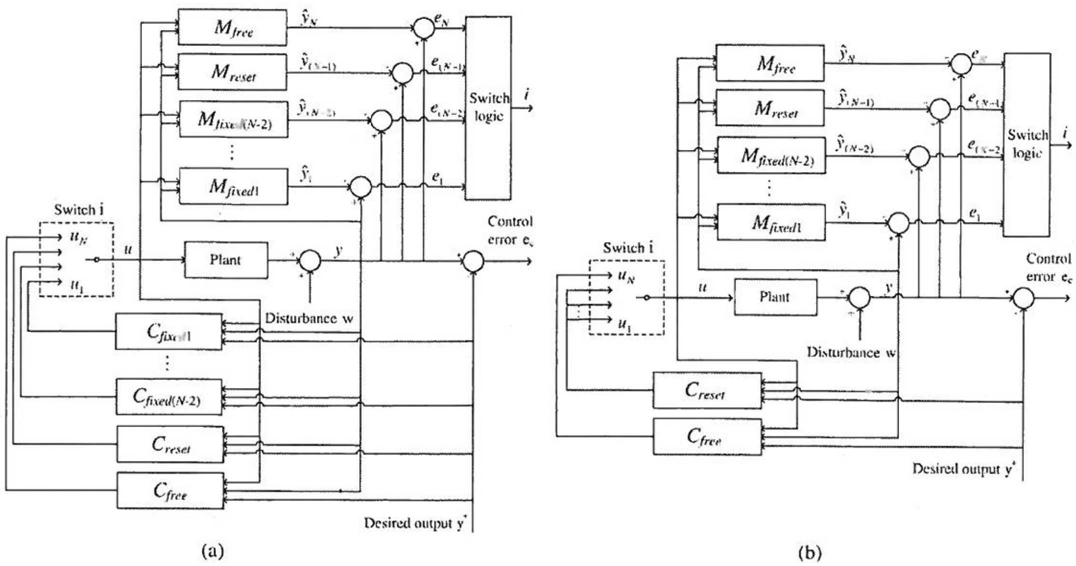

Figure 1. (a) The classical model arrangement and (b) modified model arrangement.

matched controller. This combination of fixed and adaptive models is considered a good compromise between computational complexity and performance [1]. The model that contributes most to performance is the resetting adaptive model. It strategically reinitializes whenever a fixed model is selected and uses initial estimates taken from that fixed model's parameters. The classical model arrangement is shown in Fig. 1(a), where the models are denoted as follows: M_{fixed1} is fixed model #1, $M_{fixed(N-2)}$ is fixed model $\#N-2$, M_{reset} is the adaptive resetting model, and M_{free} is the adaptive free-running model. Model M_i, where $i \in \{1, \ldots, N\}$, is associated with controller C_i. When model M_i is selected, controller C_i is used to control the plant. Controller C_i uses parameter estimates taken from M_i and the desired plant output, $y^*(k)$, to generate its certainty equivalence control input, $u_i(k)$. If the newly selected model, M_i, is a fixed model, then M_{reset} will reinitialize, using initial estimates taken from the parameter vector, θ_i^{T}, of model M_i.

4. Hypothesis Test Switching

An n-sample hypothesis test is proposed to detect plant parameter jumps and enable model switching. If a switch is allowed, the model that has the least plant output estimation error during the n-sample test is selected. This new method is suitable for use with both the classical model arrangement and the modified model arrangement (proposed in Section 5). The null hypothesis is stated as follows:

$$H_0 - \text{no parameter jump has occurred}$$

When the stochastic plant is controlled by means of a perfectly matched controller, the control error, $e_c(k) = y(k) - y^*(k)$, is a Gaussian statistic with zero mean. In the event of a parameter jump, the change in statistics can be observed by the t-test result, $t_0(k)$, calculated as:

$$t_0(k) = \frac{\sum_{\tau=0}^{n-1} |e_c(k-\tau)|}{S(k)/\sqrt{n}} \tag{11}$$

The user-defined constant, n, is the sample size of past control error and the running standard deviation, $S(k)$, is calculated as:

$$S^2(k) = \frac{\sum_{\tau=0}^{r-1} [e_c(k-\tau) - \bar{e}_c(k)]^2}{r-1} \tag{12}$$

The sample mean control error, $\bar{e}_c(k)$, is calculated as:

$$\bar{e}_c(k) = \frac{1}{r-1} \sum_{\tau=0}^{r-1} e_c(k-\tau) \tag{13}$$

The running standard deviation sample size, r, increases by one with each time-step, k. When the null hypothesis, $\mathbf{H}_0$, is rejected, r is reset to the hypothesis testing sample size, n (where $r \geq n$, always). If $\mathbf{H}_0$ is never accepted, $S(k)$ will saturate at a user-defined maximum, S_{sat}. A binary decision rule is used to compare the t-test result, $t_0(k)$, to a user-defined rejection threshold, T_r, which corresponds to a significance level, α, using statistical tables. This is a one-tailed test of significance. The decision rule for the hypothesis test is stated as:

1. if $t_0(k) \leq T_r$, accept $\mathbf{H}_0$ (no parameter jump);
2. if $t_0(k) > T_r$, reject $\mathbf{H}_0$ (a parameter jump).

When $\mathbf{H}_0$ is accepted, switching is not permitted and no change occurs. When $\mathbf{H}_0$ is rejected, the model M_i, where $i \in \{1, \ldots, N\}$, with the smallest associated performance index (14) is selected. The effect of the switch depends on the type of model arrangement (classical arrangement is defined in Section 3.2 and modified arrangement is defined in Section 5). In the event that the newly selected model is the same as the previously selected model, no change will have occurred (M_{reset} only resets when a new fixed model is selected). The performance index is defined as:

$$J_i(k) = \frac{1}{n}\sum_{\tau=0}^{n-1} e_i^2(k-\tau) \tag{14}$$

where n is the number of samples used in the t-test (11). Index $J_i(k)$ utilizes the past n timesteps' model estimation errors, $e_i(k)$, $e_i(k-1), \ldots, e_i(k-n+1)$. Using a larger sample size, n, would increase the accuracy of the hypothesis test, at a cost of reducing the speed in which a jump can be detected [11, 12].

In the context of this hypothesis test implementation, the following statistical terms are defined: α is the probability of falsely detecting a parameter jump (a Type I error) and β is the probability of missing a parameter jump (a Type II error). With the proposed binary decision rule, a reduction in α will result in an increase in β. It is worth emphasizing that a falsely detected parameter jump enables, but does not force, a model switch. This is because the enabled performance index (14) switch may determine that the current model is still the most suitable model. As a result, it is best to allow a large α when forced to implement hypothesis test switching without *a priori* knowledge of the control problem.

5. Modified Model Arrangement

The proposed modified model arrangement differs from the classical model arrangement because it excludes all controllers associated with the $N-2$ fixed models. As shown in Fig. 1(b), only the controllers corresponding to the two adaptive models are available to control the plant. The benefit of the fixed models is still realized, as they provide an initialization point from which model M_{reset} can adapt. When model M_{free} or M_{reset} is selected, it operates as defined in the classical model arrangement, and the corresponding controller, C_{free} or C_{reset}, is applied. Selection of any fixed model, M_{fixedi}, where $i \in \{1, \ldots, N-2\}$, causes the immediate re-initialization of model M_{reset} and application of the matched controller, C_{reset}. Model M_{reset} uses initial estimates taken from parameter vector $\hat{\theta}^{T}_{fixedi}$ of the selected fixed model, M_{fixedi}. As a result, the fixed models are an extension to the adaptive model M_{reset} and only serve as strategic re-initialization points. The following example illustrates how the modified model arrangement behaves during two possible switches:

- At time-step $k-1$, adaptive model M_{reset} is the selected model and its corresponding controller, C_{reset}, is controlling the plant.
- At time-step k, the switching logic selects fixed model M_{fixed1}. Because a fixed model was selected, controller C_{reset} remains applied and continues to control the plant. And for the same reason, model M_{reset} reinitializes and takes initial estimates from parameter $\hat{\theta}^{T}_{fixed1}$ of model M_{fixed1}. The parameters of C_{reset} have changed because it is a controller matched to M_{reset}.
- At time-step $k+1$, the switching logic selects adaptive model M_{free}. Because this is not a fixed model, the associated controller, C_{free}, becomes the newly applied controller.

6. Stability and Switching Stability

The plant parameter vector (2) is assumed to be time-invariant. The stability arguments made in this paper require the following theorem as a foundation.

Theorem 1 (adapted from [2, Theorem 2]): The system, given by (1)–(3) and adaptive control laws (4)–(9), with $N > 1$ arbitrarily switching adaptive models yields mean-square bounded signals and minimum variance control.

Proof of Theorem 1 is given by Narendra and Xiang [2].

With the addition of fixed models, further analysis is required. Any applied switching method must be analyzed to show that no unstable, fixed controller-plant combination can become "frozen" [2, 3]. This condition (denoted in this paper as *switching stability*) requires that the switching signal, $i(k)$, converges such that a fixed controller cannot be applied. Once switching stability is shown, Theorem 1 provides proof of stability. Switching stability is discussed for performance index (10) switching in Section 6.1, hypothesis test switching in Section 6.2, and the modified model arrangement in Section 6.3.

6.1 Classical Arrangement Switching Stability with Performance Index Switching

Before discussing the conditions for switching stability with performance index (10), the more simple performance index must be considered:

$$J_i(k) = \frac{1}{k}\sum_{\tau=1}^{k} e_i^2(\tau) \tag{15}$$

Performance index (15) is equal to (10) with $d_1 = 0$, $d_2 = 1$, and $d_3 = 1$. The indices for the $N-2$ fixed models are denoted by $J_{fixed1}(k)$, $J_{fixed2}(k), \ldots, J_{fixed(N-2)}(k)$, and those of the two adaptive models are denoted by $J_{reset}(k)$ and $J_{free}(k)$.

Theorem 2 (adapted from [2, Section IV-C-II and IV-C-III]): There exists a finite K and lower bound $\epsilon > 0$, such that performance index (15) converges as follows:

$$J_{fixedi}(k) - max\{J_{reset}(k), J_{free}(k)\} > \epsilon > 0,$$
$$1 \leq i \leq N-2 \text{ and } \forall k \geq K$$

Proof of Theorem 2 is given by Narendra and Xiang [2]. An incremental approach is used to exhaustively analyze all possible placements of fixed models. Theorem 2 states that at some time-step, K, the switching scheme converges such that no fixed controller can be applied (this is switching stability). Reference [4] states that Theorem 2 be extended to hold for performance index (10) given the following.

Assumption (A): The SNR is sufficiently large or performance index (10) uses $d_3 \approx 1$.

It is worth noting that using $d_3 \approx 1$ defeats the purpose of this forgetting factor. As described in Section 3.1, index (10) was defined for improving performance in time-varying environments. Finding a stable lower bound for d_3 in stochastic environments remains an open area of research.

6.2 Classical Arrangement Switching Stability with Hypothesis Test Switching

The purpose of the hypothesis test is to enable and disable switching according to performance index (14). Analysis is first performed for *Case (1)*, where the user-defined α is "too large" and switching is always enabled. Then *Case (2)* is considered, where α is "too small" and an unstable model selection could go unnoticed.

Case (1) – Large α: The null hypothesis is always rejected and as such, switching is always enabled. Performance index (14) is analyzed for the following.

Assumption (B): The SNR is sufficiently large or performance index (14) uses $n \approx k$.

Assumptions *(B)* and *(A)* are similar and (14) becomes equal to (15), for which Theorem 2 holds. *Assumption (B)* is no more desirable than *(A)*, however, with *Case (1)*, the system reduces to performance index switching alone, which is not the method's intended purpose.

Case (2) – Small α: An incorrectly applied model could create a long-lasting, unstable controller-plant combination. The ensuing proof serves to show that no such combination could become "frozen." In other words, $\mathbf{H}_0$, would be rejected at some time-step, K, permitting a model switch. As defined in Section 4, $\mathbf{H}_0$ is rejected when the t-test result, $t_0(k)$, is greater than the threshold, T_r (associated with the significance level, α).

Assumptions:

(C) An unstable controller-plant combination yields control error that is increasing in magnitude, such that $|e_c(k)| \rightarrow \infty$, as $k \rightarrow \infty$.

(D) After one or more rejections of the null hypothesis, $\mathbf{H}_0$, an adaptive model is selected.

Theorem 3: There exists a time-step K, such that:

$$t_0(K) \geq T_r, \quad 0 < K < \infty$$

Proof: With *Assumption (C)*, the running standard deviation (12) increases as follows: $S(k) \rightarrow S_{sat}$, as $|e_c(k)| \rightarrow \infty$, where Section 4 defines $S_{sat} < \infty$. With $S(k) = S_{sat}$, the t-test (11) increases as follows:

$$\frac{\sum_{\tau=0}^{n-1} |e_c(k-\tau)|}{S_{sat}/\sqrt{n}} \rightarrow \infty, \quad \text{as} \quad |e_c(k)| \rightarrow \infty \qquad (16)$$

Thus, there exists some time-step, K, where $t_0(K) \geq T_r$, and proof of Theorem 3 is given. As a result, a model switch is permitted and there can be no unstable and "frozen" controller-plant combination. Bounded control is guaranteed and upon the selection of an adaptive model (*Assumption (D)*), convergence occurs. Because Theorem 3 applies for any user-defined sample size $n > 0$, *Assumption (B)* and Theorem 2 provide for switching stability. □

6.3 Modified Model Arrangement Switching Stability with Any Switching Logic

Switching stability, as defined in the beginning of Section 6, requires that the switching signal, $i(k)$, converges such that no fixed controller is applied. Without the presence of fixed controllers, there is no risk of there being an unstable, "frozen" controller-plant combination and convergence occurs for each model selection. Thus, the requirement for switching stability can be relaxed to only require that switching converges to any one model. This condition guarantees that M_{reset} is not continually reset. Note that it is also acceptable if there is random switching between the two adaptive models. Simulation studies show that *Assumption (A)* with Theorem 2 can be relaxed for performance index (10) switching: switching stability was achieved using lower bounds for d_3 (simulation studies were also necessary for such an analysis in [4]). When using the modified model arrangement with hypothesis test switching, *Assumptions (C)* and *(D)* are no longer necessary. Given a chosen switching method satisfies these relaxed conditions for switching stability, Theorem 1 can be applied.

Comment 1: Considering that any model selection results in convergence and unnecessary switching can introduce performance degrading re-initialization of that process, stability can be alternately discussed given the following condition.

Assumption (E): Switching occurs at a frequency less than some maximum, f_{max}.

Provided that a switching method satisfies *Assumption (E)*, model M_{reset} will adapt and bounded control can be achieved. This is demonstrated using simulations in Section 7.2. Defining a stable upper bound for f_{max} is an open area for research.

Comment 2: In [3] it was stated that "mathematically, fixed models only serve to provide better initial points from which the resetting model can adapt." However, analysis in [2] and simulations in Section 7 demonstrate that incorrectly selected fixed models actually cause instability of the classical model arrangement. This stated purpose of fixed models is truly realized with the modified arrangement, for which it is intrinsic that any controller-plant combination utilizes an adaptive model's controller and is convergent.

7. Simulations

The stochastic plant (1) has the parameter vector:

$$\theta^{\mathrm{T}}(k) = [a_1(k) \quad a_2(k) \quad b_1(k) \quad b_2(k) \quad c_1(k) \quad c_2(k)] \qquad (17)$$

where the actual parameter values are defined uniquely in Sections 7.1 and 7.2. The adaptive models M_{reset} and M_{free} use forgetting factor values $\lambda_{reset} = 0.97$ and $\lambda_{free} = 0.98$, respectively. The reference signal is persistently exciting and defined as:

$$y^*(k+1) = \sin\left(\frac{2\pi k}{55}\right) + \sin\left(\frac{2\pi k}{165}\right) \qquad (18)$$

7.1 Comparing Performance Index (10) and Hypothesis Test Switching

Using random numbers, δ_1, $\delta_2, \ldots, \delta_{33}$, uniformly distributed on the interval $[-1,\ 1]$, the plant parameters and three fixed models are defined:

$$\theta^{\mathrm{T}}(k) = \begin{cases} \theta_1^{\mathrm{T}} = [\delta_1 \quad \delta_2 \quad 1.0 \ \delta_3 \quad \delta_4 \quad \delta_5], \ \ k \leq 500 \\ \theta_2^{\mathrm{T}} = [\delta_6 \quad \delta_7 \quad 0.5 \ \delta_8 \quad \delta_9 \quad \delta_{10}], \ 500 < k \leq 1000 \\ \theta_3^{\mathrm{T}} = [\delta_{11} \ \delta_{12} \ 1.5 \ \delta_{13} \ \delta_{14} \ \delta_{15}], \\ \qquad\qquad 1000 < k \leq 1500 \text{ (end)} \end{cases} \tag{19}$$

$$\hat{\theta}_{fixed1}^{\mathrm{T}} = \theta_1^{\mathrm{T}} + 0.20[\delta_{16} \quad \delta_{17} \quad \delta_{18} \quad \delta_{19} \quad \delta_{20} \quad \delta_{21}] \tag{20}$$

$$\hat{\theta}_{fixed2}^{\mathrm{T}} = \theta_2^{\mathrm{T}} + 0.20[\delta_{22} \quad \delta_{23} \quad \delta_{24} \quad \delta_{25} \quad \delta_{26} \quad \delta_{27}] \tag{21}$$

$$\hat{\theta}_{fixed3}^{\mathrm{T}} = \theta_3^{\mathrm{T}} + 0.20[\delta_{28} \quad \delta_{29} \quad \delta_{30} \quad \delta_{31} \quad \delta_{32} \quad \delta_{33}] \tag{22}$$

The classical arrangement's fixed models (20)–(22) are defined to be in the neighborhood of the three plant states (19) with a maximum parameterization error of 0.20. A unique control scenario is created by using a unique seed for generating $w(k)$ and $\delta_1, \delta_2, \ldots, \delta_{33}$. Fifty such unique control scenarios are generated, and in each, the performance index (10) and hypothesis test switching method are independently tuned. In all scenarios, $w(k)$ is zero-mean Gaussian with variance $\sigma^2 = 0.01$.

Tuning performance index (10) involves simulating once using a certain forgetting factor d_3, recording the MSE, then iteratively trying again with an adjusted value for d_3. One hundred tuning attempts are allowed for each control scenario, using values for d_3 between 0.01 and 1.0 (tuning attempt j uses $d_3 = 0.01j$, where $j = 1, 2, \ldots,$ 100). Tuning the hypothesis test method involves adjusting the rejection threshold, T_r, in the same methodical manner. These 100 tuning attempts use values for T_r between 0.1 and 10.0 (tuning attempt j uses $T_r = 0.1j$, where $j = 1, 2, \ldots, 100$).

A two-dimensional filled contour presents the MSE values from the 100 tuning attempts of each control scenario. Fig. 2(a) represents this whole tuning process for the performance index switching method and Fig. 2(b) represents the same for the hypothesis test method. The unique control scenario numbers are labelled on the left vertical axis as #1, #2,…, #50. Each control scenario number in (a) is the same control scenario as in (b). The values being changed during the 100 tuning attempts (d_3 for performance index (10) or T_r for the hypothesis test method) is labelled on the bottom horizontal axis. The MSE control error resulting from each tuning attempt (i.e. for each control scenario # and switching method's value, d_3 or T_r) is plotted on the contours using grayscale.

The MSE calculation employs a user-defined threshold of 1 to identify, and act as a ceiling to, what is considered outliers. As a result, each tuning attempt's MSE is without bias from extraordinarily large transient samples (allowing convergence times to affect the MSE). Using a larger threshold would place more quantitative value on the magnitude of transient error. Using a smaller threshold would place more value on the speed at which the transient error converges.

The tuning process for the performance index method was unique for each control scenario (i.e. different MSE values were obtained in each scenario). For example, in control scenario #30 (labelled on the left vertical axis) a low MSE (shaded black) was achieved using $d_3 = 0.4$. However, using this same d_3 value in scenario #29 resulted in a large MSE (shaded white) and $d_3 = 0$ was necessary to produce a low MSE. Because there is no way of knowing what value for d_3 is suitable for a new control scenario without performing tuning, this switching method is heuristic.

The tuning process for the hypothesis test method achieved very consistent MSE results for all control scenarios. For example, in control scenario #10 (labelled on the left vertical axis) a low MSE (shaded black) was achieved using $T_r \leq 8$. And in scenario #20, using $T_r \leq 8$ also produced a low MSE. For all control scenarios a low MSE was achieved using $T_r = 5$ (shown as a vertical dashed line in Fig. 2(b)). This line was drawn to illustrate how a low control MSE could be achieved for all control scenarios without adjustment of the threshold T_r (no such line could be drawn in Fig. 2(a) because heuristic switching was used). This trend demonstrates that the hypothesis test switching method actually does not require tuning when plant parameter jumps and model placements are unique or unknown. For example, if a new control scenario, #51, was generated, the sequence $w(k)$ and the parameters of $\theta^{\mathrm{T}}(k)$, $\hat{\theta}_{fixed1}^{\mathrm{T}}$, $\hat{\theta}_{fixed2}^{\mathrm{T}}$, and $\hat{\theta}_{fixed3}^{\mathrm{T}}$, would be unknown, but $T_r = 5$ would produce a low MSE. This switching method is not heuristic because, *a priori* to implementation, it is known what value for T_r is best for such new control scenarios.

7.2 Incorrect Switching of the Classical and Modified Model Arrangement

Switching can be incorrect if it is improperly applied (e.g., not tuned) or if disturbances are significantly large. Such an incorrect switching signal, $i(k)$, is independently applied to both model arrangements. For each arrangement's simulation, the plant parameters and fixed models (19)–(22) use the same seed for generation of the random values. Because this simulation is only of length $k = 500$, the parameters (19) are time-invariant with $\theta^{\mathrm{T}} = \theta_1^{\mathrm{T}} = [0.6 \quad 0.2 \quad 1.0 \quad 0.5 \quad -0.4 \quad 0.4]$. The disturbance, $w(k)$, is zero-mean Gaussian with variance $\sigma^2 = 0.20$.

The classical and modified arrangements' control error sequences, $e_{c(clas)}(k)$ and $e_{c(mod)}(k)$, are plotted in Fig. 3. Model M_{reset} was initially selected. In response, both arrangements applied controller C_{reset}, causing $e_{c(clas)}(k)$ and $e_{c(mod)}(k)$ to converge. Model M_{fixed2} was then selected at time-step $k = 50$. The control error $e_{c(clas)}(k)$ became unbounded because the unmatched controller, C_{fixed2}, was applied by the classical arrangement. This same switch caused different behaviour for the modified arrangement: M_{reset} reinitialized (using $\hat{\theta}_{reset}^{\mathrm{T}}(50) = \hat{\theta}_{fixed2}^{\mathrm{T}}$ and $\lambda_{reset} = 0.97$) and C_{reset} remained applied (parameters of C_{reset} changed because it is matched to M_{reset}). Little

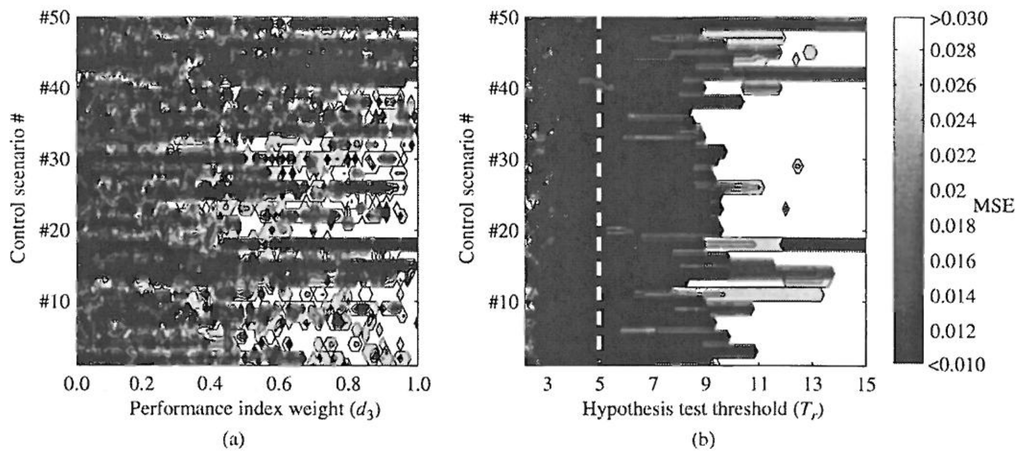

Figure 2. (a) Performance index switching and (b) hypothesis test switching, two-dimensional filled contour plots of the repetitive tuning for each of the 50 control scenarios with each switching method.

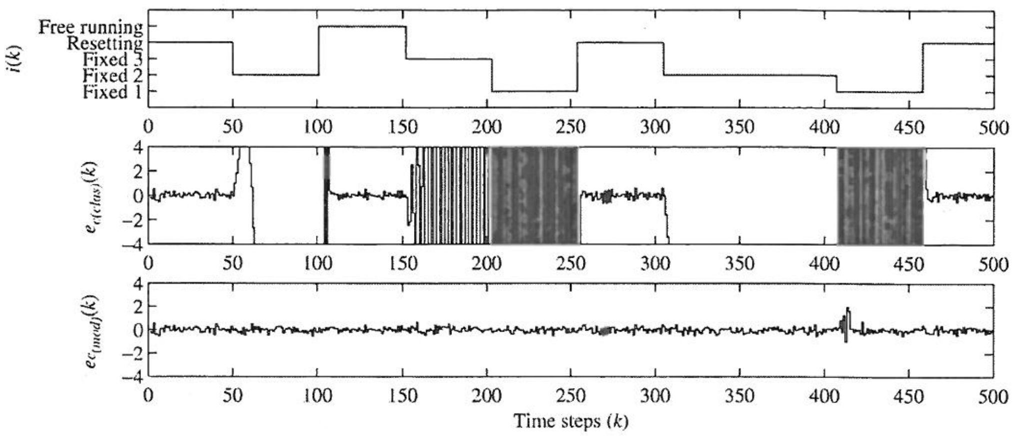

Figure 3. The classical model arrangement's control error, $e_{c(clas)}(k)$, and the modified model arrangement's control error, $e_{c(mod)}(k)$, due to arbitrary model switching.

transient error was observed in $e_{c(mod)}(k)$ because M_{reset} adapted quickly. When M_{fixed1} was selected at $k = 406$, $e_{c(mod)}(k)$ had its most apparent transient error. This increase was partly due to M_{fixed1} being the most unmatched model (from observing $e_{c(clas)}(205)$ and $e_{c(clas)}(406)$), and so $\hat{\theta}^{\mathrm{T}}_{reset}$ had a large initial parameterization error. The large $e_{c(mod)}(k)$ was also due to the reference signal (18) being large during the switch ($y^*(406) \approx 0.9$, whereas when M_{fixed1} was last selected, $y^*(206) \approx 0$).

These two simulations demonstrate that the classical arrangement's convergence is completely dependent upon switching and such a constraint does not exist for the modified arrangement. The incorrect switching of the modified arrangement caused, at worst, performance degradation. Section 6.3 provides a stability discussion relating to this.

8. Conclusions and Recommendations

The paper demonstrated that the proposed hypothesis test switching method could achieve little control error without user tuning or *a priori* knowledge of the control problem. In comparison, the performance index switching method did not possess this ease-of-implementation attribute and it required repetitive heuristic tuning. In additional simulations which could not be included due to considerations for space, it was found that, in general, the hypothesis test switching method yielded lower control error. Future work will compare the hypothesis test method to other switching methods. In particular, it may be found to have behaviour similar to that of a hysteresis-based switching method.

The arguments for stability of the classical model arrangement required performance-hindering assumptions and intricate analyzes of applied switching methods. Simulation showed instability with this arrangement when the switching logic was improperly tuned. No such behaviour was observed for the proposed modified arrangement. The modified arrangement's stability analysis showed that the definition of switching stability and undesirable assumptions could relaxed. As a result, switching methods that would compromise the classical arrangement's stability can be applied. In future work, such new and existing switching methods could be implemented to benefit from this arrangement. When comparing the modified model arrangement to the classical model arrangement, computational complexity was the same, stability was improved, and performance was increased.

Both hypothesis test switching and the modified model arrangement contributed to improve the control of time-varying stochastic systems. The most significant performance and stability improvements can be achieved when combining the two methods to form a new multiple model adaptive control method.

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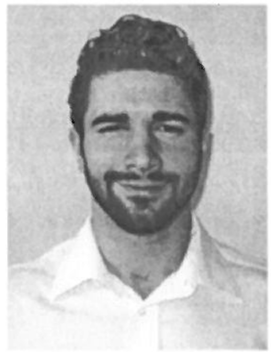
Biographies



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