NOTE: **Write your answers in box provided below. Use the back of the page for calculations. Calculations at the back of page or outside the box will not be checked or graded.**

Student Name:	Student #:

Preliminary Exam Solutions.

Carleton University

Department of Systems and Computer Engineering

94.521 Computer Networks, Preliminary Exam

January 15 2002, 7:30-8:00pm

Preliminary Exam

Messages arrive at a processing node according to a Poisson Process with a rate of 15messages per minute on the average. Find the probability that in a 1 minute period, 3 messages arrive during the first 10 seconds and 2 messages arrive during the last 15 seconds. PLEASE JUSTIFY ASSUMPTIONS AND METHODOLOGY USED. CALCULATIONS WITH NO EXPLANATIONS WILL NOT COUNT!

Formula of Poisson probability mass function with parameter λ: $P(N=n) = \frac{\lambda^n}{n!} e^{-\lambda}$

The average arrival rate in seconds is $\lambda = 15/60 = 1/4$ packets/sec.

There are two acceptable solutions to the problem:

a) During 1 minute 3 message, arrive during the first 10 seconds and 2 messages arrive during the last 15 secs.

Let N_t = # of arrivals in $(0,t]$. Then

$$\mathbb{P}(N(10)=3 \text{ AND } \{N(60)-N(45)\}=2) =$$

$$= \mathbb{P}(N(10)=3)\,\mathbb{P}(N(60)-N(45)=2) \quad \text{(independence of increments)} \quad \text{(important!)}$$

$$= \mathbb{P}(N(10))\,\mathbb{P}(N(60-45)=2) \quad \text{(stationarity of increments)}$$

$$= \frac{(10/4)^3 e^{-10/4}}{3!} \cdot \frac{(15/4)^2 e^{-15/4}}{2!} \quad \text{(Justify!!)}$$

Any of the two approaches is acceptable.

b) Or you may calculate the probability of 3 mesages arrive in first 10 secs, 2 in the last 10 secs and 0 (nothing) in between. In this case:

$$\mathbb{P}(N(10)=3 \text{ AND } (N(45)-N(10)=0) \text{ AND } (N(60)-N(45)=2) =$$

$$= \frac{(10/4)^3 e^{-10/4}}{3!} \cdot \frac{(35/4)^0 e^{-35/4}}{0!} \cdot \frac{(15/4)^2 e^{-15/4}}{2!}$$