### 4.8.2 *Efficiency of CSMA/CD*

We define the efficiency as the fraction of time that the nodes using the protocol can transmit new packets under a heavy load imposed by all the nodes. We show that

$$\eta_{\text{CSMA.CD}} \approx \frac{1}{1+5a} \quad \text{with} \quad a := \frac{\rho}{\tau} \tag{4.1}$$

where $\tau$ designates the transmission time of a packet. (Recall that $\rho$ is the maximum propagation time across the shared transmission channel.)

Note that if a single node is active on the network, then that node transmits without ever colliding with other transmissions. In that case, the fraction of time that the node transmits new packets can be close to 100 percent. The efficiency (4.1) is for a congested network where many nodes compete for transmitting on the common channel. It is under such difficult operating conditions that we need to verify how well a protocol performs.

### 4.8.3 *Analysis*

In this section we derive the efficiency (4.1). The analysis consists of the following steps. We consider that the nodes attempt to transmit at discrete times, in time slots with a duration of $2\rho$ each. This discretization simplifies the analysis and does not depart significantly from the actual operations of the protocol. The slot duration of $2\rho$ is used to guarantee that, if nodes select to transmit at the beginning of two different slots, then they cannot collide. We then perform two calculations. First we assume that the system is idle and we determine the probability $\alpha$ that the next time slot is the start of a successful transmission. That is, $\alpha$ is the probability that during that time slot there is no collision and that one node starts transmitting. We find that $\alpha \approx 0.4$. Second, we use the value of $\alpha$ to determine the average number $A$ of time slots that are wasted before a successful transmission. We show that $A \approx 1.5$. To use this result, we argue that the average fraction of the time that the nodes transmit successfully is $\tau$ out of $\tau + A \times 2\rho$, which yields an estimate close to Equation (4.1). Finally, we justify (4.1) from that estimate.

First, we calculate $\alpha$. Say that $N$ ($N \geq 2$) nodes compete for a time-slotted channel by transmitting packets with probability $p \in (0, 1)$, independently of one another, in any given time slot. Denote by $\alpha(p)$ the probability that exactly one node transmits in a given time slot. The claim is that

$$\alpha(p) = Np(1-p)^{N-1} \tag{4.2}$$

To see (4.2), consider a given time slot. The probability that, during this time slot, a specific node among the $N$ nodes transmits and that the other $N-1$ nodes do not transmit is equal to

$$p \times (1-p) \times \cdots \times (1-p) = p(1-p)^{N-1}. \tag{4.3}$$

Indeed, the probability that a collection of independent events all occur is the product of the probabilities of the individual events and each node transmits with probability $p$ and does not transmit with probability $1-p$. (Appendix A explains these notions of probability theory.)

Consequently, the probability that any one of the $N$ nodes transmits and that the other $N-1$ nodes do not transmit is $N$ times the probability (4.4), which is $\alpha(p)$ given in (4.2). Indeed, there are $N$ mutually exclusive ways for exactly one node to transmit, depending on which of the $N$ nodes transmits; also, the probability that one of a collection of mutually exclusive events occurs is the sum of the probabilities of the individual events.

If the nodes knew the number $N$ of competing nodes, then they could determine the value of $p$ that maximizes $\alpha(p)$ and the efficiency of the protocol. The value of $p$ that maximizes $\alpha(p)$ is $p = 1/N$. Indeed, this value is obtained by setting to zero the derivative of (4.2) with respect to $p$. One finds

$$\frac{d}{dp}\alpha(p) = N(1-p)^{N-1} - N(N-1)p(1-p)^{N-2},$$

so that the value of $p$ that makes this derivative equal to zero is $p = 1/N$, as claimed.

The corresponding maximum value of $\alpha(p)$ is

$$\alpha\left(\frac{1}{N}\right) = \left(1 - \frac{1}{N}\right)^{N-1} \approx 40\%. \tag{4.4}$$

To see the approximation, you can verify that $\alpha(1/N) \to 1/e \approx 36\%$ as $N \to \infty$. For instance,

$$\alpha(1/4) = 42\%, \qquad \alpha(1/10) = 39\%, \qquad \alpha(1/20) = 38\%.$$

In the CSMA/CD protocol, a node does not know the number $N$ of nodes that have packets to transmit and are competing for the channel. Consequently, the nodes cannot use the value $p = 1/N$. For the purpose of the analysis, we assume that the CSMA/CD protocol selects the random backoff times almost as well as they would if they knew the number of competing nodes. (The exponential backoff mechanism of IEEE 802.3 is effective; attempts to improve it substantially are not convincing.) Thus, we conclude that the probability that a time slot is the start of a successful transmission is $\alpha := 40\%$.

Second, we calculate the average number $A$ of time slots that CSMA/CD wastes before it succeeds in transmitting a packet. With probability $\alpha = 0.4$, the first time slot is successful, so that no time slot is wasted. With probability $1-\alpha$, the first slot is wasted. In the latter case, after the first wasted slot, we are essentially back to the initial situation, so that CDMA/CD will waste an average number $A$ of slots in addition to the first one before the first successful transmission. Hence,

$$A = \alpha \times 0 + (1-\alpha)(1+A).$$

Solving for $A$, we find $A = \alpha^{-1} - 1$. With $\alpha = 0.4$, this gives $A = 1.5$. (The above calculation is an application of the regenerative method that we explain in Appendix A.)

Using this value $A = 1.5$, we conclude that every successful transmission (with duration $\tau$) is accompanied by a wasted amount of time with an average value equal to 1.5 time slots, or $1.5 \times 2 \times \rho = 3 \times \rho$. Consequently, the efficiency $\eta_{CSMA.CD}$, i.e., the fraction of time when the CSMA/CD protocol transmits successfully, is given by

$$\eta_{CSMA.CD} \approx \frac{\tau}{\tau + 3 \times \rho} = \frac{1}{1 + 3a}.$$

In actuality, the CSMA/CD protocol is not really optimal, and it wastes a larger amount of time than $3 \times \rho$ per packet transmission. Simulations show that the amount of time wasted is closer to $5 \times \rho$. Hence, the efficiency of the CSMA/CD protocol is approximately given by (4.1). We should not be surprised that a simplified analysis gives a result that is not quite exact. The opposite would be miraculous. Nevertheless, the analysis has the merit of explaining how the efficiency is reduced when the parameter $a$ of the network increases.

### 4.8.4 Examples

To develop a concrete feel for the efficiency of the CSMA/CD protocol, let us calculate $\eta_{CSMA.CD}$ for nodes attached to a 2.5-km-long coaxial cable, with a 10-Mbps transmission rate and 620-bit packets. We find that $\rho$, the one-way propagation time of a signal from one end of the cable to the other, is given by

$$\rho = \frac{2500 \text{ m}}{2.3 \times 10^8 \text{ m/s}} \approx 1.09 \times 10^{-5} \text{ s}.$$

(We used the transmission speed $2.3 \times 10^8$ m/s in a coaxial cable.)

The transmission time $\tau$ of a packet is

$$\tau = \frac{620 \text{ bits}}{10 \times 10^6 \text{ bps}} = 6.2 \times 10^{-5} \text{ s}.$$

From these two values we conclude that $a = \rho/\tau \approx 0.176$ and, therefore,

$$\eta = \frac{1}{1 + 5a} = \frac{1}{1 + 0.176 \times 5} \approx 53\%.$$

Consequently, the effective transmission rate of this 10-Mbps network is only 53 percent of 10 Mbps, i.e., 5.3 Mbps, when the network is heavily loaded by many nodes. If the network transmits TCP/IP frames, about 30 bytes, i.e., 240 bits, of the 620 frame bits are not user data. Thus, only a fraction $(620 - 240)/620 \approx 61\%$ of the frame bits is user data bits. Therefore, the maximum rate at which the network can transmit user data is $61\% \times 5.3$ Mbps $= 3.2$ Mbps. Note that this efficiency result is very sensitive to the length of packets, as you can verify easily.

As another example, let us consider a network with twisted pairs that are up to 200 m in length and with a transmission rate of 1 Gbps. These assumptions would correspond to a hypothetical implementation of shared Gigabit Ethernet. Assuming that the average packet length is again 620 bits, we adapt the above calculations and we find

$$\rho = 0.2 \text{ km} \times 3.3\ \mu\text{s/km} \approx 0.66 \times 10^{-6} \text{ s},$$

and

$$\tau = \frac{620 \text{ bits}}{10^9 \text{ bps}} = 0.62 \times 10^{-6} \text{ s.}$$

Combining these values, we find $a = \rho/\tau \approx 1$, so that

$$\eta = \frac{1}{1+5a} = \frac{1}{1+5} \approx 16\%,$$

which shows that a shared Gigabit Ethernet would not be an efficient backbone technology.