

# Network Virtualization with Dynamic Resource Pooling and Trading Mechanism

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**Abstract**—Network virtualization is a promising technology for the next generation network, which is required to offer a more dynamic and flexible network infrastructure. Virtual network embedding plays a vital role in the resource allocation of network virtualization. Current virtual network embedding allocates resources in an exclusive and excessive manner. For example, the whole bandwidth amount of the virtual network's peak traffic demand is allocated to the virtual network with full availability. However, such excessive resource allocation may result in resource under-utilization, leading to high user cost and low carrier revenue. To address this problem, we propose a new dynamic resource pooling and trading mechanism. The proposed mechanism is formulated as a Stackelberg game, using bandwidth as an example of the resources. We compare the user cost and the carrier revenue under the proposed mechanism against those under the exclusive resource allocation schemes. Our results show that under certain conditions, the "win-win" situation, in which the user saves cost and the carrier increases its revenue, exists and the optimal Subgame Perfect Equilibrium (SPE) point can be found.

## I. INTRODUCTION

The current ossified Internet is facing problems supporting high-bandwidth heterogeneous applications with the emergence of cloud-based services and big data. A more dynamic computing and network infrastructure is required for the future growth of Internet. In recent years, network virtualization [1] has attracted the attention from both the academia and industry as a long-term solution to enhance the existing Internet. In network virtualization, each virtual network (VN) is a partition or aggregation of the underlined physical network (PN) resources, tailored to the specific requirements of the applications. Network virtualization is the key technology to enable multiple isolated VNs to share a common substrate.

In network virtualization, the PN provider (carrier) allocates a part of its infrastructure resources for the VNs. The problem of mapping a virtual node to a physical node and mapping a virtual link to a physical path is called VN embedding or VN mapping problem [2]. In the traditional VN embedding problems, the resources are allocated *exclusively*. The VN provider (user/tenant) requests the bandwidth amount as its peak bandwidth needs, and it does not share the unused bandwidth with other VNs. However, if the peak traffic demand only happens occasionally, the bandwidth utilization will be low. The recent advancements in software-defined networking (SDN) [3] provide a new opportunity for fast and

self-service based network resource provisioning. For instance, the OpenFlow-based SDN decouples the control plane and data plane, and offers a flexible network automation and management framework, making it possible to develop tools that automate the network resource provisioning which is done manually today. Moreover, SDN can allow dynamic network resource provisioning and re-provisioning to be performed on demand and application driven.

Based on the SDN technology, we propose a dynamic resource pooling and trading mechanism in network virtualization, which has the potential to greatly improve the current resource utilization, reduce the users' cost, and increase the carriers' revenue. In our new mechanism, the user requests a long-term resource amount at the beginning, which can be lower than the peak demand. The resources may be any of the network, computing, or storage resources. As time goes on, the user sells its unused resource to the carrier at a reduced rate. The carrier creates and manages a *resource pool* to gather the unused resource. If the user needs extra resource to accommodate its peak demand, the user may search the resource pool for the resource and buy the resource from the carrier at an increased rate. In this way, the user does not need to exclusively reserve the resource at its peak demand. In this paper, we use bandwidth as an example of the resources to analyze the user cost and carrier revenue from a theoretical perspective. The proposed mechanism is formulated as a Stackelberg game [4-6]. The Subgame Perfect Equilibrium (SPE) is found and the numerical results are shown to prove the benefits of the new mechanism.

The rest of this paper is organized as follows: in Section II, we give a review of the related research works. Section III describes our dynamic resource pooling and trading mechanism in detail and explains how to formulate it as a Stackelberg game. Section IV shows the process of finding SPE. Numerical results are shown in Section V. Finally, Section VI concludes the paper.

## II. RELATED WORK

Most of the current research works on network virtualization assume exclusive resource allocation [7-10]. The user's requested resource amount is equal to its peak demand. There are a few studies [11-13] analyzing the overbooking problem in network virtualization. In [11], some users require full availability of the resources, while others can accept limited availability of the resources as long as the service level agreement (SLA) is not violated. [12] extends [11] by

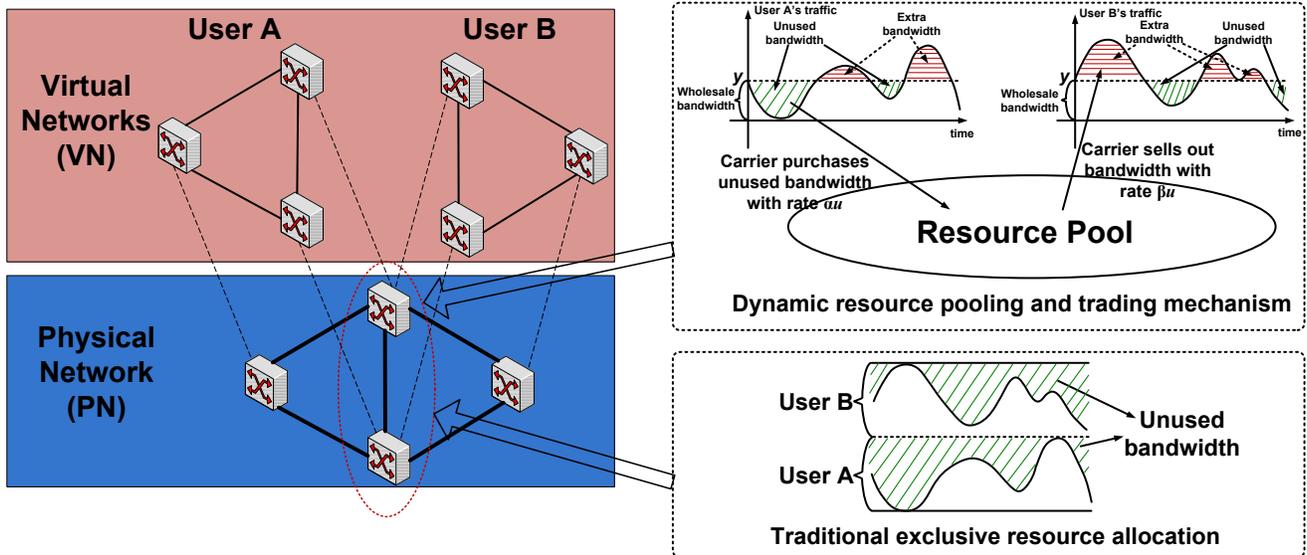


Fig. 1. Dynamic resource pooling and trading mechanism vs. traditional exclusive resource allocation

formulating a mixed integer linear programming (MILP) to minimize the virtual network cost, given the diverse degrees of availability of the users. In [13], an analytic model is formulated to quantify the performance impact of overbooking virtualized resources as a function of the relevant environment and usage parameters.

However, these papers on overbooking mainly focus on analyzing the performance impact of overbooking and how many more users may be multiplexed without violating the SLA. Although the users can share the resources to a certain degree, their requested resources are still statically provisioned. They don't have a dynamic resource exchange mechanism to specify how to distribute the unused resource dynamically. This sets the study in this paper apart from the prior works on overbooking problems. In addition, we analyze the user cost and the carrier revenue under the overbooking scenario, which is still largely missing in the literature.

### III. DYNAMIC RESOURCE POOLING AND TRADING MECHANISM

In this section, we firstly illustrate our proposed dynamic resource pooling and trading mechanism. Then, we explain how to formulate it as a Stackelberg game, using bandwidth as an example of the resources to be provisioned.

#### A. Introduction on Dynamic Resource Pooling and Trading Mechanism

In this section, we use bandwidth as an example of the resources to introduce our new dynamic resource pooling and trading mechanism. The other network, computing, or storage resources can be pooled and traded in the same way. As shown in Fig. 1, the users' traffic fluctuates over time. In our mechanism, the user purchases a certain amount of bandwidth  $y$  at a certain rate  $u$  (per unit bandwidth and per unit time) at the beginning. The user can occupy the bandwidth up to  $y$  at the guaranteed rate  $u$  at any time. We call the bandwidth  $y$  as the *wholesale bandwidth* and the guaranteed rate  $u$  as *wholesale rate*.

As time goes on, the user's traffic may fall below the wholesale bandwidth  $y$ . The bandwidth below the wholesale bandwidth  $y$  and above the user's current traffic is unused, so we call it the *unused bandwidth*. The user can sell the unused bandwidth to the carrier at a reduced rate  $\alpha u$ ,  $0 \leq \alpha \leq 1$ . We call the ratio  $\alpha$  as the *reduced rate ratio*. The exact value of  $\alpha$  is agreed upon negotiation between the users and the carrier. The carrier purchases back the unused bandwidth, puts it in the resource pool, and sells the bandwidth to other users in need, who share the same physical link as the user selling the unused bandwidth. Note that the unsold bandwidth of the carrier is also in the resource pool.

When the user's traffic is above the wholesale bandwidth  $y$ , the user will not have enough bandwidth to accommodate its traffic demand. Under this situation, the user has to purchase extra bandwidth in the resource pool from the carrier at an increased rate  $\beta u$ ,  $\beta \geq 1$ . We call the ratio  $\beta$  as the *reselling rate ratio*. There is a chance that the resource pool does not have enough bandwidth to meet all the buyers' extra bandwidth demand, and thus the congestion happens. The congestion probability is agreed by both the user and the carrier in the SLA.

Fig. 1 shows an example of our proposed dynamic mechanism and the traditional exclusive resource allocation method. In this example, User A and User B share a common physical link. In our proposed mechanism, User A's unused bandwidth can be purchased by the carrier and resold to User B, and vice versa. Thus, User A and User B do not have to purchase long-term bandwidth amount at their peak traffic demands. On the contrary, in the traditional exclusive resource allocation, User A and User B purchase the same bandwidth amount as their peak traffic demands and they are not able to sell their unused bandwidth to the carrier.

#### B. A Two-Stage Stackelberg Game

Stackelberg game is a strategic game including one leader and one or more followers. The leader takes action first, and

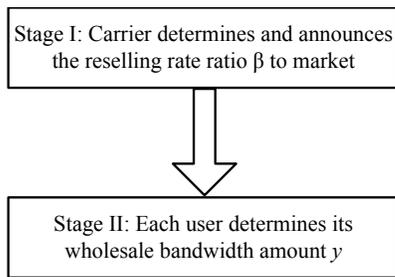


Fig. 2. Two-stage Stackelberg game

the followers response sequentially. The Stackelberg game can be solved to find the Subgame Perfect Equilibrium (SPE), which is the strategy that is best for each player, given the strategies of the other players. In Stackelberg game, the leader usually has some advantages enabling it to move first. In the carrier-user relation, the carrier sets the price first, and the users find minimum cost solutions to satisfy their requirements.

The proposed new mechanism can be formulated as a two-stage Stackelberg game using bandwidth as an example of the resources, as shown in Fig. 2. The carrier is the leader in the Stackelberg game, while the users are the followers. In Stage I, the carrier first decides the reselling rate ratio  $\beta$ , which maximizes its revenue, and announces it to the users. In Stage II, each user decides the amount of wholesale bandwidth  $y$ , which minimizes its long-term cost.

The two-stage Stackelberg game can be analyzed by exploiting the SPE. To search for SPE, backward induction [14] is a general technique often used. It starts with Stage II, then proceeds to Stage I, where each of these two stages can be formulated as an independent optimization problem. We will search the SPE in our dynamic resource pooling and trading mechanism and show that the “win-win” situation exists under SPE. The notations used in our analysis are shown in Table. I.

TABLE I. NOTATIONS

Symbol	Meaning
$u$	Wholesale rate
$x$	User's traffic demand
$y$	Amount of wholesale bandwidth
$\alpha$	Reduced rate ratio
$\beta$	Reselling rate ratio
$C$	The capacity of a physical link
$N$	The number of users sharing a physical link
$X$	The random variable representing user's traffic
$\lambda$	The parameter of the user's traffic distribution
$b$	The peak rate of the user's traffic
$T$	The total time period
$P_{new}$	The total cost a user pays to the carrier using the new mechanism
$P_{old}$	The total cost a user pays to the carrier using the old mechanism
$P_{new\_avg}$	The average cost a user pays to the carrier using the new mechanism
$P_{old\_avg}$	The average cost a user pays to the carrier using the old mechanism
$R_{new\_avg}$	The average revenue of the carrier using the new mechanism
$R_{old\_avg}$	The average revenue of the carrier using the old mechanism

In the following sections, the *new mechanism* refers to our dynamic resource pooling and trading mechanism, while the *old mechanism* refers to the traditional exclusive resource allocation.

#### IV. BACKWARD INDUCTION OF THE TWO-STAGE GAME

In this section, we show how to determine the SPE through backward induction. In the first part, we show how the users determine their wholesale bandwidth amount to minimize their cost; in the second part, we show how the carrier determines the reselling rate ratio  $\beta$  to maximize its revenue.

##### A. Determining wholesale bandwidth amount $y$ in Stage II

In Stage II, the user decides the wholesale bandwidth amount to minimize its cost, given the reselling rate ratio  $\beta$  announced by the carrier in Stage I.

We assume the user's traffic follows the truncated exponential distribution with parameters  $\lambda$  and  $b$ . The probability density function (pdf) is:

$$f_X(x) = \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda b}}, \quad 0 \leq x \leq b \quad (1)$$

This assumption of traffic distribution enables us to obtain closed-form solution of wholesale bandwidth amount  $y$ . Note that the benefits of our new mechanism do not come from any particular traffic distribution. The “win-win” situation should exist for almost any traffic distributions, as long as the traffic is not at its peak all the time. The reason is that the user can save cost by purchasing certain amount of wholesale bandwidth at the beginning, and purchasing extra bandwidth only when needed, regardless of the traffic distribution. Also, the users can have additional income from selling the bandwidth it does not use. The carrier can have higher revenue because it can support more users by reselling the unused bandwidth. Besides, the carrier earns the difference between the higher rate  $\beta u$  and the lower rate  $\alpha u$ . We use the truncated exponential distribution in this paper just for the purpose of simplifying the analysis.

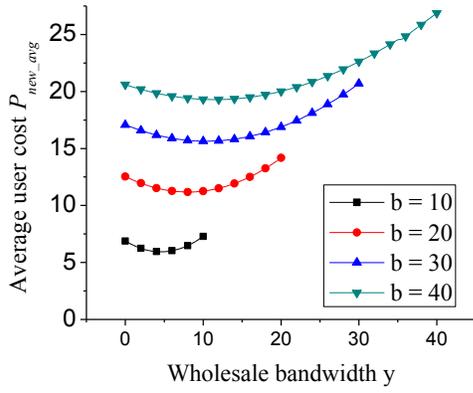
The user cost  $P_{new}$  under the new mechanism over a time period  $T$  can be obtained by the following formula:

$$P_{new}(\beta, y) = yuT - \alpha uT \int_0^y (y - x) f_X(x) dx + \beta uT \int_y^b (x - y) f_X(x) dx, \quad 0 \leq y \leq b \quad (2)$$

where the first part is the total payment for the wholesale bandwidth, the second part is the income the user obtains from the carrier for selling the unused bandwidth, and the third part is the total payment for buying the extra bandwidth from the carrier.

Let  $P_{new\_avg}(\beta, y) = \frac{P_{new}(\beta, y)}{uT}$ , then

$$P_{new\_avg}(\beta, y) = y - \alpha \int_0^y (y - x) f_X(x) dx + \beta \int_y^b (x - y) f_X(x) dx, \quad 0 \leq y \leq b \quad (3)$$


 Fig. 3. Average user cost  $P_{new\_avg}$  vs. wholesale bandwidth amount  $y$ 

Put  $f_X(x) = \frac{\lambda e^{-\lambda x}}{1-e^{-\lambda b}}$  into Eqn. (3), we get

$$P_{new\_avg}(\beta, y) = y - \frac{1}{1-e^{-\lambda b}} [(\alpha - \beta e^{-\lambda b})y + \left(\frac{\alpha}{\lambda} - \frac{\beta}{\lambda}\right) e^{-\lambda y} - \frac{\alpha}{\lambda} + \beta \left(\frac{1}{\lambda} + b\right) e^{-\lambda b}], \quad 0 \leq y \leq b \quad (4)$$

Fig. 3 shows the average user cost under different wholesale bandwidth amount  $y$  and different peak traffic demand  $b$ , where  $\lambda = 0.05$ ,  $\alpha = 0.5$ , and  $\beta = 1.5$ . From Fig. 3, we can see that a minimum cost exists under a unique value of  $y$ . We denote the unique  $y$  that minimizes the user cost as

$$y^*(\beta) = \arg \min_{0 \leq y \leq b} P_{new\_avg}(\beta, y) \quad (5)$$

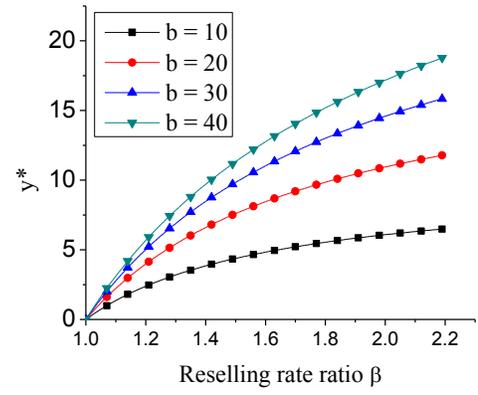
If the wholesale bandwidth amount is less than  $y^*$ , the user may have to pay more for the extra bandwidth; else, if the wholesale bandwidth is greater than  $y^*$ , the user pays less for the extra bandwidth, but there will be more unused bandwidth and the penalty of reselling the unused bandwidth is larger. In order to find out  $y^*(\beta)$ , we take the partial derivative of Eqn. (4) over  $y$ :

$$\frac{\partial P_{new\_avg}(\beta, y)}{\partial y} = 1 - \frac{\alpha - \beta e^{-\lambda b}}{1-e^{-\lambda b}} - \frac{(\beta - \alpha)e^{-\lambda y}}{1-e^{-\lambda b}} \quad (6)$$

Let  $\frac{\partial P_{new\_avg}(\beta, y)}{\partial y} = 0$ , we have

$$y^*(\beta) = -\frac{1}{\lambda} \ln \frac{(\beta - 1)e^{-\lambda b} - \alpha + 1}{\beta - \alpha} \quad (7)$$

From  $0 \leq y^*(\beta) \leq b$ , we get  $\beta \geq 1$  and  $\alpha \leq 1$ , given that  $\lambda > 0$  and  $b \geq 0$ . Thus, as long as  $\beta \geq 1$  and  $\alpha \leq 1$  hold, we can find the unique  $y^*(\beta)$  that minimizes the user cost. Fig. 4 shows  $y^*$  under different  $\beta$  and different  $b$ , where  $\lambda = 0.05$ ,  $\alpha = 0.5$ . It shows that  $y^*$  increases when  $\beta$  increases. When  $\beta$  increases, the extra bandwidth becomes more expensive, thus the user prefers to buy more wholesale bandwidth in order to reduce the need for extra bandwidth. When  $\beta$  is 1.0, the user does not buy any wholesale bandwidth because the wholesale rate and the extra bandwidth rate are the same, and thus there is no need to purchase long-term wholesale bandwidth.


 Fig. 4.  $y^*$  vs. reselling rate ratio  $\beta$ 

Finally, the minimum average user cost under the new mechanism is only determined by  $\beta$  and can be expressed as

$$P_{new\_avg}^*(\beta) = y^*(\beta) - \frac{1}{1-e^{-\lambda b}} [(\alpha - \beta e^{-\lambda b})y^*(\beta) + \left(\frac{\alpha}{\lambda} - \frac{\beta}{\lambda}\right) e^{-\lambda y^*(\beta)} - \frac{\alpha}{\lambda} + \beta \left(\frac{1}{\lambda} + b\right) e^{-\lambda b}] \quad (8)$$

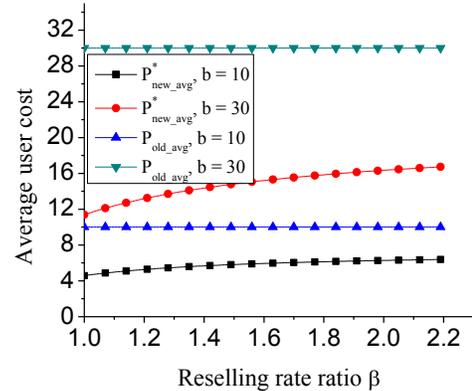
In the old mechanism, the user purchases wholesale bandwidth at its peak traffic demand, which is  $b$  in this case. Thus, the user cost using the old mechanism is

$$P_{old} = buT \quad (9)$$

Similar to  $P_{new\_avg}$ , let  $P_{old\_avg} = \frac{P_{old}}{uT}$ , we have

$$P_{old\_avg} = b \quad (10)$$

Fig. 5 shows the average user cost under different  $\beta$  and different  $b$ , where  $\lambda = 0.05$ ,  $\alpha = 0.5$ . We can see that under the new mechanism, the user cost increases along with the increase of  $\beta$ . Although the user can increase the amount of wholesale bandwidth to make up for the rising price of extra bandwidth, it is not enough to compensate for the increasing payment for the extra bandwidth that it needs to buy. This is why the new average user cost increases non-linearly. The old average user cost is constant since it is not affected by the  $\beta$  values. It is


 Fig. 5. Average user cost vs. reselling rate ratio  $\beta$

shown that users pay less under the new mechanism. The cost savings are up to 54.2% when  $b = 10$ , and 62.1% when  $b = 30$ . As  $\beta$  value is decided by the carrier in Stage I, in the next step, we study how the carrier should decide its  $\beta$  value, and whether or not  $P_{new\_avg}^*(\beta) < P_{old\_avg}$  still holds under that  $\beta$  value.

### B. Determining reselling rate ratio $\beta$ in Stage I

In Stage I, the carrier needs to determine the reselling rate ratio  $\beta$  to maximize its total revenue. For simpler analysis, we assume all the users follow the same truncated exponential distribution with the same parameters  $\lambda$  and  $b$ . Also, the traffic distributions of different users are independent. Assume the traffic demands of User 1, User 2, ..., User  $n$  are represented as random variables  $X_1, X_2, \dots, X_n$ , and the sequence of variables  $\{X_1, X_2, \dots, X_n\}$  is i.i.d.

The carrier needs to solve the following problem

$$\begin{aligned} \beta^* &= \arg \max_{\beta \geq 1} R_{new\_avg}(\beta) \\ &= \arg \max_{\beta \geq 1} N_{new}(\beta) P_{new\_avg}^*(\beta) \end{aligned} \quad (11)$$

where  $N_{new}(\beta)$  is the number of users the carrier can accommodate under  $\beta$  using the new mechanism. When  $\beta$  is small, the carrier earns less from reselling the unused bandwidth. On the other hand, when  $\beta$  is large, the carrier earns more from reselling the unused bandwidth; however, the number of users it can accommodate is smaller. The reason is that  $y^*(\beta)$  is larger when  $\beta$  is larger and the carrier has to guarantee each user's wholesale bandwidth amount  $y^*(\beta)$  under given capacity  $C$ , i.e.,  $N_{new}(\beta)y^*(\beta) \leq C$ , and thus  $N_{new}(\beta)$  becomes smaller when  $\beta$  increases. From the analysis above, there should be a  $\beta^*$  which maximizes the carrier revenue.

Note that  $N_{new} \neq N_{old}$ . In the old mechanism, the user buys the same bandwidth amount as its peak traffic demand and it does not resell its unused bandwidth. Thus,  $N_{old}$  is obtained by

$$N_{old} = \left\lfloor \frac{C}{b} \right\rfloor \quad (12)$$

In the new mechanism, the carrier buys the unused bandwidth from one user and sells it to other users in need. Thus, the new mechanism may support more users than the old mechanism. However, the multiplexing of users in the new mechanism may result in congestion, while there is no congestion in the old mechanism. In order to make the new mechanism and old mechanism comparable, we have to limit the congestion probability to a very small value, such that it is negligible. To measure the congestion probability, we firstly need to study the aggregated traffic of all the users and its distribution.

Suppose the aggregated traffic of  $n$  users is denoted by a random variable  $S_n$ , i.e.,  $S_n = X_1 + X_2 + \dots + X_n$ . According to the classical central limit theorem [15], given that  $\{X_1, X_2, \dots, X_n\}$  is a sequence of i.i.d random variables with  $E[X_i] = \mu$  and  $\text{Var}[X_i] = \sigma^2 < \infty$ , then for large  $n$ , the random variable  $S_n$  is close to a normal distribution  $N(n\mu, n\sigma^2)$ . In our analysis, the

random variable  $X_i$  follows a truncated exponential distribution with parameters  $\lambda$  and  $b$ , and its mean is given by

$$\mu = \frac{1}{\lambda} - \frac{be^{-\lambda b}}{1-e^{-\lambda b}} \quad (13)$$

and the variance is

$$\sigma^2 = \frac{1}{e^{-\lambda b} - 1} \left[ b^2 e^{-\lambda b} + \frac{2}{\lambda} \left( b e^{-\lambda b} + \frac{1}{\lambda} e^{-\lambda b} - \frac{1}{\lambda} \right) \right] - \mu^2 \quad (14)$$

Assume we have a large number of users,  $S_n$  then approximately follows the normal distribution  $N(n\mu, n\sigma^2)$ . According to the 68-95-99.7 rule [16], approximate 99.7% of the values of  $S_n$  lie within 3 standard deviations of the mean, i.e.,  $\text{Prob}(n\mu - 3\sqrt{n}\sigma \leq s_n \leq n\mu + 3\sqrt{n}\sigma) \approx 0.9973$ . Then we have  $\text{Prob}(s_n \leq n\mu + 3\sqrt{n}\sigma) \approx 0.999$ . Thus, if we let  $n\mu + 3\sqrt{n}\sigma \leq C$ , then the congestion probability is at most 0.1%, which is negligible. Meanwhile, the carrier has to guarantee each user's wholesale bandwidth amount  $y^*(\beta)$  at any time, i.e.,  $ny^*(\beta) \leq C$ . Together we have two constraints for  $N_{new}(\beta)$ , both of which need to be satisfied:

$$N_{new}(\beta) \leq \left\lfloor \left( \frac{-3\sigma + \sqrt{9\sigma^2 + 4\mu C}}{2\mu} \right)^2 \right\rfloor \quad (15)$$

$$N_{new}(\beta) \leq \left\lfloor \frac{C}{y^*(\beta)} \right\rfloor \quad (16)$$

Overall, we have

$$N_{new}(\beta) = \min \left( \left\lfloor \left( \frac{-3\sigma + \sqrt{9\sigma^2 + 4\mu C}}{2\mu} \right)^2 \right\rfloor, \left\lfloor \frac{C}{y^*(\beta)} \right\rfloor \right) \quad (17)$$

Fig. 6 shows the number of users under different  $\beta$  and different  $b$ , where  $\lambda = 0.05$ ,  $\alpha = 0.5$ , and  $C = 10^4$ . From Fig. 6, we can see that under new mechanism, the number of users is constant when  $\beta$  is small, and starts to decrease after a certain inflection point. When  $\beta$  is small,  $y^*$  is small, and thus  $N_{new}(\beta)$  obtained by Eqn. (16) is large. Under this situation,

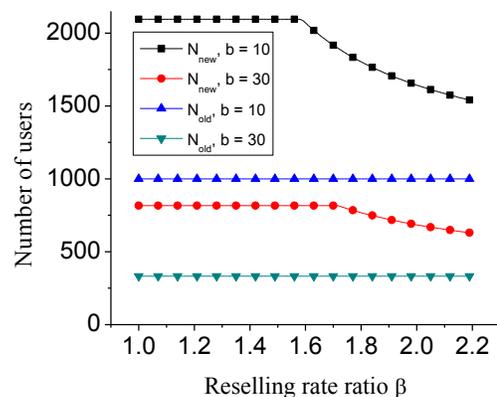


Fig. 6. Number of users vs. reselling rate ratio  $\beta$

Eqn. (15) is dominant over Eqn. (16). Since Eqn. (15) does not depend on  $\beta$ , the  $N_{new}$  remains unchanged. However, when  $\beta$  increases to a certain value,  $N_{new}(\beta)$  obtained by Eqn. (16) is smaller than that obtained by Eqn. (15), and thus Eqn. (16) dominates. This is why  $N_{new}$  starts to decrease when  $\beta$  is greater than a certain value. As shown in Fig. 6, the new mechanism supports more users than the old mechanism. The increase in the number of users is up to 109.5% when  $b = 10$ , and 145.0% when  $b = 30$ .

Finally, the maximum carrier revenue under the new mechanism is

$$R_{new\_avg}^* = N_{new}(\beta^*)P_{new\_avg}^*(\beta^*) \quad (18)$$

and the maximum carrier revenue under the old mechanism is

$$R_{old\_avg} = N_{old}P_{old\_avg} = \left\lfloor \frac{C}{b} \right\rfloor b \quad (19)$$

Fig. 7 shows the carrier revenue under different  $\beta$  and  $b$ , where  $\lambda = 0.05$ ,  $\alpha = 0.5$ , and  $C = 10^4$ . It is shown that under the new mechanism, the carrier revenue increases before a certain value of  $\beta$ , and decreases after that. As discussed in Fig. 6, the number of users is constant before the inflection point, thus the revenue increase comes from the rising price of extra bandwidth. When  $\beta$  increases to the inflection point, the number of users starts to drop due to the increase of  $y^*$ . Although the price of the extra bandwidth continues to increase, it is not sufficient to make up for the reducing number of users. On the other hand, the carrier revenue under the old mechanism remains constant since it does not depend on  $\beta$ . Also, the  $R_{old\_avg}$  under  $b = 10$  and  $b = 30$  are almost the same, since  $R_{old\_avg} \approx C$ . From Fig. 7 and Fig. 5 together, when  $b = 10$ ,  $\beta^* = 1.58$ ,  $R_{new\_avg}^* = 12406.6$ ,  $R_{old\_avg} = 10000$ ,  $P_{new\_avg}^* = 5.9$ ,  $P_{old\_avg} = 10$ . Using the new mechanism, the carrier revenue is increased by 24.1%, and the user cost is decreased by 41.0%. When  $b = 30$ ,  $\beta^* = 1.72$ ,  $R_{new\_avg}^* = 12723.8$ ,  $R_{old\_avg} = 9990$ ,  $P_{new\_avg}^* = 15.6$ ,  $P_{old\_avg} = 30$ . Using the new mechanism, the carrier revenue is increased by 27.4%, and the user cost is decreased by 48.0%. Consequently, in this case, the “win-win” situation exists under SPE. In the next section, we test other cases to show that the “win-win” situation exists in other cases as well.

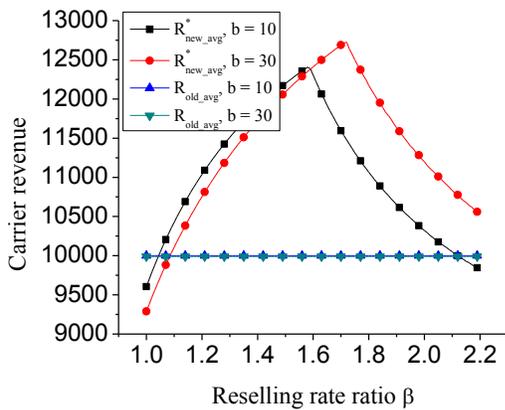


Fig. 7. Carrier revenue vs. reselling rate ratio  $\beta$

## V. NUMERICAL RESULTS

In this section, we present some numerical results to compare the new mechanism and the old mechanism. We firstly fix the capacity  $C$  and test the results under different  $\alpha$  and  $\beta$ , and then we fix  $\alpha$  and test the results under different  $C$  and  $\beta$ .

### A. Varying $\alpha$ and $\beta$

We compare the user cost and carrier revenue of the new mechanism with the old mechanism under different  $\alpha$  and  $\beta$ . In the experiment,  $\lambda = 0.05$ ,  $b = 10$ ,  $C = 5 \times 10^4$ . Under these settings, the average user cost and the carrier revenue under the old mechanism are 10 and  $5 \times 10^4$ , respectively. Fig. 8 shows the average user cost of the new mechanism under different  $\alpha$  and  $\beta$ . From Fig. 8, we can see that the average user cost of the new mechanism is always lower than 10, and thus always lower than the old mechanism, when  $\alpha$  and  $\beta$  vary.

Fig. 9 shows the carrier revenue of the new mechanism under different  $\alpha$  and  $\beta$ . The values of  $R_{new\_avg}$  greater than  $R_{old\_avg}$  are in green color, while the values smaller than  $R_{old\_avg}$  are in red color. It is shown that no matter how  $\alpha$  varies, the carrier revenue of the new mechanism is always greater than the old mechanism under  $\beta^*$ . Since the average user cost of new mechanism is also lower under  $\beta^*$ , the “win-win” situation always exists under SPE, no matter how  $\alpha$  changes.

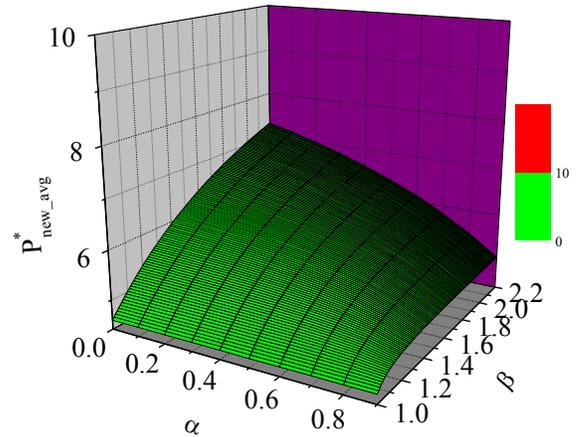


Fig. 8.  $P_{new\_avg}^*$  vs.  $\alpha$  and  $\beta$

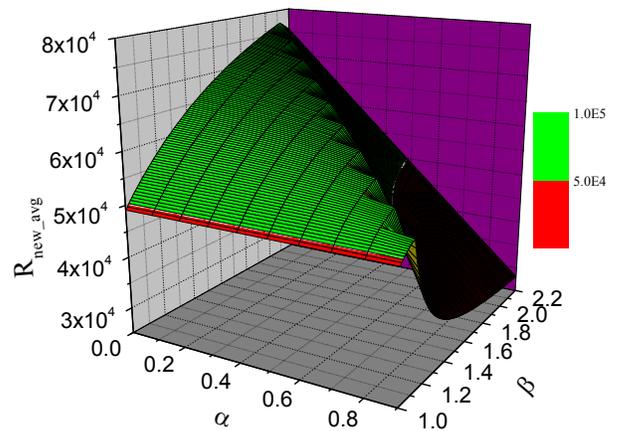
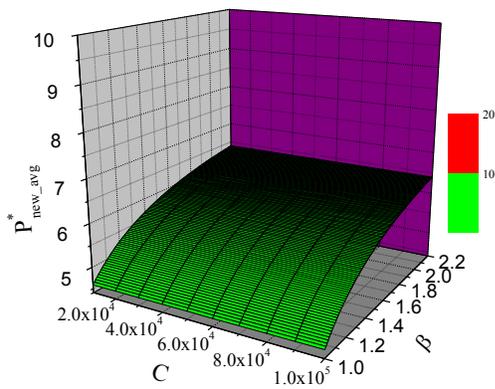
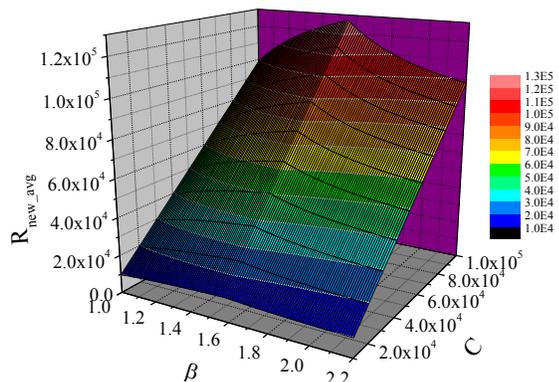


Fig. 9.  $R_{new\_avg}$  vs.  $\alpha$  and  $\beta$

Fig. 10.  $P_{new\_avg}^*$  vs.  $C$  and  $\beta$ Fig. 11.  $R_{new\_avg}$  vs.  $C$  and  $\beta$ 

### B. Varying $C$ and $\beta$

In this experiment, we compare the user cost and carrier revenue of the new mechanism with the old mechanism under different  $C$  and  $\beta$ . In the experiment,  $\lambda = 0.05$ ,  $b = 10$ ,  $\alpha = 0.5$ . Under these settings, the average user cost under the old mechanism is 10. The carrier revenue under the old mechanism varies according to the capacity  $C$ . In this experiment,  $C = \{10^4, 2 \times 10^4, \dots, 10^5\}$ , so  $R_{old\_avg} = \{10^4, 2 \times 10^4, \dots, 10^5\}$  accordingly. Fig. 10 shows the average user cost of the new mechanism under different  $C$  and  $\beta$ . From Fig. 10, we can see that the average user cost of the new mechanism is always lower than the old mechanism ( $P_{new\_avg}$  is lower than 10), when  $C$  and  $\beta$  vary.

Fig. 11 shows the carrier revenue of the new mechanism under different  $C$  and  $\beta$ . Different  $R_{old\_avg}$  values are represented with different colors shown in the legend. It is shown that  $R_{new\_avg}$  achieves its maximum value under almost the same  $\beta$  (around 1.56), when the capacity  $C$  varies. The maximum revenue  $R_{new\_avg}^*$  is always greater than the corresponding  $R_{old\_avg}$  under the same capacity  $C$ . Since the average user cost of new mechanism is also lower under  $\beta^*$ , the “win-win” situation always exists under SPE, no matter how the capacity  $C$  changes.

## VI. CONCLUSION

The current exclusive resource allocation in virtual network embedding, in which the users reserve resources at their peak

demands, has led to resource under-utilization, especially when the peak demand rarely happens. The user has to pay for the resources it does not actually use, and the carrier has no way to make extra income from the users’ unused resources, since the resources are allocated exclusively and excessively. In this paper, we proposed a dynamic resource pooling and trading mechanism for the users and carrier to exchange the unused resource. The proposed mechanism was then formulated as a Stackelberg game, using bandwidth as an example of the resources. The Subgame Perfect Equilibrium (SPE) was found through backward induction. The numerical results showed that the “win-win” situation, in which the user saves cost and the carrier increases its revenue, exists under the Subgame Perfect Equilibrium, while keeping 99.9% bandwidth availability to the users.

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