Sequential Detection Techniques for Unresolved Multipath Ricean/Rayleigh Fading Channels

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Abstract

With proliferation of wireless applications, detection techniques for multipath fading channels have received a growing interest. This thesis presents several sequential detection schemes for multipath mixed mode Ricean/Rayleigh channels, as well as multipath Rayleigh fading channels. The multipath delays are assumed known but unresolved and the phase information of the paths is not available (non-coherent). Although the closed form log-likelihood function for mixed mode channels and multipath Rayleigh channels can be obtained, the non-linearities of the functions make the optimal approach based on Maximum Likelihood Sequence Estimation (MLSE) not suitable for large sequence lengths or large multipath spreads. We propose a sub-optimal pilot-aided receiver structure based on the expectation-maximization (EM) algorithm, which provides a non-exhaustive approach to achieve close to optimal performance by iteratively finding the ML estimate. Using a decorrelation matrix, the algorithm resolves received samples at each iteration and thus diversity-like gains can be obtained even if the multipath is unresolved, and the channel is only known statistically. For the purpose of comparison, We also derive and implement two types of Known-channel Minimum Mean Square Error receiver (K-MMSE), one of which is with Single-sided tap delay line (KS-MMSE) and the other is with Double-sided tap delay line (KD-MMSE). Simulations are performed for BPSK and 4QAM DS-CDMA system and it is seen that the EM algorithm yields good performance. Simulation results are provided to demonstrate the superiority of the EM-based structure over the KS-MMSE scheme that assumes instantaneous knowledge of the channel is illustrated. It is also shown that the performance of the EM-based receiver is very close to the performance of KD-MMSE approach with complete channel knowledge, although the EM-based structure does not know the channel. We can see the double-sided tap de-
lay line outperforms the single-sided one greatly for our K-MMSE structures. Finally, it is shown that the performance of the EM algorithm may depend on the particular choice of spreading codes, especially for small spreading gains. Better performance is obtained for spreading codes with a low correlation at the channel interpath delays.
Acknowledgments

First of all, I would like to express my deep gratitude to my supervisor, Dr. Florence Danilo-Lemoine, for all of the time and patience she has given me throughout this research. She has provided a solid basis to my studies. Without her careful guidance, continuous support and critical insight, I would not have reached this point. I would also like to thank my colleagues in BCWS lab for many discussions we did while doing my work. Finally, I am really thankful to my family and friends for their love and supports during my graduate studies.
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<td>Binary Phase Shift Keying</td>
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<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>DDFSE</td>
<td>Delayed Decision-Feedback Sequence Estimation</td>
</tr>
<tr>
<td>DFE</td>
<td>Decision-Feedback Equalization</td>
</tr>
<tr>
<td>DPSK</td>
<td>Differential Phase Shift Keying</td>
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<tr>
<td>DS-CDMA</td>
<td>Direct Sequence Code-Division Multiple Access</td>
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<tr>
<td>EM</td>
<td>Expectation-Maximization</td>
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<td>EM-PS</td>
<td>Expectation-Maximization with Pilot Symbol</td>
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<tr>
<td>FSK</td>
<td>Frequency Shift Keying</td>
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<tr>
<td>GMLSDE</td>
<td>Generalized ML Sequential Detection and Estimation</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol-Interference</td>
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<tr>
<td>JDE</td>
<td>Joint multiuser Detection and multichannel Estimation</td>
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<tr>
<td>K-MMSE</td>
<td>Known channel and Minimum Mean-Square Error</td>
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<tr>
<td>KD-MMSE</td>
<td>Known channel and Double-sided tapped delay line Minimum Mean-Square Error</td>
</tr>
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<td>KS-MMSE</td>
<td>Known channel and Single-sided tapped delay line Minimum Mean-Square Error</td>
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<td>LE</td>
<td>Linear Equalization</td>
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<td>MLSDE</td>
<td>ML Sequential Detection/Estimation</td>
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<td>Abbreviation</td>
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<tr>
<td>MLSE</td>
<td>Maximum Likelihood Sequence Estimation</td>
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<td>MMSE</td>
<td>Minimum Mean Square Error</td>
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<td>MPEM</td>
<td>&quot;Missing Parameter&quot; EM</td>
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<td>MS</td>
<td>Mobile Station</td>
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<td>PCS</td>
<td>Personal Communication Systems</td>
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<td>pdf</td>
<td>Probability density function</td>
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<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<td>QDR</td>
<td>Quadratic Decorrelation Receiver</td>
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<td>RSSE</td>
<td>Reduced-State Sequence Estimation</td>
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<td>SAGE</td>
<td>Space-alternating Generalized Expectation-maximization</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>WLAN</td>
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<td>$T$</td>
<td>Transposition</td>
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<td>*</td>
<td>Complex conjugation</td>
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<td>†</td>
<td>Hermitian conjugation</td>
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<td>$\mathbf{v}, \overline{B}$</td>
<td>Mean of a vector $\mathbf{v}$ or mean of a matrix $B$</td>
<td>30</td>
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<td>$\langle x(t), y(t) \rangle$</td>
<td>Inner product of $x(t)$ and $y(t)$</td>
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<td>$[\mathbf{v}]_i$</td>
<td>$i^{th}$ entry of a column vector $\mathbf{v}$</td>
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<tr>
<td>$[B]_{ij}$</td>
<td>$ij^{th}$ entry of a matrix $B$</td>
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<tr>
<td>${B}_d$</td>
<td>Diagonal matrix composed of the main diagonal entries of $B$</td>
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</tr>
<tr>
<td>${B}_l$</td>
<td>Lower triangular matrix composed of the lower triangular elements of $B$ with zero main diagonal entries</td>
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<td>$</td>
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<td>$(\Delta f)_c$</td>
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<td>$h(t)$</td>
<td>Low-pass complex impulse response of the channel (assumed to be static)</td>
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<td>$L \times L$ identity matrix</td>
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<td>$\Im {\cdot}$</td>
<td>Imaginary part of</td>
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<td>$J(a^m)$</td>
<td>Part of $f(r(a^m)</td>
<td>\theta_1,a^m)$ independent of $\theta_1$</td>
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<td>$K$</td>
<td>Ricean parameter</td>
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<td>$K(t,u;a)$</td>
<td>Covariance function of $v(t,a)$ conditioned on $\theta$</td>
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<td>$K_p$</td>
<td>Constant present in $P(f)$ related to the local mean power</td>
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<td>$\Lambda(a_m</td>
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<td>Conditional likelihood ratio given $\theta$ under $H_m$ see (3.15)</td>
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<td>$L$</td>
<td>Number of (major) paths (finite time resolution)</td>
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<td>Part of $\Lambda(a_m</td>
<td>\theta)$ that is independent of $\theta$ see (3.21)</td>
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<td>$M_{ij}(a^m)$</td>
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<td>$M$</td>
<td>Number of paths (infinite time resolution)</td>
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<td>$M_l$</td>
<td>Number of subpaths in a major path</td>
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<td>$M_\Delta$</td>
<td>Number of delay elements in the K-MMSE structures</td>
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<td>$N$</td>
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<td>Column vector with entries $n_k(\mathbf{a})$</td>
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<td>Noise, modeled as a white Gaussian circularly complex random process see (3.2)</td>
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<td>$n_k(\mathbf{a})$</td>
<td>Projection of $n(t)$ on $\phi_k(t; \mathbf{a})$</td>
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Chapter 1

Introduction

1.1 Thesis objectives

In the past decades, to reach the goal of communicating with anybody at anytime and anywhere, lots of attention has been given to both indoor and outdoor wireless communications including Wireless Local Area Networks (WLAN), Personal Communication Systems (PCS) and so on. Due to the characteristics of the radio wave propagation, i.e., reflection, diffraction and scattering, signal strength will be reduced not only by the large-scale path loss, but also by small-scale fading caused by multipath propagation. Multiple versions of the transmitted signal arrive at the receiver from different transmission paths. The phase, amplitude and time delay of these versions are all different due to the different transmission paths, e.g., one is line-of-sight and another is a reflection from a building. The sum of them causes signal distortion and fluctuation of the amplitude of the received signal, which is called multipath fading. In addition, the phase and the amplitude of each path component may vary with time because of the movement of the transmitter or receiver or the surrounding objects, and there is also channel noise. How to detect signals from such complicated
random wireless channels accurately and economically is a big challenge.

Different techniques such as equalization, diversity and channel coding [1] are used to compensate for different signal impairments caused by multipath fading. The characteristics of the channel and the bandwidth of the transmission signal decide the choice of a particular channel model. One characteristic of a multipath channel is the time spread introduced by the multiple delayed versions of the transmitted signal. The relationship between the signal duration and the inter-path delays, or equivalently, (as the transmitted signal and channel coherence bandwidths can be respectively approximated by the reciprocals of the signal duration and the delay spread) the relationship between the transmitted signal and channel coherence bandwidths determines whether a channel is flat or frequency selective.

Flat fading occurs when the channel coherence bandwidth is much larger than the signal bandwidth, corresponding to narrow-band transmissions. All versions from different paths cannot be separated from one to another due to the small inter-path delays. No Inter-Symbol-Interference (ISI) exists in such a channel and the signal is only corrupted by the channel attenuation and channel noise. Many diversity techniques such as spatial diversity [2][3] and frequency diversity [4] are used to combat the effects of the depth and duration of the fades in such channels, and yield diversity gains.

When the bandwidth of the signal is much wider than the channel coherence bandwidth corresponding to wide-band transmissions, the amplitude and phase of the signal at different frequencies may experience different attenuations and shifts through the channel. This kind of channel is called a frequency selective channel. In the time domain, the received signal is composed of several attenuated versions of the transmitted signal, each delayed by a different path delay. The maximum amount of delay is referred to as the delay spread. Each path consists of subpaths which cannot
be separated (i.e., cannot be resolved). When the delay spread is larger than the symbol duration, ISI exists in such channels because the received signal at a sampling time not only contains the information of current symbol, but also the information of last and next few symbols. Therefore, in that case estimating the transmitted signal sequentially instead of symbol-by-symbol may improve the estimation performance.

Traditionally, Rake receivers [5]-[7] have been applied to multipath fading channels for wide-band signals to achieve the performance of an equivalent diversity communication system. Inter-path delays are required to be large enough to avoid overlap of signals from different paths after matched filtering (resolved multipath) so that the receiver can use a weighted sum of these time-shifted versions to obtain a higher Signal-to-Noise Ratio (SNR). Such resolvability condition is satisfied whenever the inter-path delays are much larger than the transmitted signal autocorrelation time. The autocorrelation time is defined as the width of the autocorrelation function (or the width containing essentially non-zero energy), and is approximately equal to the reciprocal of the signal bandwidth. Therefore, the path resolvability assumption is satisfied for spread-spectrum systems in outdoor environments, but is generally not satisfied in dense urban or indoor environments due to the very small inter-path delays (of the order of one to tens of μs).

This thesis will assume a wide-band channel model with known multipath delays such that the path resolvability assumption is not satisfied. Since this work is applicable more particularly to dense urban or indoor environments characterized by a small Doppler spread, this thesis will assume that the known multipath delays are constant for one transmission block. They can be estimated by probing the channel with a wide-band signal or by using super-resolution techniques [8]-[10]. Similarly, since no significant moves of the transmitter and receiver, and changes in the environment are expected within one transmission block, this thesis will assume a slow fading channel,
i.e. the channel fadings (including amplitudes and phases) are assumed fixed during one transmission block.

A generalization of the Rake receiver for narrow-band systems when the multi-path is not entirely resolved has been presented in [11] for multipath Rayleigh fading channels and in [12, 13] for multipath Rayleigh/Ricean fading channels, both based on a single shot transmission approach. In [14], multi-shot case was considered and a decorrelating filter was used to mitigate ISI and resolve the inter-path cross-correlation for frequency selective Rayleigh fading channels. Performance of the decorrelator over two-path Rayleigh fading channels was compared to the Rake receiver and the linear equalizer. Following [15], where the EM algorithm was used to extract parameter estimates from superimposed signals, Spasojevic and Georghiades introduced Expectation-Maximization with Pilot Symbol (EM-PS) approach (pilot symbols are inserted into transmitted data symbols in order to reduce the detection error rate) in [16] for sequence estimation in time-invariant unresolved multipath Rayleigh fading channels not neglecting ISI. Signal resolution was performed based on the general autocorrelation structure of the signal and hence its performance is close to the Maximum-likelihood (ML) receiver (optimal receiver) even when path cross-correlations are high. Various other works based on the EM algorithm or variations, such as the Space-alternating Generalized EM (SAGE) [17] can be found in the literature, and are summarized in section 2.2.2. However none of these works were developed for multipath Ricean fading channels.

This thesis considers sequential detection techniques for unresolved slow fading mixed mode Ricean/Rayleigh multipath (i.e. frequency selective) fading channels. A mixed mode Ricean/Rayleigh channel is a channel whose first path gain is Ricean distributed and the other path gains are Rayleigh. This channel may represent line-of-sight transmission. Multipath Rayleigh channels will be treated as a special case
of the mixed mode channels. Non-coherent detection will be considered in this work and the corresponding channel model will be introduced in section 3.1. In this thesis, the term non-coherent means that the magnitude of the specular component of the Ricean path is known and the phase information is unknown to the receiver.

The goal of our detection technique is to achieve close to optimal performance by a non-exhaustive approach. It is well known that the Maximum Likelihood Sequence Estimation (MLSE) is the optimal receiver for multipath fading channels in the sense of minimizing the bit error probability. But, the MLSE is hard to implement due to its complexity, especially for long sequence lengths. Therefore, we propose a sequential receiver structure based on the EM algorithm to estimate the transmitted sequence which maximizes the log likelihood function. For purpose of comparison, the performance of MLSE will still be obtained by simulation using an exhaustive method with short transmitted sequences. Performance of the proposed scheme will be also compared to two Minimum Mean-Square Error (MMSE)-based equalizer receivers that assume knowledge of the channel. The first one estimates the current transmitted data by using only the current and past received samples and is referred to as the Known channel and Single-sided tapped delay line Minimum Mean Square Error receiver (KS-MMSE). The second one uses past, current and future received samples and is referred to as the Known channel and Double-sided tapped delay line Minimum Mean-Square Error receiver (KD-MMSE). In this thesis, we consider uncoded linear modulation systems. Other modulations and the effect of coding are left for future consideration.
1.2 Thesis contributions

The main contribution of the thesis is the derivation of the non-coherent sequential receiver structure based on the EM\(^1\) algorithm for mixed mode Ricean/Rayleigh channels. By using the EM algorithm, which is an iterative two-steps algorithm (Expectation step and Maximization step), an estimate of the ML sequence is found without relying on an exhaustive search. Furthermore, it is shown that for linear modulations, the maximization step reduces to a symbol-by-symbol maximization, where the solution can be found in closed-form. Performance of the proposed scheme is illustrated using Direct Sequence Code-Division Multiple Access (DS-CDMA), demonstrating in particular the effects of different spreading codes. MLSE and Known channel and Minimum Mean-Square Error (K-MMSE) approaches are also considered for comparison. A list of the contributions of the thesis is as follows:

- Derivation of the non-coherent sequential receiver structure based on the EM algorithm for mixed mode Ricean/Rayleigh channels (section 3.3.1). It is shown that the proposed scheme reduces to the structure of [16] for multipath Rayleigh channels (appendix G).

- Demonstration that for linear modulations, the maximization step of the EM for mixed mode Ricean/Rayleigh channels reduces to a symbol-by-symbol maximization and derivation of the closed-form solution (section 3.3.1).

- Derivation of the log likelihood function for MLSE over unresolved multipath mixed mode Ricean/Rayleigh and Rayleigh channels (section 3.2).

---

\(^1\)The EM algorithm and its applications will be reviewed in detail in section 2.4.
1 Introduction

- Performance study of the EM-based receiver for various multipath Ricean/Rayleigh fading channels for Binary Phase Shift Keying (BPSK) and Quadrature Amplitude Modulation (QAM) (section 4.1.2).

- Presentation of performance comparison among the EM approach, MLSE and K-MMSE approach (section 3.4) over mixed mode Ricean/Rayleigh channels and multipath Rayleigh fading channels (section 4.2).

- Study of the effects of spreading codes on the performance of DS-CDMA using the EM approach, and outline of guidelines towards the design of good spreading codes for improved performance over unresolved multipath fading channels using the EM approach (section 4.3).

1.3 Thesis organization

- Chapter 2 gives in depth theoretical background on multipath fading channels explaining in particular flat versus frequency selective channels and resolved multipath versus unresolved multipath. It also reviews works that have been done for detection techniques over such channels and presents the remaining sequential detection problems for unresolved multipath fading channels and the thesis objectives. Finally, this chapter gives an overview of the EM algorithm, which will be used in our proposed receiver structure.

- Chapter 3 introduces the non-coherent mixed mode Ricean/Rayleigh channel model. The likelihood ratio function for the MLSE (the optimal receiver) and the goal function for the EM approach over such a channel are derived. It is shown that for linear modulations, the maximization step of the EM algorithm reduces to a symbol-by-symbol maximization and the solution is derived in
closed-form, thus providing a receiver with low complexity. The receiver structure based on the EM algorithm, and also the likelihood ratio function for the MLSE for multipath Rayleigh channels are derived as a special case of mixed mode channels. MMSE-based equalizer receivers (with single-sided or double-sided tapped delay lines) for unresolved multipath fading channels that assume known channel coefficients (K-MMSE) are also developed in this chapter.

- Chapter 4 describes the implementation details of the channels and receivers for computer simulations. Simulation results of the performances of the EM approach for BPSK and QAM over various multipath fading channels (including both mixed mode channels and Rayleigh fading channels) are presented. Performance comparisons of K-MMSE approach, our proposed EM-based receiver and the MLSE (optimal scheme) are illustrated. Finally, we analyze the performance of the EM-based receiver using different spreading codes with small spreading gains and show that the performance depends on the particular choice of spreading codes.

- Chapter 5 summarizes the thesis and proposes future works.
Chapter 2

Background knowledge and rationale

Wireless communications occur over multipath fading channels. Multipath fading channel is one of the most challenging channels experienced by communications. Lots of research on detection techniques in such channels have been done over decades. This chapter describes multipath fading channels in detail and also provides a review of existing detection techniques applied for such multipath channels. The thesis objective is formulated. In the last section, an overview of the EM algorithm is provided, as the novel receiver we propose in Chapter 3 of this thesis will be based on that algorithm.

2.1 Multipath fading channels

In some situations such as mobile communications, signals sent by the transmitter arrive at the receiver by direct propagation, reflection, diffraction and scattering by ground, buildings or mountains, i.e, through different paths, yielding to the so-called multipath phenomenon. Fig. 2.1 illustrates a typical scenario of multipath propa-
gation (MS and BS in Fig. 2.1 represent mobile station and base station). Thus, the receiver may get different versions of the transmitted signal with different delays. Mathematically, assuming no noise, the complex envelope of the received signal can be expressed as [18, section 2.1.1]

\[ r(t) = \sum_{i=1}^{M} c_i(t) e^{j\gamma_i(t)} s(t - \tau_i(t)) \]  \hspace{1cm} (2.1)

where \( M \) is the number of paths (assuming an infinite time resolution), \( c_i(t) \) is the amplitude of the \( i^{th} \) path, \( \gamma_i(t) \) is the phase of the \( i^{th} \) path, \( \tau_i(t) \) is the delay of the \( i^{th} \) path and \( s(t) \) is the complex envelope of the transmitted signal. Equivalently the channel low-pass complex impulse response is given by

\[ h(\tau, t) = \sum_{i=1}^{M} c_i(t) e^{j\gamma_i(t)} \delta(\tau - \tau_i(t)) \]
where the coefficients \{\varsigma_i(t)\}, \{\gamma_i(t)\} and \{\tau_i(t)\} can be estimated and/or characterized statistically. The most common assumptions for multipath are those of Wide-Sense stationary impulse response and Uncorrelated Scattering (WSSUS) [19]. The autocorrelation of the impulse response of a WSSUS channel is given by

\[
E \left[ h^*(\tau, t) h(\tau', t + \Delta t) / 2 \right] = R_h(\tau, \Delta t) \delta(\tau - \tau')
\]

where \(R_h(\tau, 0)\) is called the channel multipath intensity profile. Let \(R_H(\Delta f)\) be the Fourier transform of \(R_h(\tau, 0)\). The frequency range where \(R_H(\Delta f)\) is essentially non-zero is called the coherence bandwidth, usually denoted as \((\Delta f)_c\). Let \(H(f, t)\) be the Fourier transform of \(h(\tau, t)\) with respect to \(\tau\). Let \(R_H(\Delta f, \Delta t)\) be the autocorrelation function of \(H(f, t)\). The Fourier transform of \(R_H(0, \Delta t)\) is called the Doppler power spectrum \(P(f)\). The frequency range where \(P(f)\) is essentially non-zero is called the Doppler spread of the channel. The time range where \(R_H(0, \Delta t)\) is essentially non-zero is called the coherence time of the channel, usually denoted as \((\Delta t)_c\). Multipath channels can be classified according to the relationship between the channel coherence time and the duration of the signaling pulse, and the relationship between the channel coherence bandwidth and the transmission bandwidth. In particular, one can distinguish slow versus flat fading, and flat versus frequency selective channels.

### 2.1.1 Fast fading vs slow fading

Equation (2.1) represents the mathematical model of a time-varying channel as \(\varsigma_i(t)\), \(\gamma_i(t)\) and \(\tau_i(t)\) are changing over time. This kind of channel happens when the transmitter, receiver or the environment is moving. Since only time-invariant channels (for a transmitted block) are studied in this thesis, we will not go into details on that topic. However, the time-variant channel will be introduced briefly in the following...
two paragraphs in order to give a full overview of the multipath fading channel.

The time varying nature of the channel can be measured by the Doppler shift. If the speed of the mobile is much greater than the environment, the time variations of the channel are dominated by the movement of the mobile. Considering a mobile moving at speed \( v \), and receiving a signal at frequency \( f_c \), the angle between the moving direction and the signal arriving direction is \( \xi \) and the light speed is \( c \), the Doppler shift is \( f_d = f_m \cos \xi \), where \( f_m \) is the maximum Doppler shift and given by \( f_m = \frac{v}{c} f_c \). Assuming the power received by the antenna is uniformly distributed in every direction, the Doppler power spectrum of the received signal (Fig. 2.2) is given by

\[
P(f) = \frac{K_p}{f_m \sqrt{1 - \left( \frac{f-f_c}{f_m} \right)^2}}
\]

where \( K_p \) is some constant related to the local mean power. From Fig. 2.2, it is seen that when a sinusoidal signal is transmitted, we will receive a power spectrum that is spread after transmission over a multipath fading channel. The frequency range where the power spectrum is nonzero defines the Doppler spread. In this example, the Doppler power spectrum is in the range \( f_c - f_m \) to \( f_c + f_m \), i.e. the Doppler spread is \( 2f_m \).

Coherence time is another parameter to measure time-varying characteristics of the channel, which can be approximated by the reciprocal of the Doppler spread. Coherence time is the time interval over which the channel impulse responses at different times are highly correlated. If the transmitted symbol period is greater than the channel coherence time, i.e., the channel variations are faster than the baseband signal variations, the fading is called fast fading (time selective fading). Otherwise, it is referred to as slow fading. Since this thesis’s work will focus on dense urban or indoor environments characterized by small mobile mobilities (and hence small
Doppler spreads), we will consider a slow fading channel, i.e. the channel is considered to be time-invariant for the duration of one transmission block.

2.1.2 Flat fading vs frequency selective fading

Assuming a static random multipath fading channel without noise, the complex envelope of the received signal (2.1) reduces to

\[ r(t) = \sum_{i=1}^{M} \gamma_i e^{j \gamma_i} s(t - \tau_i) \]  

(2.2)

and the low-pass complex impulse response of the channel is given by

\[ h(t) = \sum_{i=1}^{M} \gamma_i e^{j \gamma_i} \delta(t - \tau_i) \]

and from [18, section 2.1.1],

\[ r(t) = h(t) * s(t) = \int_{-W/2}^{W/2} H(f) S(f) e^{j 2 \pi f t} df \]  

(2.3)

Fig. 2.2 Doppler power spectrum for an unmodulated carrier with frequency \( f_c \)
where $H(f)$ and $S(f)$ are the Fourier transforms of $h(t)$ and $s(t)$, and $W$ is the bandwidth of $s(t)$. When $W$ is much less than the channel coherence bandwidth, the channel is said to exhibit flat fading and (2.3) can be approximately expressed as

$$r(t) \approx \frac{1}{W/2} \int_{-W/2}^{W/2} H(0) S(f) e^{j2\pi ft} df = H(0) s(t)$$

where $H(0)$ is the Fourier transform of $h(t)$ at $f = 0$ given by $H(0) = \sum_{i=1}^{M} \gamma_i$. When $M$ is large, by virtue of the Central Limit Theorem, $H(0)$ is Gaussian distributed. So for a flat fading channel, the received signal is

$$r(t) = \alpha s(t)$$

(2.4)

where the magnitude of $\alpha$ is either Rayleigh distributed when its mean is zero (Rayleigh fading channel) or Ricean distributed when the mean is not zero (Ricean fading channel).

Physically, a channel is modeled as Rayleigh when none of the sub-paths is significantly stronger than the others, while the Ricean model is used when there is a specular component in the channel, which is much stronger than the others. In practice, there certainly exist channels that have more than one strong component along with the scattered components. For those channels, other models such as Nakagami, Weibull and Suzuki [20, 21] may be better in comparison with Rayleigh/Ricean models. Unlike Rayleigh/Ricean models, which have a theory behind them, Nakagami, Weibull and Suzuki are mainly empirical. This thesis is mainly based on Rayleigh/Ricean models as they are the most commonly used ones.

When the transmitted signal bandwidth is larger than the channel coherence bandwidth, the channel is said to be frequency selective since the amplitude and phase of
the signal at different frequencies may experience different attenuations and shifts through the channel. Several models can be derived for frequency selective channels depending whether the transmitted signal is strictly band-limited or how the multipath delays are characterized. One popular model is the sampling channel model that assumes a strictly band-limited transmitted signal [19]. It can be shown [19, 18] that under this condition, the channel can be modeled as a tapped-delay-line with tap spacing equal to the reciprocal of the bandwidth \((1/W)\), corresponding to a time resolution of \(1/W\). Another model, which we will consider in this thesis, assumes that the multipath delays are known at the receiver. Theoretically, the channel model is given by (2.2), where \(\{\tau_i\}\) are assumed known. Such a model corresponds to an infinite time resolution of the multipath delay channel estimator. However, in practice only a finite time resolution \(\tau_R\) is used. Thus rewriting (2.2) as a double summation, the complex envelope of the noiseless received signal is given by

\[
    r(t) = \sum_{l=1}^{L} \sum_{i=1}^{M_l} s_{il} e^{j\gamma_i} s(t - \tau_{il}) \approx \sum_{l=1}^{L} \left( \sum_{i=1}^{M_l} s_{il} e^{j\gamma_i} \right) s(t) = \sum_{l=1}^{L} \alpha_l s(t - \tau_l) \quad (2.5)
\]

where all the “sub-paths” within a cluster that cannot be resolved by the multipath delay estimation technique (when \(|\tau_{il} - \tau_{i}l| \ll \tau_R\) have been grouped together into major paths, \(L\) is the number of (major) paths, and \(\alpha_l = \sum_{i=1}^{M_l} s_{il} e^{j\gamma_i}\) and \(\tau_l \approx \tau_{il}\) are the channel complex attenuation and delay of the \(l\)th major path (referred as the \(l\)th path from now on). Fig. 2.3 shows the impulse responses of the channel and gives an idea of what sub-paths and major paths are.

For coherent detection, from (2.5), the complex envelope of the noiseless received signal can be modeled as

\[
    r(t) = \sum_{l=1}^{L} \alpha_l s(t - \tau_l) \quad (2.6)
\]
where $\alpha_l$ is the fading coefficient for the $l^{th}$ path and is formed by all the signals that arrive at the receiver with approximately the same delay (i.e., the signals from the group of sub-paths which form the $l^{th}$ path), and it is Gaussian distributed with zero mean or non-zero mean, while $\tau_l$ is the delay of the $l^{th}$ path. This model can be applied for both Rayleigh and Ricean channels.

For non-coherent detection, due to the lack of the phase information, the complex envelope of the noiseless received signal is modeled as

$$r(t) = \sum_{l=1}^{L} \alpha_l e^{j\theta_l} s(t - \tau_l)$$

(2.7)

where $\theta_l$ is the phase shift for the $l^{th}$ path and is uniformly distributed between $-\pi$ and $\pi$. For multipath Rayleigh channels with uncorrelated multipath gains, the random variables $\alpha_l$ are modeled as independent circularly complex zero mean Gaussian random variables [22], so the statistic of $\alpha_l e^{j\theta_l}$ is the same as $\alpha_l$. Therefore, for Rayleigh channels, $e^{j\theta_l}$ can be dropped and (2.6) is still valid. But for Ricean models, the random variables $\alpha_l$ are not zero mean so the term $e^{j\theta_l}$ cannot be absorbed. We employ (2.7) instead of (2.6) to model Ricean channels with non-coherent detection.
2.1.3 Path resolvability

Let \( s(t) \) be the signal transmitted over a multipath fading channel, the path components are said to be resolved if the following condition is satisfied:

\[
\int_{T_l}^{T_f} s(t - \tau_l) s^*(t - \tau_m) dt = 0 \quad l \neq m
\]

(2.8)

where \([T_l, T_f]\) is the observation interval. When one symbol is transmitted, if the symbol is time-limited, in order to include all the received signal energy, the observation interval

\[
[T_l, T_f] = [\tau_{\text{min}}, T + \tau_{\text{max}}]
\]

where \( T \) is the symbol duration, and \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \) are the minimum and maximum path delays. For time-unlimited (i.e., band-limited) signal, the observation interval is \((-\infty, \infty)\). When a sequence is transmitted,

\[
[T_l, T_f] = [\tau_{\text{min}}, NT + \tau_{\text{max}}]
\]

where \( N \) is the length of the sequence.

The path resolvability condition can also be expressed in another way related to the signal autocorrelation function defined as

\[
R(\tau) = \int_{-\infty}^{\infty} s^*(t) s(t - \tau) dt
\]

If the observation interval is such that it contains all the received signal energy, (2.8) is equivalent to

\[
R(\tau_l - \tau_m) = 0 \quad l \neq m
\]
which means the path components will be resolved as long as

\[ |\bar{\tau}_l - \tau_m| \gg T_R \]  \hfill (2.9)

where \( T_R \) is the width of the autocorrelation function (i.e., the range over which the autocorrelation is nonzero) or roughly the reciprocal of the signal bandwidth. For bandlimited signals in theory of infinite duration, equivalent conditions can be defined, where equality signs are replaced by approximate equality signs. In that case, \( T_R \) is defined as the range over which the autocorrelation is essentially nonzero.

The path resolvability assumption can also be visualized graphically as follows. Assuming that \( s(t) \) is transmitted (using complex envelope representation), the output of the matched filter, matched to \( s(t) \) is given by

\[
y(t) = r(t) * s^*(-t) = \int_{-\infty}^{\infty} r(\tau) s^*(\tau - t) d\tau
\]
\[
= \int_{-\infty}^{\infty} \left[ \sum_{l=1}^{L} \alpha_l s(\tau - \bar{\tau}_l) + n(\tau) \right] s^*(\tau - t) d\tau
\]
\[
= \sum_{l=1}^{L} \alpha_l \int_{-\infty}^{\infty} s(\tau - \bar{\tau}_l) s^*(\tau - t) d\tau + \int_{-\infty}^{\infty} n(\tau) s^*(\tau - t) d\tau
\]
\[
= \sum_{l=1}^{L} \alpha_l \int_{-\infty}^{\infty} s(u - (\bar{\tau}_l - t)) s^*(u) du + \int_{-\infty}^{\infty} n(\tau) s^*(\tau - t) d\tau
\]

Assuming no noise, \( y(t) \) is given by

\[
y(t) = \sum_{l=1}^{L} \alpha_l R(\bar{\tau}_l - t) = \sum_{l=1}^{L} \alpha_l R^*(t - \bar{\tau}_l)
\]

where \( R(t) \) is the autocorrelation function of \( s(t) \). The output of the matched filter \( y(t) \) is illustrated in Fig. 2.4 for a two-path noiseless channel. Fig. 2.4(a) shows that
if \(|\tau_2 - \tau_1| \geq T_R\), the two autocorrelation functions \(\textit{approximately}\) do not overlap. In other words, the two paths are separated or resolved and one can be distinguished from the other.

Since the width of the autocorrelation is approximately equal to the reciprocal of the transmission bandwidth, it is seen that the path resolvability can be satisfied for spread spectrum signals (such as CDMA systems) in outdoor environments but may not be satisfied in dense urban or indoor environments characterized by smaller interpath delays. In this thesis, the path resolvability will assumed to be not satisfied, that is this work is applicable for narrowband signals, whose bandwidth is not sufficient to resolve the multipath. This thesis assumes that the multipath delays are known. These could have been estimated using a sounding signal with a bandwidth much larger than the information signal, or using super-resolution techniques (i.e. techniques that yield a time resolution higher than the reciprocal of the sounding signal) [18].
2.2 Receivers for multipath fading channels

2.2.1 Receiver types

With proliferation of wireless applications, detection techniques for multipath fading channels have received a growing interest. Receiver structures depend on the channel model and the available channel knowledge. For example, the channel coefficients may be known, unknown but deterministic or even stochastic (with known first or second order statistic). Receivers for multipath fading channels can be classified in terms of different criteria.

**Single-user systems vs. multi-user systems**

In a single-user system, only one user’s signal is received, or equivalently interference from other users is considered negligible at the receiver. Otherwise, it is a multi-user system. Note that a single-user system should not be confused with a single-user receiver operating in a multi-user environment. In that case, the receiver receives signals from other users but considers them as noise. Information-theoretic and communication aspects of both single-user case and multi-user case are addressed in [23]. In this thesis, we only consider a single-user system. In that case, besides the desired signal, the receiver of single-user system may still get noise and ISI from the signal itself which may degrade the performance.

**One-shot receiver vs. multi-shot receiver**

According to the number of the symbol transmitted, two types of receivers can be identified, one-shot receivers and multi-shot receivers. A one-shot receiver means only one symbol is transmitted, and the decision is based on the observation obtained in the duration of one symbol period (for one-path channels) or long enough to contain as
much signal energy as possible (for multipath channels). No ISI needs to be considered in this case. Multi-shot receivers, also referred as sequential receivers, assume a sequence of symbols is transmitted and hence, decision is made based on the whole received sequence (the observation interval is the duration of the entire sequence or longer).

**Optimal receiver vs. sub-optimal receiver**

For multipath fading channels, ISI occurs in most multi-shot cases. Receivers that obtain the best performance in some criterion (such as MMSE) under certain conditions are called optimal receivers in the sense of that criterion. The most often used criterion in digital communications is maximum-likelihood sequence estimation (MLSE), aiming at minimum average probability of symbol error. However, in many situations where the computational complexity is limited, sub-optimal receivers such as Delayed Decision-Feedback Sequence Estimation (DDFSE) [24], Reduced-State Sequence Estimation (RSSE) [25] or Decision-Feedback Equalization (DFE) [26] or even Linear Equalization (LE) [27, 28] have to be employed.

**2.2.2 Previous work**

Traditionally, Rake receivers [5]-[7] are applied to multipath fading channels with wide-band signals to achieve the performance of an equivalent diversity communication system. In that case, path cross correlations are ignored by assuming the signal path components are resolved. The counter-part of the Rake receiver for narrow-band systems when the multipath is not entirely resolved has been explored in [12, 13], where non-coherent and specular coherent optimum, and sub-optimum structures such as the Quadratic Decorrelation Receiver (QDR) were proposed for unresolved multipath Rayleigh/Ricean fading channels based on a one shot transmission approach.
And single-pulse performance of these structures were studied for commonly used binary modulation schemes such as Frequency Shift Keying (FSK) and Differential Phase Shift Keying (DPSK). In [14], multi-shot case was considered and a decorrelating filter was used to mitigate ISI and resolve the interpath cross-correlation for frequency selective Rayleigh fading channels. The decorrelator was compared to a set of schemes, the matched filter bound, the Rake receiver and the linear equalizer, in terms of Bit Error Rate (BER) performance over a two-ray Rayleigh fading channel.

The EM algorithm was applied by Georgiades [29] to sequence estimation in the presence of random disturbances and additive white Gaussian noise. Both random phase channels and Rayleigh fading channels were considered as examples. Following [15], where the EM algorithm was used to extract parameter estimates from superimposed signals, Spasojevic and Georgiades introduced Expectation-Maximization with Pilot Symbol (EM-PS) approach (pilot symbols are inserted into transmitted data symbols in order to reduce the detection error rate) in [16] for sequence estimation in time-invariant unresolved multipath Rayleigh fading channels not neglecting ISI. Signal resolution was performed based on the general autocorrelation structure of the signal and hence its performance is close to the ML receiver (optimal receiver) even when path cross-correlations are high.

A Generalized ML Sequential Detection and Estimation criterion (GMLSDE) is derived based on the EM algorithm in [30], which makes it possible detecting the data sequence and estimating the unknown channel parameters simultaneously by alternating the EM steps (expectation and maximization) between estimation and detection. Real-time ML Sequential Detection/Estimation (MLSDE) receivers were developed from GMLSDE for time-variant multipath Rayleigh fading channels with different available channel knowledge. A tapped delay line model was used and the estimation and detection parts were, separately, implemented by Titterington’s approach and the
Viterbi algorithm.

A multistage detector based on the EM algorithm was derived in [31] for multiuser synchronous CDMA system over AWGN channels. The multiuser detection problem is decomposed into a series of single-user problems and thus make the multiuser detection computationally feasible. It was shown that by carefully choosing the parameters, good performance can be achieved for both strong and weak users. Furthermore, the EM algorithm was employed for multiuser detection problems for downlink synchronous CDMA over frequency selective Rayleigh channels [32], where the orthogonality of the codes is destroyed and significant distortion occurs. The channel were assumed known in order to get the maximal diversity gain. Convergence to local maxima is avoided by the deterministic annealing technique and high performance can be obtained by the multiple-stage interference cancellation.

Nelson and Poor [17] applied the EM algorithm and its extended algorithms, the SAGE algorithm and the “Missing Parameter” EM (MPEM) algorithm to multiuser detection for additive noise CDMA channels. The SAGE algorithm is a twofold generalization of the EM algorithm and it has higher convergence rate than the EM algorithm. The MPEM receiver, which treats all but one user’s sequence as missing data at each iteration, incorporates both the E-step soft-decisions of the EM receiver and the sequential bit estimate updates of the SAGE receiver and achieves good performance for a variety of initializations. In [33], the EM algorithm and the SAGE algorithm were applied to the problem of Joint multiuser Detection and multichannel Estimation (JDE) over flat Rayleigh fading channels. Two iterative receivers, the EM-JDE receiver and the SAGE-JDE receiver were derived, separately, based on each of the algorithms for synchronous DS-CDMA system. It was demonstrated that the EM-JDE scheme can be optimized by optimizing the weight coefficients and the best performance is achieved by the SAGE-JDE receiver when the user’s bit sequences are
cyclically updated in the order of increasing signal strength. Monte-Carlo simulations which employ a few preamble bits for convergence show that the JDE schemes are more robust in unknown channel multiuser systems than those that consider data detection and channel estimation separately. We note that none of these works were developed for multipath Ricean fading channels or mixed mode Ricean/Rayleigh fading channels.

2.3 Thesis goals

The objective of this thesis is to design non-coherent sequential detection techniques for unresolved mixed mode Ricean/Rayleigh fading channels. Two approaches are proposed. The first one (which constitutes a major part of this thesis) is based on the EM-algorithm which provides an estimate of the ML sequence, thus tends toward optimal performance. The second one is based on MMSE principles. After the derivations of the receiver structures, their BER are to be presented using simulations for various mixed mode multipath Ricean/Rayleigh fading channels and multipath Rayleigh fading channels. In particular, the effect of the choice of spreading codes in DS-CDMA on performance of the EM approach will be illustrated. For small sequences, performance of the EM-based receiver will also be compared to the optimal receiver (ML sequence obtained via an exhaustive search).

2.4 Expectation-Maximization (EM) algorithm

The EM algorithm was originally introduced in the statistics literatures and had been known to statisticians for a long time. A general approach for iteratively maximizing likelihood functions via the EM algorithm was developed by Dempster, Laird and Rubin [34, 35]. The convergence properties of the EM algorithm was further discussed
by Wu in [36]. A broad review of applications of the EM algorithm can be found in [37]-[41].

In digital communications, the EM algorithm has been employed for solving ML estimation problems where maximizing the likelihood function directly is overly complicated. The EM algorithm decreases significantly the computational complexity by finding iteratively the ML estimate. We believe that many detection and estimation problems can be solved by the EM algorithm. Various receivers based on the EM algorithm were reviewed in section 2.2. In the following we present the EM algorithm in details.

Let \( r \) be the observed data and \( a \in A \) be the parameter that needs to be estimated. Then the ML estimate of \( a \) is

\[
\hat{a} = \arg \max_{a \in A} f(r|a)
\]

where \( f(r|a) \) is the conditional probability density function (pdf) of \( r \) given \( a \). In most cases, maximizing the likelihood function directly is difficult or even impossible. Under some conditions, the EM algorithm provides a way to find iteratively the ML estimate \( \hat{a} \) by assuming that another set of observation, \( u \in U \) is available [34, 36]. We refer to \( r \) and \( u \) as respectively incomplete and complete data. The complete data is such that it can be obtained from the incomplete data via a non-invertible, many-to-one mapping \( u \rightarrow r(u) \). And the relationship of the likelihood function of the complete data and that of the incomplete data is as follows:

\[
f(r|a) = \int_{U(r)} f(u|a) \, du
\]

where \( U(r) \) is the subset of \( U \) determined by the one-to-many mapping \( r \rightarrow u(r) \).
The two steps of the EM algorithm at the $m^{th}$ iteration are as follows [29]:

- **Expectation-step (E-step)** is to compute the goal function of the EM algorithm
  
  \[ Q(a|a^m) = E[\log f(u|a)|r,a^m] \]

- **Maximization-step (M-step)** is to solve
  
  \[ a^{m+1} = \arg\max_a Q(a|a^m) \]

where $f(u|a)$ is the conditional pdf of $u$ given $a$, and $a^m$ is the parameter vector estimate at the $m^{th}$ iteration. Since the complete data is not actually part of the initial problem, and thus not readily available, the algorithm maximizes the conditional expectation of the conditional density instead. Each new estimate has a non-decreasing likelihood and the approximate ML estimate can be achieved at the convergence. The algorithm may start with any initial estimate $a^0$ and continues with the two steps iteratively until it converges. The initial guess of the parameters is important as a bad choice may lead to a local maximum instead of a global one.

One of the typical applications of the EM algorithm to parameter estimation problems is the one with channels having undesirable random parameters, for example, channel gains or phases. Let the undesirable random parameters be a vector $\alpha$, and assume $\alpha$ is independent of the estimated parameters $a$. A natural choice for the complete data is $u = r, \alpha$. The goal function is

\[ Q(a|a^m) = E[\log f(r,\alpha|a)|r,a^m] \]  \hspace{1cm} (2.10)

Since $\alpha$ and $a$ are independent,

\[ f(r,\alpha|a) = f(r|\alpha,a)f(\alpha) \]  \hspace{1cm} (2.11)

From (2.10) and (2.11), the goal function of the EM algorithm can be simplified by
retaining only the term which is function of $a$,

$$Q(a|a^n) = E[\log f(r|\alpha, a)|r, a^n]$$

where $f(r|\alpha, a)$ can be more easily obtained in most cases.
Chapter 3

Sequential detection for mixed mode Ricean/Rayleigh and Rayleigh fading channels

This chapter studies several sequential detection techniques for \( L \)-path mixed mode Ricean/Rayleigh channels. An \( L \)-path Rayleigh channel is treated as a special case of an \( L \)-path mixed mode channel. The first section presents the channel model for non-coherent detection and gives a note on notations used in the following mathematical derivations. The following sections contain derivations of the ML receiver, the EM-based receiver and the K-MMSE receivers (including KS-MMSE and KD-MMSE) for both multipath mixed mode Ricean/Rayleigh and Rayleigh fading channels.

3.1 Channel model

Assuming \( a \) is the transmitted symbol sequence of length \( N \), the transmitted signal is represented by its complex envelope \( s(t; a) \). Supposing \( s(t; a) \) is transmitted over an
3 Sequential detection for mixed mode Ricean/Rayleigh and Rayleigh fading channels

$L$-path mixed mode Ricean/Rayleigh channel (a channel with $L$ independent paths for which the first path is Ricean distributed and the rest $L - 1$ paths are Rayleigh distributed), the complex envelope of the received signal is given by

$$r(t) = \sum_{i=1}^{L} \alpha_i e^{j\theta_i} s(t - \tau_i; a) + n(t)$$  \hspace{1cm} (3.1)$$

where $\alpha_i$ are independent circularly complex Gaussian random variables with variance $\sigma_i^2 \triangleq E[(\alpha_i - \overline{\alpha}_i)(\alpha_i - \overline{\alpha}_i)^*]/2$, the mean of $\alpha_1$ is $\overline{\alpha}_1$, and the means of the rest $\alpha_i$ are all zero. We denote $\theta_i$ as an additional multipath component phase shift associated with each path representing the lack of reference phase information at the receiver. The phase $\theta_1$ is a random variable uniformly distributed between $-\pi$ and $\pi$, and the rest $\theta_i$ are equal to zero. Let $\{\tau_i\}_i$ be the delays of all paths, supposed to be known and $\tau_i \neq \tau_j$ if $i \neq j$. The channel noise is modeled as an additive zero-mean circularly complex Gaussian process $n(t)$ satisfying

$$E[n(t)n^*(u)] = 2N_0 \delta(t - u)$$  \hspace{1cm} (3.2)$$

Note that modeling the channel noise of the equivalent low pass channel as a zero mean circularly complex white Gaussian process with power spectral density $2N_0$ corresponds to model the noise of the channel as a zero mean band pass white Gaussian process with two sided power spectral density $N_0/2$ [42]. Table 3.1 summarizes which channel parameters are known and unknown to the receiver.

It is helpful to use vector and matrix algebra to study receiver structures for multipath fading channels. We use the following notation: bold capital letters denote matrices and bold lowercase letters denote vectors, $^T$, $^*$ and $^\dagger$ denote, respectively, the transposition, complex conjugation, and Hermitian conjugation of a matrix or vector.
3 Sequential detection for mixed mode Ricean/Rayleigh and Rayleigh fading channels

<table>
<thead>
<tr>
<th>Channel parameter</th>
<th>Known/unknown to the receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$ (instantaneous value)</td>
<td>unknown</td>
</tr>
<tr>
<td>$E[\alpha_i] \triangleq \bar{\alpha}_i$</td>
<td>known</td>
</tr>
<tr>
<td>$E[(\alpha_i - \bar{\alpha}_i)(\alpha_i - \bar{\alpha}_i)^*/2]$</td>
<td>known</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>unknown</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>known</td>
</tr>
<tr>
<td>$n(t)$ (instantaneous value)</td>
<td>unknown</td>
</tr>
<tr>
<td>$E[n(t)] = 0$</td>
<td>known</td>
</tr>
<tr>
<td>power spectral density of $n(t) = 2N_0$</td>
<td>known</td>
</tr>
</tbody>
</table>

Table 3.1 Knowledge of the receiver of the channel parameters

The $i^{th}$ entry of a column vector $\mathbf{v}$ is denoted as $[\mathbf{v}]_i$ and the $ij^{th}$ entry of a matrix $\mathbf{B}$ is denoted as $[\mathbf{B}]_{ij}$. The diagonal matrix composed of the main diagonal entries of $\mathbf{B}$ is denoted as $\{\mathbf{B}\}_d$, and the lower triangular matrix composed of the lower triangular elements of $\mathbf{B}$ is $\{\mathbf{B}\}_l$ with zero main diagonal entries. The determinant of a matrix $\mathbf{B}$ is denoted $|\mathbf{B}|$.

In this thesis, $\Re\{\cdot\}$ denotes the real part of a complex number and $\Im\{\cdot\}$ is the imaginary part. And an overline denotes the mean, e.g., the mean of $\alpha$ is denoted as $\bar{\alpha}$. The covariance matrix of a circularly complex Gaussian random vector $\mathbf{z}$ is defined by

$$R_{\mathbf{z}} = \frac{1}{2}E[(\mathbf{z} - \bar{\mathbf{z}})(\mathbf{z} - \bar{\mathbf{z}})^\dagger]$$

(3.3)

and the inner product of $x(t)$ and $y(t)$ is denoted as

$$<x(t), y(t)> = \int x(t)y^*(t) \, dt$$

In the following discussions, we assume that the delayed signals $s(t - \tau_1; \mathbf{a}), s(t - \tau_2; \mathbf{a}), \ldots, s(t - \tau_l; \mathbf{a})$ are continuous and linearly independent on the observation interval $[T_l, T_F]$. The delays ($\tau_1, \tau_2, \ldots, \tau_l$) are distinct, i.e., $\tau_i \neq \tau_j$, if
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\( i \neq j \). The continuity hypothesis ensures that the discrete representation of \( r(t) \) exists [12, 18]. The linearity hypothesis (called \( L \)-path linear independency assumption in [18]) is used when we discuss the eigenvalues and eigenfunctions of the covariance of the noiseless received signal. The requirement for linearity is much easier to satisfy than the orthogonality condition of multipath resolvability (in particular it is satisfied when the signal is time-limited). We define the column vectors \( \theta \) and \( \alpha \) as

\[
\theta = (\theta_1, \theta_2, \ldots, \theta_L)^T \\
\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_L)^T
\]

Conditioned on \( \theta \), the noiseless received signal

\[
v(t, \mathbf{a}) = \sum_{i=1}^{L} \alpha_i e^{j\theta_i} s(t - \tau_i; \mathbf{a})
\]

is a random Gaussian process and is assumed to be independent of the noise \( n(t) \). Its covariance function is given by

\[
K(t, u; \mathbf{a}) = \frac{1}{2} E \left[ (v(t; \mathbf{a}) - \overline{v}(t; \mathbf{a}))(v(u; \mathbf{a}) - \overline{v}(u; \mathbf{a}))^* \right] \\
= \sum_{i=1}^{L} \sigma_i^2 s(t - \tau_i; \mathbf{a}) s^*(u - \tau_i; \mathbf{a})
\]

As explained in [12], the covariance function of the signal process has at most \( L \) eigenvalues and \( L \) eigenfunctions. The eigenvalues \( \lambda_i(\mathbf{a}) \) \((i = 1, 2, \ldots, L)\) and eigenfunctions \( \phi_i(t; \mathbf{a}) \) \((i = 1, 2, \ldots, L)\) are solutions of the equation

\[
\lambda_i(\mathbf{a}) \phi_i(t; \mathbf{a}) = \int_{T_f}^{T_r} K(t, u; \mathbf{a}) \phi_i(u; \mathbf{a}) \, du
\]
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Under the assumption of $L$-path linear independence, the eigenfunctions $\phi_k(t; a)$ can be expressed as a linear combination of $s(t - \tau_1; a), s(t - \tau_2; a), \ldots, s(t - \tau_L; a)$ and given by [18]

$$
\phi_k(t; a) = \frac{1}{\sqrt{\xi(a)}} \sum_{s=1}^{L} \frac{y_{ks}(a)}{\sqrt{\epsilon_s(a)}} s(t - \tau_s; a) \quad T_I \leq t \leq T_F \tag{3.5}
$$

where $\xi(a)$ denotes the total transmitted energy and $\epsilon_s(a)$ denotes the ratio of the energy of the signal $s(t - \tau_s; a)$ on the observation interval to the total energy, and are respectively given by

$$
\xi(a) = \int_{-\infty}^{\infty} |s(t; a)|^2 dt \tag{3.6}
$$

$$
\epsilon_s(a) = \frac{1}{\xi(a)} \int_{T_I}^{T_F} |s(t - \tau_s; a)|^2 dt \tag{3.7}
$$

When $T_F$ is greater than the length of the sequence plus the delay $\tau_s$, $\epsilon_s(a) = 1$. In (3.5), $y_{ks}(a)$ is the $ks^{th}$ entry of $Y(a)$, which is an $L \times L$ decorrelation matrix that satisfies the equations

$$
Y^*(a) \Gamma^*(a) Y^T(a) = I \tag{3.8a}
$$

$$
2\epsilon(a) R \Gamma^*(a) Y^T(a) = Y^T(a) D(a) \tag{3.8b}
$$

where $I$ is the $L \times L$ identity matrix, $\epsilon(a)$ is an $L \times L$ diagonal matrix with $[\epsilon(a)]_{ij} = \epsilon_i(a) \delta_{ij}$. When the observation interval $[T_I, T_F]$ is long enough to contain all the energy transmitted, i.e., when all the path components are received in this duration, $\epsilon(a)$ becomes $I$. The signal cross correlation matrix $\Gamma(a)$ is an $L \times L$ matrix with

$$
[\Gamma(a)]_{ij} = \frac{1}{\xi(a) \sqrt{\epsilon_i(a) \epsilon_j(a)}} \int_{T_I}^{T_F} s(t - \tau_i; a) s^*(t - \tau_j; a) \, dt \tag{3.9}
$$
The matrix $\xi(a) \mathcal{E}^{1/2}(a) \Gamma(a) \mathcal{E}^{1/2}(a)$ is the Grammian in the $L_2$ space. The matrix $\mathcal{E}^{1/2}(a) \Gamma(a) \mathcal{E}^{1/2}(a)$ is positive definite Hermitian due to the $L$-path linear independence assumption [43, p. 74]. Therefore, $\Gamma(a)$ is also positive definite matrix ($\mathcal{E}^{1/2}(a)$ is a diagonal matrix with positive entries). In (3.8b), $R_\alpha$ is the covariance matrix of $\alpha$, which is a diagonal positive-definite matrix (due to the independence of the $L$ paths), and is given by

$$
R_\alpha = \begin{pmatrix}
\sigma_1^2 \\
\sigma_2^2 \\
\vdots \\
\sigma_L^2
\end{pmatrix}
$$

where from (3.3), the $i^{th}$ diagonal entry $\sigma_i^2$ is given by

$$
\sigma_i^2 = \frac{1}{2} E [(\alpha_i - \overline{\alpha}_i)(\alpha_i - \overline{\alpha}_i)^*]
$$

In (3.8b), $D(a)$ denotes the diagonal matrix with diagonal entries [12]

$$
[D(a)]_{ii} = \lambda_i(a)/\xi(a)
$$

where $\lambda_i(a)$ is the $i^{th}$ eigenvalue of $K(t, u; a)$ the covariance function of the signal process when $\theta$ is held fixed. And it is also the $i^{th}$ eigenvalue of the matrix $2\mathcal{E}(a) R_\alpha \Gamma^*(a)$. Hence $\{\lambda_i(a)\}_i$ are solutions of a classical eigenvalue problem.

Suppose the sequence $a$ is to be estimated based on the observation $r(t)$ on the observation interval $[T_i, T_f]$. Let $r(a)$ be a sufficient statistic for the channel model given by (3.1) when $\theta$ is held fixed. Similarly to [12], $r(a)$ is the column vector
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composed of the projection of \( r(t) \) on the signal space and is defined as

\[
r(a) = (r_1(a), r_2(a), \ldots, r_L(a))^T
\]

since one basis of the signal space can be formed by the \( L \) eigenfunctions \( \{\phi_k(t; a)\}_k \) given by (3.5). For any sequence \( a \), from (3.1) the \( k^{th} \) element of \( r(a) \) is given by

\[
r_k(a) = \int_{T_1}^{T_T} r(t)\phi_k^*(t; a) \, dt
\]

\[
= \int_{T_1}^{T_T} \sum_{i=1}^L \alpha_i e^{j\theta_i} s(t - \tau_i; a) \phi_k^*(t; a) \, dt + \int_{T_1}^{T_T} n(t)\phi_k^*(t; a) \, dt \tag{3.10}
\]

where the first term is the projection of the signal part, and the second term is the noise part. When \( \theta \) is held fixed, both terms are circularly complex Gaussian random variables.

Let us define

\[
n_k(a) = \int_{T_1}^{T_T} n(t)\phi_k^*(t; a) \, dt
\]

\[
[G(a)]_{ik} = \int_{T_1}^{T_T} s(t - \tau_i; a)\phi_k^*(t; a) \, dt \tag{3.12}
\]

where \( G(a) \) is the matrix with entries \([G(a)]_{ik}\) that can also be expressed as

\[
G(a) = \sqrt{\xi(a)} \mathcal{E}_1^{1/2}(a) \Gamma(a) Y^\dagger(a) \tag{3.13}
\]

by substituting (3.5) into (3.12) and using (3.9). From (3.11),

\[
r_k(a) = \sum_{i=1}^L \alpha_i e^{j\theta_i}[G(a)]_{ik} + n_k(a)
\]
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\[ r_k(a) = \sum_{i=1}^{L} |\alpha_i| |P(\theta)G(a)|_{ik} + n_k(a) \]

\[ r(a) = [P(\theta)G(a)]^T \alpha + n(a) \]

(3.14)

where \( P(\theta) \) is the diagonal matrix with diagonal entries \( [P(\theta)]_{ii} = e^{i\theta_i} \), and

\[ n(a) = (n_1(a), n_2(a), \ldots, n_L(a))^T \]

3.2 Maximum Likelihood (ML) approach

3.2.1 ML approach for mixed mode Ricean/Rayleigh fading channels

Suppose there are \( M \) possible transmitted sequences, we define \( a_m \) as the sequence sent over the multipath fading channel under the \( m^{th} \) hypothesis [44, p. 46]. We also define \( H_0 \) as the \( 0^{th} \) hypothesis when no signal is transmitted. Conditioned on \( \theta \), under each hypothesis, including \( H_0 \), the received signal \( r(t) \) is a complex Gaussian random process and its sufficient statistic \( r(a_m) \) is a circularly complex Gaussian random vector. From [12], the conditional likelihood ratio given \( \theta \) under \( H_m \) is

\[ \Lambda(a_m|\theta) = \frac{|R_0| \exp\{-\frac{1}{2}(r(a_m) - \mu(a_m))^\dagger R_{r|\theta,a_m}^{-1}(r(a_m) - \mu(a_m))\}}{|R_{r|\theta,a_m}| \exp\{-\frac{1}{2}r^\dagger(a_m)R_0^{-1}r(a_m)\}} \]

(3.15)

where \( \mu(a_m) \) and \( R_{r|\theta,a_m} \) are, respectively, the conditional mean vector and covariance matrix of \( r(a_m) \) given \( \theta \) and \( a_m \) and \( R_0 \) is the covariance matrix of \( r(a_m) \) under \( H_0 \). From (3.14) and (3.13), \( \mu(a_m) \) can be expressed as

\[ \mu(a_m) \triangleq E[r(a_m)|\theta,a_m] = \sqrt{\xi(a_m)}Y^*(a_m)\Gamma^T(a_m)\epsilon^{1/2}(a_m)g(\theta) \]

(3.16)
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where \( \varphi(\theta) \) is a column vector and defined as

\[
\varphi(\theta) \triangleq \mathbf{P}(\theta) \overline{\mathbf{x}} = (\pi_1 e^{i\theta_1}, 0, \ldots, 0)^T
\]

(3.17)

And similar to [12], using (3.14)-(3.13) and (3.8), we obtain

\[
R_{r|\theta,a_m} \triangleq \frac{1}{2} E \left[ (r(a_m) - \mu(a_m)) (r(a_m) - \mu(a_m))^\dagger | \theta, a_m \right]
\]

\[
= N_0 I + \frac{\xi(a_m)}{2} D(a_m)
\]

(3.18)

\[
R_0 \triangleq \frac{1}{2} E [r(a_m) r^\dagger(a_m) | H_0] = N_0 I
\]

(3.19)

We write (3.15) as

\[
\Lambda(a_m | \theta) = L(a_m) g(\theta; a_m)
\]

where \( g(\theta; a_m) \) includes all factors involving \( \theta \) and \( L(a_m) \) contains everything left over. The conditional log-likelihood \( \ln(\Lambda(a_m | \theta)) \) can be obtained by

\[
\ln(\Lambda(a_m | \theta)) = \ln(L(a_m)) + \ln(g(\theta; a_m))
\]

(3.20)

From (A.3),

\[
\ln(L(a_m)) = \ln \left( \frac{|R_0|}{R_{r|\theta,a_m}} \right) - \frac{1}{2} r^\dagger(a_m) (R_{r|\theta,a_m}^{-1} - R_0^{-1}) r(a_m) - \frac{\xi(a_m)}{2}
\]

\[
\cdot \varphi^\dagger(\theta) \{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_{r|\theta,a_m}^{-1} \Gamma^*(a_m) Y^*(a_m) \epsilon^{1/2}(a_m) \} d \varphi(\theta)
\]

(3.21)

\[
g(\theta; a_m) = \exp \left\{ \frac{1}{2} r^\dagger(a_m) R_{r|\theta,a_m}^{-1} \mu(a_m) + \frac{1}{2} \mu^\dagger(a_m) R_{r|\theta,a_m}^{-1} r(a_m) \right\} - \xi(a_m)
\]

\[
\cdot \Re \left\{ \varphi^\dagger(\theta) \{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_{r|\theta,a_m}^{-1} \Gamma^*(a_m) Y^*(a_m) \epsilon^{1/2}(a_m) \} \varphi(\theta) \right\}
\]

(3.22)
For convenience, let us define the diagonal matrix $B(a_m)$ as

$$B(a_m) \equiv \left\{ \mathbf{e}_{1/2}(a_m) \Gamma^*(a_m) \mathbf{Y}^T(a_m) R_{r|\theta,a_m}^{-1} \Gamma^*(a_m) \mathbf{e}_{1/2}(a_m) \right\}_d$$

For mixed mode Ricean/Rayleigh channels, we note that

$$\theta = (\theta_1, 0, \ldots, 0)^T$$
$$\alpha = (\alpha_1, 0, \ldots, 0)^T$$

Therefore,

$$g^*(\theta) \left\{ \mathbf{e}_{1/2}(a_m) \Gamma^*(a_m) \mathbf{Y}^T(a_m) R_{r|\theta,a_m}^{-1} \Gamma^*(a_m) \mathbf{e}_{1/2}(a_m) \right\}_d \alpha \theta = [\alpha_1]^2 [B(a_m)]_{11}$$

(3.25)

$$g^*(\theta) \left\{ \mathbf{e}_{1/2}(a_m) \Gamma^*(a_m) \mathbf{Y}^T(a_m) R_{r|\theta,a_m}^{-1} \Gamma^*(a_m) \mathbf{e}_{1/2}(a_m) \right\}_d \alpha \theta = 0$$

(3.26)

Using (3.8b), (3.18), (3.19), and (3.23)-(3.26), (3.21) and (3.22) can be written as (detailed derivations in Appendix B)

$$\ln(L(a_m)) = \frac{1}{2N_0} r^*(a_m) Q(a_m) r(a_m) - \ln \left( \left| I + \frac{\xi(a_m)}{2N_0} D(a_m) \right| \right)$$
$$- \frac{\xi(a_m)}{2} [\alpha_1]^2 [B(a_m)]_{11}$$

(3.27)

$$g(\theta; a_m) = g(\theta_1; a_m) = \exp \left\{ \frac{[\alpha_1]}{\sigma_1 \sqrt{\epsilon_1(a_m)}} b_1(a_m) \cos(\theta_1 - \varphi_1(a_m)) \right\}$$

(3.28)

where $b_1(a_m)$ is the first term of the column vector $b(a_m)$, which is defined as

$$b(a_m) \equiv \frac{1}{\sqrt{\xi(a_m)}} \mathbf{Y}^T(a_m) Q(a_m) r(a_m)$$

(3.29a)

$$= \left( b_1(a_m), b_2(a_m), \ldots, b_L(a_m) \right)^T$$

(3.29b)
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and \( Q(a_m) \) and \( \varphi_1(a_m) \) are, respectively, given by

\[
Q(a_m) = \frac{\xi(a_m)}{2N_0} \left( D^{-1}(a_m) + \frac{\xi(a_m)}{2N_0} I \right)^{-1} 
\]

(3.30)

\[
\varphi_1(a_m) = \arg[b_1(a_m)] - \arg[\alpha_1] 
\]

(3.31)

as in [12].

Therefore, from (3.20) and (3.28) the closed form of the log-likelihood for the mixed mode channel can be obtained from the expectation of the conditional log-likelihood over \( \theta_1 \) (which is uniformly distributed in \([-\pi, \pi]\)) and is given by

\[
\ln \left( \Lambda(a_m) \right) = \ln \left( L(a_m) \right) + \ln \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} g(\theta_1; a_m) \, d\theta_1 \right) 
\]

\[
= \ln \left( L(a_m) \right) + \ln \left[ I_0 \left( \frac{|\alpha_1|}{\sigma^2 R_1} \right) \right] 
\]

(3.32)

It is shown by (3.32) that for mixed mode Rayleigh/Ricean channels, the ML solution is a highly non-linear problem, which results in its hard realization. The optimal sequence estimation is to maximize the log-likelihood function over all possible transmitted sequences. So even for the exhaustive method, it is unrealistic to implement the optimal estimation with long sequences, since the number of the possible sequences increases exponentially with the length of the sequences.

3.2.2 ML approach for Rayleigh channels

For Rayleigh channels, \( \alpha_i \) in the channel model (3.1) is a zero mean complex Gaussian random variable and \( \theta_i \) satisfies

\[
\theta_1 = \theta_2 = \ldots = \theta_L = 0
\]
Under the $m^{th}$ hypothesis, the log-likelihood function (3.32) reduces to

$$\ln(L(a_m)) = \ln(L(a_m))$$

where from (3.27), $\ln(L(a_m))$ is given by

$$\ln(L(a_m)) = \frac{1}{2N_0} r^*(a_m) Q(a_m) r(a_m) - \ln \left( \left| I + \frac{\zeta(a_m)}{2N_0} D(a_m) \right| \right)$$

where the definitions of $r(a_m), Q(a_m)$ and $\zeta(a_m)$ are the same as for the mixed mode Ricean/Rayleigh fading channels. It is seen that although the likelihood function is simplified for Rayleigh channels, the ML-solution in this case is still a non-linear problem, which makes the optimal approach based on MLSE not suitable for large sequence lengths or a large multipath spread.

### 3.3 EM approach

From last section, we see that direct implementation of the ML is not an easy task since it is difficult to evaluate the gradient of the log-likelihood function. In this section, we show how the EM algorithm can be used to simplify the sequence estimation by iteratively finding the ML estimate.

#### 3.3.1 EM approach for mixed mode Ricean/Rayleigh fading channels

**Derivation of the goal function $Q(a|a^m)$**

The first step is to choose the complete data set and find the goal function. Following [15], let us define

$$x_i(t) = \alpha_i e^{j\theta_i} s(t - \tau_i; a) + n_i(t)$$

(3.33)
where $a$ is the sequence to be estimated and $n_i(t)$ are independent complex zero mean white Gaussian processes with

$$E\left[n_i(t)n_j^*(u)\right] = \beta_i 2N_0 \delta(t - u)\delta_{ij} \quad (3.34)$$

such that $n(t) = \sum_{i=1}^L n_i(t)$. Clearly, $\beta_i$ satisfies $\sum_{i=1}^L \beta_i = 1$. In [15], the complete data set for the EM algorithm is composed of all $x_i(t)$. Here, our complete data set also includes the channel fading $\alpha$ and the channel phase shift $\theta$ and is set as

$$u = (x, \alpha, \theta) = (x_1(t), \ldots, x_L(t), t \in [T_L, T_F], \alpha, \theta)$$

where $[T_L, T_F]$ is the observation interval.

The goal function of the EM algorithm in this case is given by

$$Q(a|a^m) = E[\log f(x, \alpha, \theta|a)r, a^m] \quad (3.35)$$

From Bayes’s rule [45], we know

$$f(x, \alpha, \theta|a) = \frac{f(x, \alpha, \theta, a)}{f(a)} = \frac{f(x|\alpha, \theta, a)f(\alpha, \theta, a)f(\theta|a)f(a)}{f(a)}$$

So from (3.35), the goal function is

$$Q(a|a^m) = E[\log f(x|\alpha, \theta, a)|r, a^m] + E[\log f(\alpha|\theta, a)|r, a^m] + E[\log f(\theta|a)|r, a^m]$$

The M-step of the EM algorithm involves maximization of the goal function with respect to $a$. Since $\alpha$ and $\theta$ are independent of $a$, the second and third terms can be
dropped for the purpose of the M-step. And the goal function now becomes

$$Q(a|a^m) = E \left[ \log f(x|\alpha, \theta, a) | r(a^m), a^m \right]$$  \hspace{1cm} (3.36)$$

where \( r(a^m) \) is a column vector representing a sufficient statistic for the channel model given by (3.1) when \( \theta \) is held fixed and its elements are defined as (3.11).

A discrete representation of \( x = (x_1(t), x_2(t), \ldots, x_L(t), t \in [T_l, T_f]) \) is

$$x(a) = [x_1^T(a), x_2^T(a), \ldots, x_L^T(a)]^T$$  \hspace{1cm} (3.37)$$

where \( x_l(a) \) itself is a column vector defined as

$$x_l(a) = (x_{l1}(a), x_{l2}(a), \ldots, x_{lL}(a))^T$$  \hspace{1cm} (3.38)$$

The \( k \text{th} \) element of \( x_l(a) \) is the projection of \( x_l(t) \) on the \( k \text{th} \) base function of the signal space formed by the \( L \) eigenfunctions of the signal process when \( \theta \) is held fixed, and is given by

$$x_{lk}(a) = \int_{T_l}^{T_f} x_l(t) \phi_k^*(t; a) dt$$  \hspace{1cm} (3.39)$$

$$= \int_{T_l}^{T_f} \alpha_l e^{j\theta_l s(t - \tau_l; a)} \phi_k^*(t; a) dt + \int_{T_l}^{T_f} n_l(t) \phi_k^*(t; a) dt$$

where \( \phi_k(t; a) \) is the \( k \text{th} \) eigenfunction of the signal process as defined in (3.5).
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Given \((\alpha, \theta, \alpha)\), \(x(t)\) is a complex Gaussian random vector with mean

\[
E[x(t)|\alpha, \theta, \alpha] = \begin{pmatrix}
\int_{T_I}^{T_F} \alpha e^{j\theta_i s(t - \tau; \alpha)} \phi_1^*(t; \alpha) \, dt \\
\int_{T_I}^{T_F} \alpha e^{j\theta_i s(t - \tau; \alpha)} \phi_2^*(t; \alpha) \, dt \\
\vdots \\
\int_{T_I}^{T_F} \alpha e^{j\theta_i s(t - \tau; \alpha)} \phi_L^*(t; \alpha) \, dt
\end{pmatrix}
\]  \quad (3.40)

and covariance matrix \(R_{x(t)|\alpha, \theta, \alpha}\) given by

\[
R_{x(t)|\alpha, \theta, \alpha} \triangleq \frac{1}{2} E\left[ (x(t) - E[x(t)|\alpha, \theta, \alpha]) (x(t) - E[x(t)|\alpha, \theta, \alpha])^\dagger \right]
\]

\[
= \frac{1}{2} E\left[ \begin{pmatrix}
\int_{T_I}^{T_F} n_i(t) \phi_1^*(t; \alpha) \, dt \\
\int_{T_I}^{T_F} n_i(t) \phi_2^*(t; \alpha) \, dt \\
\vdots \\
\int_{T_I}^{T_F} n_i(t) \phi_L^*(t; \alpha) \, dt
\end{pmatrix}
\cdot \left( \int_{T_I}^{T_F} n_i^*(u) \phi_1(u; \alpha) \, du, \int_{T_I}^{T_F} n_i^*(u) \phi_2(u; \alpha) \, du, \ldots, \int_{T_I}^{T_F} n_i^*(u) \phi_L(u; \alpha) \, du \right) \right]
\]

With (3.34) and the orthogonality of the eigenfunctions \(\phi_k(t; \alpha)\), \(R_{x(t)|\alpha, \theta, \alpha}\) is a diagonal matrix with diagonal entries \(\beta_i N_0\), i.e.,

\[
R_{x(t)|\alpha, \theta, \alpha} = \beta_i N_0 I
\]  \quad (3.41)
3 Sequential detection for mixed mode Ricean/Rayleigh and Rayleigh fading channels

The conditional pdf of $x_l(a)$ is given by [46, p. 122]

$$f(x_l(a)|\alpha, \theta, a) = \frac{1}{(2\pi)^L | R_{xx}|_{\alpha, \theta, a}} \exp \left\{ -\frac{1}{2} (x_l(a) - E[x_l(a)|\alpha, \theta, a])^t R^{-1}_{xx}|_{\alpha, \theta, a} (x_l(a) - E[x_l(a)|\alpha, \theta, a]) \right\}$$

$$= \frac{1}{(2\pi)^L (\beta L N_0)^L} \exp \left\{ -\frac{1}{2\beta L N_0} ||x_l(t) - \alpha_l e^{j\theta_l} s(t - \tau_l; a)||^2 \right\}$$

(3.42)

where the detailed derivation of the second line is given in Appendix C.

Given $(\alpha, \theta, a)$, from (3.33), $x_1(t), x_2(t), \ldots, x_L(t)$ are independent, which leads to the independence of $x_1(a), x_2(a), \ldots, x_L(a)$, therefore from (3.37) $f(x|\alpha, \theta, a)$ can be obtained by

$$f(x|\alpha, \theta, a) = f(x|\alpha, \theta, a)$$

$$= f(x_1(a)|\alpha, \theta, a) \cdot f(x_2(a)|\alpha, \theta, a) \ldots f(x_L(a)|\alpha, \theta, a)$$

$$= \prod_{l=1}^L \frac{1}{(2\pi)^L (\beta L N_0)^L} \exp \left\{ -\frac{1}{2\beta L N_0} ||x_l(t) - \alpha_l e^{j\theta_l} s(t - \tau_l; a)||^2 \right\}$$

(3.43)

From (3.36) and (3.43), the goal function of the EM algorithm is

$$Q(a|a^m) \equiv E \left[ \log f(x, \alpha, \theta|a)|r, a^m \right]$$

$$= E \left[ -\frac{1}{2\beta L N_0} \sum_{l=1}^L \beta_l ||x_l(t) - \alpha_l e^{j\theta_l} s(t - \tau_l; a)||^2 | r(a^m), a^m \right] + \log \left[ \prod_{l=1}^L \frac{1}{(2\pi)^L (\beta L N_0)^L} \right]$$

(3.44)

$$= \frac{1}{N_0} \sum_{l=1}^L \left\{ \mathbb{R} \left\{ \frac{1}{\beta_l} < \hat{x}_l(t; a^m), s(t - \tau_l; a) > \right\} - \frac{1}{2\beta_l} \hat{\gamma}_l(a^m) ||s(t - \tau_l; a)||^2 \right\}$$

(3.45)

(3.45) can be obtained by expansion of $|| \cdot ||^2$ in (3.44) and dropping the terms not
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function of $a$, and $\hat{\rho}_{il}(a^m)$ and $\hat{\chi}_l(t; a^m)$ are defined as

$$\hat{\rho}_{il}(a^m) \triangleq E[\alpha_i^* e^{-j\theta_i} \alpha_i e^{j\theta_i} | r(a^m), a^m] \quad \text{ (i.e. } \hat{\rho}_{il}(a^m) \triangleq E[|\alpha_i|^2 | r(a^m), a^m]) \quad (3.46)$$

$$\hat{\chi}_l(t; a^m) \triangleq E[\alpha_i^* e^{-j\theta_i x_l(t)} | r(a^m), a^m] \quad (3.47)$$

The derivations of the expressions of $\hat{\rho}_{il}(a^m)$ and $\hat{\chi}_l(t; a^m)$ are given respectively in Appendix E and Appendix F.

We may express $\hat{\rho}_{il}(a^m)$ separately in terms of the values of the indexes $l$ and $i$

$$\hat{\rho}_{il}(a^m) = \begin{cases} 
E[\alpha_i^* \alpha_l e^{-j\theta_1} | r(a^m), a^m] & l = 1 \text{ and } i \neq 1 \\
E[\alpha_i^* \alpha_l e^{j\theta_1} | r(a^m), a^m] & l \neq 1 \text{ and } i = 1 \\
E[\alpha_i^* \alpha_l | r(a^m), a^m] & \text{otherwise}
\end{cases} \quad (3.48)$$

Using the fact that the expectation of a random variable can be viewed as the expectation of the conditional mean given another variable [47, p. 119], we can write

$$E[\alpha_i^* \alpha_l | r(a^m), a^m]$$

as

$$E\left[\alpha_i^* \alpha_l | r(a^m), a^m\right] = E\left[E[\alpha_i^* \alpha_l | r(a^m), \theta, a^m] | r(a^m), a^m\right]$$

$$= \int_{-\pi}^{\pi} E[\alpha_i^* \alpha_l | r(a^m), \theta, a^m] f(\theta) | r(a^m), a^m \rangle d\theta \quad (3.49)$$

In the same way, $E[\alpha_i^* \alpha_l e^{-j\theta_1} | r(a^m), a^m]$ is given by

$$E[\alpha_i^* \alpha_l e^{-j\theta_1} | r(a^m), a^m] = \int_{-\pi}^{\pi} e^{-j\theta_1} E[\alpha_i^* \alpha_l | r(a^m), \theta, a^m] f(\theta) | r(a^m), a^m \rangle d\theta \quad (3.50)$$

where $f(\theta) | r(a^m), a^m \rangle$ is the conditional pdf of $\theta_1$ given $r(a^m)$ and $a^m$. And $E[\alpha_i^* \alpha_l e^{j\theta_1} | r(a^m), a^m]$ can be viewed as the conjugate of $E[\alpha_i^* \alpha_l e^{-j\theta_1} | r(a^m), a^m]$.

Calculating $f(\theta) | r(a^m), a^m \rangle$ and $E[\alpha_i^* \alpha_l | r(a^m), \theta, a^m]$ and solving the integrals in
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(3.49) and (3.50), it is shown in Appendix E (see (E.12)) that \( \hat{\rho}_t(a^m) \) can be expressed as

\[
\hat{\rho}_t(a^m) = 2|\mathbf{R}_\alpha|q - 2|\mathbf{R}_\alpha M(a^m)\mathbf{R}_\alpha|d + c_t(a^m)c_t^*(a^m) + (\overline{\alpha}_t - \overline{\alpha}_1d_t(a^m)) (\overline{\alpha}_t - \overline{\alpha}_1d_t(a^m))
\]

\[
+ (\overline{\alpha}_t - \overline{\alpha}_1d_t(a^m))c_t(a^m)e^{-j\phi_1(a^m)} \frac{I_1\left( \frac{|\overline{\alpha}_t|}{\sigma_1^2 \sqrt{\epsilon(a^m)}} |b_1(a^m)| \right)}{I_0\left( \frac{|\overline{\alpha}_t|}{\sigma_1^2 \sqrt{\epsilon(a^m)}} |b_1(a^m)| \right)}
\]

\[
+ (\overline{\alpha}_t - \overline{\alpha}_1d_t(a^m))c_t^*(a^m)e^{j\phi_1(a^m)} \frac{I_1\left( \frac{|\overline{\alpha}_t|}{\sigma_1^2 \sqrt{\epsilon(a^m)}} |b_1(a^m)| \right)}{I_0\left( \frac{|\overline{\alpha}_t|}{\sigma_1^2 \sqrt{\epsilon(a^m)}} |b_1(a^m)| \right)}
\]  

(3.51)

where \( M(a^m) \) is given by

\[
M(a^m) \equiv \sqrt{\xi(a^m)} A(a^m) Y^*(a^m) \Gamma^*(a^m) \epsilon^{1/2}(a^m)
\]  

(3.52)

\[
A(a^m) \equiv \sqrt{\xi(a^m)} \epsilon^{1/2}(a^m) \Gamma^T(a^m) Y^T(a^m) \left( N_0 I + \frac{\xi(a^m)}{2} D(a^m) \right)^{-1}
\]  

(3.53)

c_t(a^m), d_t(a^m) and \( \phi_1(a^m) \) are given by

\[
d_t(a^m) \equiv [\mathbf{R}_\alpha M(a^m)]_{t1}
\]  

(3.54)

\[
c_t(a^m) \equiv [\mathbf{R}_\alpha A(a^m) \mathbf{r}(a^m)]_t \quad (l = 1, 2, \ldots, L)
\]  

(3.55)

\[
\phi_1(a^m) = \arg[b_1(a^m)] - \arg[\overline{\alpha}_t]
\]  

(3.56)

Using the fact that the expectation of a random variable can be viewed as the expectation of the conditional mean given another variable [47, p. 119], (3.47) can be
expressed as

$$\hat{\chi}_i(t; a^m) = E[\alpha_i e^{-j\theta_i} E[x_i(t)|r(a^m), a^m, \alpha, \theta]|r(a^m), a^m]$$  \hspace{1cm} (3.57)$$
and from Appendix F, $\hat{\chi}_i(t; a^m)$ can be expressed as

$$\hat{\chi}_i(t; a^m) = \hat{\rho}_{i t}(a^m)s(t-\tau_i; a^m) + \beta_1 \left\{ r(t)E[\alpha_i e^{-j\theta_i}|r(a^m), a^m] - \sum_{i=1}^L \hat{\rho}_{it}(a^m)s(t-\tau_i; a^m) \right\}$$  \hspace{1cm} (3.58)$$
where $\hat{\rho}_{it}(a^m)$ is given by (3.51) and

$$E[\alpha_i e^{j\theta_i}|r(a^m), a^m] = \epsilon_1(a^m) + (\bar{\alpha}_i - \bar{\alpha}_i d_i(a^m)) \frac{I_1\left(\frac{\sqrt{r_i}}{\sigma_i\sqrt{\epsilon_1(a^m)}}|b_1(a^m)|\right)}{I_0\left(\frac{\sqrt{r_i}}{\sigma_i\sqrt{\epsilon_1(a^m)}}|b_1(a^m)|\right)} e^{j\phi_1(a^m)}$$  \hspace{1cm} (3.59)$$
$I_n(\cdot)$ is the $n^{th}$ order modified Bessel function, $\epsilon_1(a^m)$ and $b_1(a^m)$ are given by (3.7) and (D.8), $d_i(a^m)$ and $\epsilon_i(a^m)$ are, respectively, given by (3.54) and (3.55).

The M-step is to maximize the goal function $Q(a|a^m)$ with respect to the sequence $a$. From (3.45), the goal function depends on $a$, and the maximization is generally an $N$-order optimization problem, meaning that all the symbols in the sequence are estimated at once. However, as will be shown in next section, for linear modulations, the sequence maximization can be reduced to a symbol-per-symbol maximization, for which a closed-form solution exists for the maximum.

**Closed form for uncoded linear modulation**

The M-step is to maximize the goal function $Q(a|a^m)$ with respect to $a$. Let us consider a linear modulation with known signaling pulse $p(t)$ so that the transmitted
signal is
\[ s(t; \mathbf{a}) = \sum_{i=1}^{N} a_i p(t - iT) \] (3.60)
where \( \mathbf{a} = [a_1, a_2, \ldots, a_N]^T \) is the symbol sequence of length \( N \). The goal function (3.45) now can be rewritten as
\[ Q(\mathbf{a}|\mathbf{a}_m) = \frac{1}{N_0} \sum_{i=1}^{N} \Re \left\{ a_i \sum_{l=1}^{L} \frac{\hat{\chi}_l(t; \mathbf{a}_m)}{\beta_l} \right\} \]
\[-\frac{1}{2N_0} \sum_{i=1}^{N} a_i a_p^* \sum_{l=1}^{L} \frac{\hat{\rho}_l(\mathbf{a}_m)}{\beta_l} \int_{T_i}^{T_F} p(t - \tau_l - iT) p^*(t - \tau_l - pT) \, dt \]
Let us now assume \( p(t) \) is designed such that it satisfies
\[ \int_{T_i}^{T_F} p(t - \tau_l - iT) p^*(t - \tau_l - jT) \, dt = 0 \quad (i \neq j) \] (3.61)
This can be achieved by considering a pulse spanning only the interval \([0,T]\).

The goal function can be simplified as
\[ Q(\mathbf{a}|\mathbf{a}_m) = \frac{1}{N_0} \sum_{i=1}^{N} \Re \left\{ a_i \sum_{l=1}^{L} \frac{\hat{\chi}_l(t; \mathbf{a}_m)}{\beta_l} \right\} \]
\[-\frac{1}{2N_0} \sum_{i=1}^{N} |a_i|^2 \sum_{l=1}^{L} \frac{\hat{\rho}_l(\mathbf{a}_m)}{\beta_l} \int_{T_i}^{T_F} |p(t - \tau_l - iT)|^2 \, dt \]
In that particular case, the maximization of the goal function over \( \mathbf{a} \) (the M-step of the EM algorithm) reduces to a symbol-by-symbol maximization, where the closed-form solution for the data symbols is given by (obtained by differentiation)
\[ a_i^{m+1} = \frac{\sum_{l=1}^{L} \hat{\chi}_l(t; \mathbf{a}_m), p(t - \tau_l - iT) > /\beta_l}{\sum_{l=1}^{L} \hat{\rho}_l(\mathbf{a}_m) \int_{T_i}^{T_F} |p(t - \tau_l - iT)|^2 \, dt /\beta_l} \] (3.62)
where \( \hat{\chi}_l(t; \mathbf{a}_m) \) is given by (3.58) and \( \hat{\rho}_l(\mathbf{a}_m) \) is given by (3.51). Note if pilot symbols
are present, the values of the pilot symbols are kept unchanged at each iteration (i.e. (3.62) is not used for any known symbols).

For simplicity, we let $\beta_1 = \beta_2 = \ldots = \beta_L = 1/L$ and assume the observation interval is long enough to include all the energy of the signals (i.e. $T_l = T + \min(\tau_l)$, $T_F = NT + \max(\tau_l)$). Let the energy of the modulated signaling pulse be $\int_{-\infty}^{\infty} |p(t)|^2 = \xi_p$. Thus, (3.62) can be simplified as

\[
\hat{a}_i^{m+1} = \frac{\sum_{l=1}^{L} \langle \hat{\chi}_i(t; a^m) , p(t - \tau_l - iT) \rangle}{\xi_p \sum_{l=1}^{L} \hat{p}_l(a^m)}
\]  

Symbol-by-symbol maximization is achieved as seen from (3.63). Before convergence, $a_i$ can be any value in complex field. At the convergence, $a_i$ is ended up with the point in the signal constellation closest to the convergence point. For uncoded BPSK,

\[
\hat{a}_i = \begin{cases} 
1, & \text{if } \Re\{a_i^\infty\} > 0 \\
-1, & \text{if } \Re\{a_i^\infty\} < 0 
\end{cases}
\]  

**Receiver structure based on the EM approach**

The sequential receiver structure using the EM algorithm with uncoded linear modulations for mixed mode Ricean/Rayleigh channels is illustrated in Fig. 3.1. Let us define

\[
u(a^m) = (u_1(a^m), \ldots, u_L(a^m))^T
\]

as the received samples. And $u_l(a^m)$ is defined as

\[
u_l(a^m) = \frac{1}{\sqrt{\xi(a^m)}} \int_{T_l}^{T_F} r(t) s^*(t - \tau_l; a^m) dt
\]
Fig. 3.1 Receiver structure based on the EM approach.
and can be obtained by passing the received signal $r(t)$ through a matched filter with impulse response $s^*((N + 1)T - t; \mathbf{a}^m) / \sqrt{\xi(\mathbf{a}^m)}$ and sampled at time $(N + 1)T + \tau_i$, where $N$, $T$ are, respectively, the sequence length and the symbol period, and $s(t)$ is the signal of transmitted sequence given by (3.60). Before further processing, a decorrelation matrix $\mathbf{Y}(\mathbf{a}^m)$ is employed to resolve the signal samples from the different paths. From the definition of $\mathbf{r}(\mathbf{a}^m)$ and (3.5), we get

$$\mathbf{r}(\mathbf{a}^m) = \mathbf{Y}^*(\mathbf{a}^m)\mathbf{u}(\mathbf{a}^m)$$

In Fig. 3.1, $\xi_p$ is the energy of the pulse $p(t)$ and $\mathbf{b}(\mathbf{a}^m)$, $\mathbf{Q}(\mathbf{a}^m)$ and $\mathbf{A}(\mathbf{a}^m)$ are defined by (D.8), (D.9) and (3.53) respectively. We define $\hat{\mathbf{\rho}}(\mathbf{a}^m)$ as a matrix with $i^{th}$ entry $\hat{\rho}_{ii}(\mathbf{a}^m)$, and $\hat{\zeta}(\mathbf{a}^m)$ and $\hat{\chi}(t; \mathbf{a}^m)$ as column vectors with elements $E[\alpha_i e^{j\phi_i} | \mathbf{r}(\mathbf{a}^m), \mathbf{a}^m]$ and $\hat{\chi}(\mathbf{a}^m)$ respectively. Therefore, from (3.59), (3.51) and (3.58)

$$\hat{\zeta}(\mathbf{a}^m) = \mathbf{c}(\mathbf{a}^m) + \mathbf{v}(\mathbf{a}^m)\frac{I_1\left(\frac{\mu_1}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}}|b_1(\mathbf{a}^m)|\right)}{I_0\left(\frac{\mu_1}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}}|b_1(\mathbf{a}^m)|\right)} e^{j\phi_1(\mathbf{a}^m)}$$

$$\hat{\rho}(\mathbf{a}^m) = 2R_\alpha - 2R_\alpha \mathbf{M}(\mathbf{a}^m)\mathbf{R}_\alpha + \mathbf{c}(\mathbf{a}^m)\mathbf{c}^\dagger(\mathbf{a}^m) + \mathbf{v}(\mathbf{a}^m)\mathbf{v}^\dagger(\mathbf{a}^m)$$

$$+ \left[\mathbf{c}(\mathbf{a}^m)\mathbf{v}^\dagger(\mathbf{a}^m)e^{-j\phi_1(\mathbf{a}^m)} + \mathbf{v}(\mathbf{a}^m)\mathbf{c}^\dagger(\mathbf{a}^m)e^{j\phi_1(\mathbf{a}^m)}\right] \frac{I_1\left(\frac{\mu_1}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}}|b_1(\mathbf{a}^m)|\right)}{I_0\left(\frac{\mu_1}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}}|b_1(\mathbf{a}^m)|\right)}$$

$$\hat{\chi}(t; \mathbf{a}^m) = \{\hat{\mathbf{\rho}}(\mathbf{a}^m)\}_d \alpha(t; \mathbf{a}^m) + \beta \left[\mathbf{r}(t)^* \hat{\zeta}(\mathbf{a}^m)) - \hat{\mathbf{\rho}}^T(\mathbf{a}^m)\alpha(t; \mathbf{a}^m)\right]$$

where, $\beta$ is the diagonal matrix with diagonal elements $\beta_i$ and $\mathbf{s}(t; \mathbf{a}^m)$ and $\mathbf{v}(\mathbf{a}^m)$ are defined as

$$\mathbf{s}(t; \mathbf{a}^m) = (s(t - \tau_1; \mathbf{a}^m), s(t - \tau_2; \mathbf{a}^m), \ldots, s(t - \tau_L; \mathbf{a}^m))^T$$
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\[ \nu(a^m) = \overline{\alpha} - \overline{\alpha}_1 d(a^m) \]

and \( c(a^m) \) and \( d(a^m) \) are column vectors with respectively \( l^{th} \) elements \( c_l(a^m) \) and \( d_l(a^m) \) as defined by (3.55) and (3.54). And \( < \hat{\chi}_l(t; a^m), p(t - \tau_l - iT) > \) in (3.63) can be implemented by passing \( \hat{\chi}_l(t; a^m) \) through the matched filter \( p^*(T - t) \) and sampling at \((i + 1)T + \tau_l\).

Fig. 3.1 shows the receiver structure at the \( m^{th} \) iteration. The process will be iterated until it converges. The number of iterations has to be chosen such that convergence is achieved, i.e., such that

\[ |a^{m+1} - a^m| < \varepsilon \]

is satisfied for several consecutive iterations, where \( \varepsilon \) is small enough. In our simulations, the total number of iterations was chosen to be 12 and the order of \( \varepsilon \) was no higher than 10\(^{-2}\) for the last three iterations. It was verified that increasing the number of iterations did not change the estimate of the probability of error. Decision is made based on \( a^m \) at the convergence (e.g., (3.64) is used for BPSK).

3.3.2 EM approach for Rayleigh channels

Using the same approach as for mixed mode Ricean/Rayleigh fading channels, the goal function for Rayleigh fading channels is given by

\[ Q(a|a^m) = \frac{1}{N_0} \sum_{l=1}^{L} \left\{ \text{Re} \left\{ \frac{1}{\beta_l} < \hat{\chi}_l(t; a^m), s(t - \tau_l; a) > \right\} - \frac{1}{2\beta_l} \beta_l(a^m) \| s(t - \tau_l; a) \|^2 \right\} \]
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where $\chi_l(t; a^m)$ is given by (3.58). Using the fact that $\alpha = 0$ and $\theta_l = 0$ for $l = 1, 2, \ldots, L$, from (D.22) and (3.51),

$$E[\alpha_l e^{j\theta_l}|r(a^m), a^m] = c_l(a^m)$$  \hspace{1cm} (3.67)

$$\tilde{\rho}_l(a^m) = 2[R_\alpha]_{il} - 2[R_\alpha M(a^m)R_\alpha]_{il} + c_l(a^m)c^*_l(a^m)$$  \hspace{1cm} (3.68)

where $R_\alpha$ is the covariance matrix of the channel and $c_l(a^m)$ and $M(a^m)$ are, respectively, given by (3.55) and (3.52). The results here are similar to [16] (see Appendix G).

3.4 K-MMSE receiver

MMSE linear equalization is one of the most attractive suboptimum approach in dealing with band-limited channels that result in ISI due to its simplicity (computational complexity is a linear function of the channel dispersion length $L$). In this section, we analyze the structures of linear MMSE-based receivers that assume knowledge of the channel (K-MMSE).

3.4.1 K-MMSE receiver for mixed mode Ricean/Rayleigh fading channels

Fig. 3.2 represents the general block diagram of a noiseless communication system that uses a linear equalizer [19, p. 586], where $a_t$ is the data symbol transmitted and $c(t)$ is the combined complex baseband impulse response of the transmitter and the channel. The system uses an analog filter matched to the combination of the transmitter filter and channel followed by a sampler sampling at the symbol rate $(1/T)$. The complex
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envelope of the received signal without noise can be expressed as

$$r(t) = \sum_{i=1}^{N} a_i c(t - iT)$$  \hspace{1cm} (3.69)$$

where $N$ is the length of the sequence. In Fig. 3.2, $y_i$ is the discrete time complex valued samples after the matched filter and given by

$$y_i = r(t) * c^*(-t)|_{t=iT}$$  \hspace{1cm} (3.70)$$

$$= \int_{-\infty}^{\infty} r(s)c^*(s - iT) \, ds$$  \hspace{1cm} (3.71)$$

(3.71) can be obtained by replacing the convolution in (3.70) by its definition and using the condition $t = iT$.

![Fig. 3.2 Block diagram of a noiseless communication system that uses a linear equalizer](image)

From section 3.1, the complex envelope of the received signal $r(t)$ of a multipath fading channel can also be expressed as (3.1). When the channel is known, $r(t)$ can be simplified as

$$r(t) = \sum_{l=1}^{L} \alpha_l s(t - \tau_l; \mathbf{a}) + n(t)$$

since the phase shifts of the paths are known. Ignoring the noise and assuming the signal described in (3.60) is transmitted over the multipath channel, $r(t)$ is now given by

$$r(t) = \sum_{l=1}^{L} \alpha_l \sum_{i=1}^{N} a_i p(t - iT - \tau_l)$$  \hspace{1cm} (3.72)$$
where \( p(t) \) is the time limited signaling pulse spanning the interval \([0, T]\).

From (3.69) and (3.72), the low-pass complex impulse response \( c(t) \) is given by

\[
c(t) = \sum_{i=1}^{L} \alpha_i p(t - \tau_i)
\]

(3.73)

Substituting (3.73) into (3.71), we get

\[
y_i = \sum_{i=1}^{L} \alpha_i^* \int_{iT + \tau_i}^{(i+1)T + \tau_i} r(t) p^*(t - \tau_i - iT) \, dt
\]

(3.74)

**Fig. 3.3** Receiver structure based on the MMSE approach

The detailed receiver structure is illustrated in Fig. 3.3. The discrete samples \( y_i \) can be obtained by passing the continuous time received signal through a matched filter \( p^*(T - t) \) with sampling time \( t = (i + 1)T + \tau_i \) \((i = 1, 2, \ldots, L)\), weighting the samples from each branch individually by the corresponding channel coefficients \( \alpha_i \) and then adding them together, as shown in Fig. 3.3. Diversity from each fading path is taken advantage of in this structure. We realize the transversal filter by a tapped delay line with \( M_\Delta \) delay elements (the unit delay time is \( T \)), \( M_\Delta + 1 \) taps and \( M_\Delta + 1 \) weights. There are two ways to estimate the weights and the symbols using the tapped delay line.
KS-MMSE receiver

One way is to estimate the current transmitted symbol $a_i$ by only using the current and past received samples, i.e., using $y_i = (y_i, y_{i-1}, \ldots, y_{i-M_A})^T$. Since in this case, the inputs of the equalizer do not include the future received samples (i.e., the samples received after the current received sample $y_i$, which corresponds to the symbol $a_i$ that we are going to estimate), we use the term single-sided tapped delay line. The KS-MMSE receiver containing a Single-sided tapped delay line is defined as the KS-MMSE receiver in this thesis. Let $w = (w_0, w_1, \ldots, w_{M_A})^T$ be the weights of the equalizer, and $y_i$ be the input of the equalizer. The output of the equalizer is

$$\hat{a}_i = w^T y_i$$

which is the unquantized estimated symbol. In our simulations, the number of weights is chosen to be equal to the path number due to the value of the inter-path delays and the fact that the duration of the signaling pulse is $T$. Simulations showed that increasing the number of the taps did not change the performance significantly. For uncoded linear modulations, after quantization, $\hat{a}_i$ ends up with the closest point in the signal constellation. For the special case of BPSK, the quantization of $\hat{a}_i$ can be simplified by using following criterion:

$$\hat{a}_i = \begin{cases} 
1, & \Re\{\hat{a}_i\} > 0 \\
-1, & \Re\{\hat{a}_i\} < 0
\end{cases}$$

Using the same approach as [1, p. 305], the optimum weight vector $\hat{w}$ for the
KS-MMSE approach is given by

$$\hat{w} = (F^{-1}p)^*$$  \hspace{1cm} (3.75)$$

where \( p \) and \( F \) are, respectively, the cross correlation vector and the input correlation matrix given by

$$p = E[a_i^*y_k|\alpha] = (E[a_i^*y_k|\alpha] \ E[a_i^*y_{k-1}|\alpha] \ \cdots \ E[a_i^*y_{M_\Delta}|\alpha])^T$$

$$F = E[y_i^*y_i^*|\alpha] = \begin{pmatrix}
E[|y_k|^2|\alpha] & E[|y_k y_{k-1}|^2|\alpha] & \cdots & E[|y_k y_{M_\Delta}|^2|\alpha] \\
E[|y_{k-1} y_k|^2|\alpha] & E[|y_{k-1}|^2|^2|\alpha] & \cdots & E[|y_{k-1} y_{M_\Delta}|^2|\alpha] \\
\vdots & \vdots & \ddots & \vdots \\
E[|y_{M_\Delta} y_k|^2|\alpha] & E[|y_{M_\Delta} y_{k-1}|^2|\alpha] & \cdots & E[|y_{M_\Delta}|^2|\alpha]
\end{pmatrix}$$

Using (3.74), (3.72) and (3.2), the elements of \( p \) and \( F \) can be, respectively, calculated as follows:

$$E[a_i^*y_{i-m}|\alpha] = E \left[ a_i^* \sum_{j=1}^{L} \sum_{k=1}^{N} a_k \sum_{l=m}^{L} \int_{T_l}^{T_{T_l}} p(t-kT-\tau_l)p^*(t-(i-m)T-\tau_j) \, dt \right]$$

$$+ E \left[ a_i^* \sum_{j=1}^{L} \int_{T_l}^{T_{T_l}} n(t)p^*(t-(i-m)T-\tau_j) \, dt \right]$$  \hspace{1cm} (3.76)$$

$$= \sum_{j=1}^{L} \sum_{l=1}^{L} \int_{T_l}^{T_{T_l}} p(t-\tau_l)(i-m)T-\tau_j) \, dt$$  \hspace{1cm} (m = 0, 1, \ldots, M_\Delta)  \hspace{1cm} (3.77)$$
3 Sequential detection for mixed mode Ricean/Rayleigh and Rayleigh fading channels

\[ E[y_{i\rightarrow q} y_{i \rightarrow m}^* | \alpha] = E \left[ \sum_{j=1}^{L} \sum_{k=1}^{N} \alpha_j^* \sum_{l=1}^{L} \int_{T_j}^{T_F} p(t - kT - \tau_l) p^* (t - (i - q)T - \tau_j) \, dt \right. \\
+ \sum_{j=1}^{L} \alpha_j^* \int_{T_j}^{T_F} n(t) p^* (t - (i - q)T - \tau_j) \, dt \\
\left. \cdot \left( \sum_{p=1}^{L} \alpha_p^* \sum_{r=1}^{N} \sum_{s=1}^{L} \alpha_r \int_{T_j}^{T_F} p(u - rT - \tau_s) p^* (u - (i - m)T - \tau_p) \, du \right) \right. \\
\left. + \sum_{p=1}^{L} \alpha_p^* \int_{T_j}^{T_F} n(u) p^* (u - (i - m)T - \tau_p) \, du \right] \\
= \sum_{j=1}^{L} \sum_{l=1}^{L} \sum_{k=1}^{N} \sum_{s=1}^{L} \alpha_j^* \alpha_l \alpha_p \alpha_s \int_{T_j}^{T_F} p(t - kT - \tau_l) p^* (t - (i - q)T - \tau_j) \, dt \\
\cdot \int_{T_j}^{T_F} p^* (u - kT - \tau_s) p(u - (i - m)T - \tau_p) \, du \\
+ 2N_0 \sum_{j=1}^{L} \sum_{p=1}^{L} \alpha_j^* \alpha_p \int_{T_j}^{T_F} p^* (t - (i - q)T - \tau_j) p(t - (i - m)T - \tau_p) \, dt \\
(\text{m, q} = 0, 1, \ldots, M_{\Delta}) \tag{3.79} \]

In the condition of the known channel, considering \( a_i, n(t) \) are independent random variables, (3.77) and (3.79) can be obtained from (3.76) and (3.78) respectively by straightforward calculations by using the facts that \( n(t) \) is a zero mean Gaussian process satisfying (3.2) and

\[
E[a_i^* a_k] = \begin{cases} 
1, & k = i \\
0, & k \neq i 
\end{cases} \tag{3.80}
\]

for BPSK assuming the transmitted symbols are 1 or -1, and they are equally likely and independent. For other linear modulations such as 4QAM, (3.80) can still be satisfied by normalization of the transmitted symbols.
KD-MMSE

We can also estimate the current transmitted symbol \( a_i \) by using the future, current and past received samples, i.e., using \( y_{i+M\Delta} = (y_{i+M\Delta}, \ldots, y_i, \ldots, y_{i-M\Delta})^T \) \((M\Delta \text{ should be even in this case})\). The linear equalizer employed here is called double-sided tapped delay line since the inputs include both the future and the past received samples. The K-MMSE receiver containing a Double-sided tapped delay line is defined as the KD-MMSE receiver in this thesis. To estimate \( a_i \) by using \( y_{i+M\Delta} \) corresponds to estimate \( a_{i-M\Delta} \) by using \( y_i = (y_i, y_{i-1}, \ldots, y_{i-M\Delta})^T \). Therefore, the receiver structure in Fig. 3.3 can also be used for the KD-MMSE approach. Let \( \mathbf{w}' = (w'_0, w'_1, \ldots, w'_{M\Delta})^T \) be the weights of the equalizer, and \( y_i \) be the input of the equalizer. The output of the equalizer is

\[
\hat{a}_{i-M\Delta} = \mathbf{w}'^T y_i
\]

Using the similar approach as for the KS-MMSE scheme, the optimum weight vector \( \hat{\mathbf{w}}' \) for the KD-MMSE approach is given by

\[
\hat{\mathbf{w}}' = (F^{-1}\mathbf{p}')^*
\]

where \( F \) is the same input correlation matrix as that of the KS-MMSE receiver, for which the elements are given by (3.79), and the cross correlation vector \( \mathbf{p}' \) is given by

\[
\mathbf{p}' = E[a^*_{i-M\Delta} y_i | \alpha] = \left(E[a^*_{i-M\Delta} y_i | \alpha] \ E[a^*_{i-M\Delta} y_{i-1} | \alpha] \ \ldots \ E[a^*_{i-M\Delta} y_{i-M\Delta} | \alpha] \right)^T
\]
3 Sequential detection for mixed mode Ricean-Rayleigh and Rayleigh fading channels

The elements of $p'$ can be calculated using the same way as to calculate the elements of $p$ for the KS-MMSE receiver, and is given by

$$E \left[ a_i^* y_{i-m} | \alpha \right] = \sum_{j=1}^{L} \sum_{i=1}^{L} \alpha_j^* \alpha_i \int_{T_j}^{T_f} p \left( t - \left( i - \frac{M\Delta}{2} \right) T - \tau_i \right) p^* \left( t - (i - m) T - \tau_j \right) dt$$

$$(m = 0, 1, \ldots, M\Delta)$$

In our simulations, for the KD-MMSE receiver, we use the same number of weights as for the KS-MMSE approach for the purpose of comparison. Similar to the KS-MMSE receiver, the quantized $\hat{a}_{i-M\Delta}$ should be the closest point in the signal constellation for uncoded linear modulations. For BPSK, the quantization of $\hat{a}_i$ can be simplified by using the following criterion:

$$\hat{a}_{i-M\Delta} = \begin{cases} 1, & \Re \left\{ \hat{a}_{i-M\Delta} \right\} > 0 \\ -1, & \Re \left\{ \hat{a}_{i-M\Delta} \right\} < 0 \end{cases}$$

3.4.2 K-MMSE receiver for Rayleigh channels

The KS-MMSE and KD-MMSE receivers for $L$-path Rayleigh fading channels are the same as for $L$-path mixed mode channels (as discussed in section 3.4) due to the available channel knowledge.
Chapter 4

Computer simulations

In this chapter, we perform a series of simulations for both mixed mode Ricean/Rayleigh fading channels and Rayleigh fading channels using the receivers derived in chapter 3. Both highly correlated and low correlated multi-path channels are considered. We use DS-CDMA signaling and BPSK/QAM modulation for our simulations. Both preamble and postamble are employed for better estimation of the channel coefficients for the EM-based approach. Simulation results with both high and low spreading gains are presented and performance of the EM-based, ML and K-MMSE receiver are compared. By running simulations with different spreading codes, we also give insight on the design of spreading codes for better performance.

The performance of each receiver is presented by the bit-error probabilities in terms of the (average) received signal-to-noise ratio per bit $\frac{E_b}{N_0}$. From (H.1) in Appendix H, for an $L$-path mixed mode channel,

$$\frac{E_b}{N_0} = \left( \sum_{i=1}^{L} 2\sigma_i^2 + |\alpha_i|^2 \right) \frac{\xi_b}{2N_0} = \left( \sum_{i=1}^{L} 2\sigma_i^2 + 2\sigma_i^2K \right) \frac{\xi_b}{2N_0}$$

where $\xi_b$ is the energy per bit of the lowpass equivalent signal and $K$ is the Ricean
parameter and defined as $K = \frac{|\alpha|^2}{2\sigma^2}$. For convenience, $K$ is expressed in dB. We consider mixed mode channels where the Rayleigh part of the path gains has equal strength (i.e. $\sigma^2_1 = \sigma^2_2 = \ldots = \sigma^2_L$).

In this thesis, we run our simulations in such a way that a confidence interval of the estimation can be determined by the computation of the standard deviation. At one received SNR level, we run the simulation ten times (for each time, enough blocks are sent that at least 10 error bits are obtained even at a high SNR) using different random seeds to get the mean of the error rates and the standard deviation of the estimation. The data points in all the simulation plots are obtained by averaging ten simulation results at that point. Fig. 4.1 illustrates our simulation procedure. Performance of BPSK signaling over highly correlated three-path mixed mode channels with $K = 5$dB and $K = 15$dB is presented by the mean error probability together with the deviation expressed by an error bar (simulation details will be discussed in the following subsections). For the neatness of the plots, error bars are not going to be shown in the other figures.

4.1 Performance of the EM-based receiver

4.1.1 Simulation details

We make further approaches for some equations of chapter 3 in order to do the simulation. Using (3.60) and (3.61), from (3.6) and (3.7),

$$\xi(a^m) = \int_{-\infty}^{\infty} \left| \sum_{n=1}^{N} a^m_n p(t - nT) \right|^2 dt = \sum_{n=1}^{N} |a^m_n|^2 \int_{-\infty}^{\infty} |p(t - nT)|^2 dt = \sum_{n=1}^{N} |a^m_n|^2 \xi_p$$

$$\epsilon_s(a^m) = \int_{T_s}^{T_s + T_p} \left| \sum_{n=1}^{N} a^m_n p(t - nT - \tau_s) \right|^2 dt / \xi(a^m)$$
Fig. 4.1 Bit error probability with error bar of BPSK signaling over highly correlated three-path mixed mode channels with K=5dB and K=15dB

\[ P(e) = \sum_{n=1}^{N} |a_n^m|^2 \int_{T_j}^{T_F} |p(t - nT - \tau_j)|^2 \, dt / \xi(a^m) \]

Assuming the observation interval is long enough to contain all the energy of the pulses, \( \xi(a^m) = I \). Substituting (3.60) into (3.9)

\[ [\Gamma(a^m)]_{ij} = \frac{1}{\xi(a^m) \sqrt{\epsilon_i(a^m) \epsilon_j(a^m)}} \sum_{n=1}^{N} \sum_{q=1}^{N} a_n^{m^*} a_q^m \int_{T_j}^{T_F} p(t - \tau_i - sT)p^*(t - \tau_j - qT) \, dt \]
\( Y(a^m) \) and \( D(a^m) \) in (3.8a) and (3.8b) can be obtained by Cholesky decomposition.

From (3.11) and (3.5),

\[
\begin{align*}
  r_k(a^m) &= \frac{1}{\sqrt{\xi(a^m)}} \sum_{i=1}^{L} \alpha_i e^{j \theta_i} \sum_{s=1}^{L} \frac{y_{ks}(a^m)}{\sqrt{\epsilon_s(a^m)}} \sum_{n=1}^{N} a_n^m \sum_{j=1}^{N} a_j^{m*} \int_{T_I}^{T_F} p(t - \tau - nT) p^*(t - \tau - jT) dt \\
  & \quad + n_k(a^m)
\end{align*}
\]

where

\[
\begin{align*}
  n_k(a^m) &= \frac{1}{\xi(a^m)} \sum_{s=1}^{L} \frac{y_{ks}(a^m)}{\sqrt{\epsilon_s(a^m)}} \sum_{j=1}^{N} a_j^{m*} \int_{T_I}^{T_F} n(t) p^*(t - \tau - jT) dt
\end{align*}
\]

Using (3.58), \(< \tilde{\chi}_l(t; a^m), p(t - \tau - iT) > \) in (3.62) can be expressed by

\[
< \tilde{\chi}_l(t; a^m), p(t - \tau - iT) >
\]

\[
= < \hat{\rho}_l(a^m)s(t - \tau_l; a^m), p(t - \tau_l - iT) > + < \beta_l E[\alpha^* e^{-j \theta_l} | r(a^m), a^m] r(t), p(t - \tau_l - iT) >
\]

\[
\begin{align*}
  & \quad + < - \beta_l \sum_{s=1}^{L} \hat{\rho}_s(a^m)s(t - \tau_s; a^m), p(t - \tau_l - iT) > \\
  = & \hat{\rho}_l(a^m)a_l^m \int_{T_I}^{T_F} |p(t - \tau_l - iT)|^2 dt \\
  & \quad + \beta_l E[\alpha^* e^{-j \theta_l} | r(a^m), a^m] \left[ \sum_{s=1}^{L} \alpha_s e^{j \theta_s} \sum_{q=1}^{L} \alpha_q \int_{T_I}^{T_F} p(t - \tau_s - qT) p^*(t - \tau_l - iT) dt + n_{li} \right] \\
  & \quad - \beta_l \sum_{s=1}^{L} \hat{\rho}_s(a^m) \sum_{q=1}^{N} \alpha_q \int_{T_I}^{T_F} p(t - \tau_s - qT) p^*(t - \tau_l - iT) dt
\end{align*}
\]

The second equation can be obtained from the first one by using (3.60), (3.1) and (3.61), and \( n_{li} \) is defined as

\[
\begin{align*}
n_{li} &= \int_{T_I}^{T_F} n(t) p^*(t - \tau_l - iT) dt
\end{align*}
\]
which is a zero mean Gaussian random variable with covariance

\[ \frac{1}{2} E[n_i n^*_m] = N_0 \int_{T_i}^T p(t - \tau_m - jT) p^*(t - \tau - iT) \, dt \]  

(4.1)

The path fadings \( \alpha \), the path phases \( \theta \), the transmitted sequences \( a \) and the noise \( n(t) \) (realized by a zero mean complex Gaussian random vector with covariance given by (4.1)) are generated only once for each transmission.

4.1.2 Performance of the EM-based receiver for mixed mode

Ricean/Rayleigh fading channels

We first consider highly correlated channels with low spreading gains. In this case, the path components are highly correlated due to their integer inter path delays and the small spreading gain. For instance, we choose path delays as \( \tau = \{0, \frac{1}{3}T, \frac{2}{3}T\} \) for three path channels. The last two signal path components are highly correlated due to the integer inter-path delay. And the cross-correlation to the first component is also high due to the small spreading gain. To ensure convergence of the EM algorithm, following [16], one transmission includes 6 data symbols and 2 pilot symbol blocks with 3 pilot symbols each. Pilot symbol blocks \{1, -1, 1\} and \{1, -1, -1\} are inserted alternatively before and after the data symbols (preamble and postamble), as shown in Fig. 4.2. The spreading gain is 3 and the spreading code is \{1, -1, -1\}. Several numbers and placement of pilot symbols have been investigated. It has been found out that convergence of the EM algorithm is not guaranteed when only preamble pilots are used. It was also found out that this is the case when 2 preamble and postamble pilot symbols are used instead of 3. Performance of the receiver based on the EM algorithm that considers a 2 preamble and 2 postamble pilot symbols plus 2 pilot symbols inserted in the middle of the sequence is quite similar to the one with
3+3 pilot symbols. Therefore, performance results in this thesis will only be obtained with the sequence block structure of Fig. 4.2.

```plaintext
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<th>Postamble</th>
<th>Preamble</th>
<th>Data Symbols</th>
<th>Postamble</th>
</tr>
</thead>
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<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
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<td></td>
<td>(d_1)</td>
<td>(\ldots)</td>
<td>(d_6)</td>
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</tr>
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<td>-1</td>
<td></td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>(d_7)</td>
<td>(\ldots)</td>
<td>(d_{12})</td>
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<td>-1</td>
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<tr>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>
```

**Fig. 4.2** Sequence block structure

Fig. 4.3-Fig. 4.5 present respectively the performance of BPSK signaling over a two-path, three-path and four-path mixed mode fading channel with different parameters \(K\). From Fig. 4.3- Fig. 4.5, it is seen that the performance becomes better with the increase of \(K\) as expected. For instance, in Fig. 4.4, when \(K = 0\)dB, the performance over three-path mixed mode channels is almost the same as three-path Rayleigh channels found in [16]. On the other hand, when \(K = 20\)dB, the performance is very close to that of the non-coherent Gaussian channel, which provides a lower bound and the ultimate performance goal. This is expected because the increase of \(K\) indicates the dominance of the Ricean channel and the channel fluctuations in amplitude depend more on the specular component of the Ricean channel. As \(K\) becomes very large, regardless of the number of paths, a mixed mode channel tends towards a Gaussian channel. Thus as expected the performance of two-path, three-path and four-path mixed mode channels are very close for large \(K\).

Next, a long sequence block with high spreading gain is transmitted and one of the Gold sequences is employed as spreading code, which is more of practical interest. The spreading gain is 31 and the Gold sequence is \(\{0,0,0,0,0,0,0,1,1,1,1,0,1,1,0,1,1,1,1,0,1,1,1,0,0,0,1,0\}\). One transmission block includes 46 data symbols and the same pilot symbol blocks as for the previous simulations. Both low correlated
channel with $\tau = \{0, 0.7T, 1.2T\}$ and highly correlated channels with $\tau = \{0, \frac{1}{3}T, \frac{4}{3}T\}$ are considered. Fig. 4.6 and Fig. 4.7 present, respectively, the performance of BPSK signaling over low and highly correlated three-path mixed mode fading channels with different parameters $K$. The performance of the EM-based receiver for mixed mode channels with $K = 20$dB is close to the performance of the optimal receiver for non-coherent Gaussian channels. From Fig. 4.6 and Fig. 4.7, it is shown that the performance for low correlated channels is better than highly correlated ones as expected. It is seen that good performances can also be achieved even when the length of the data symbol sequence is much larger than that of the pilot symbols.
Fig. 4.4 Performance of BPSK signaling over highly correlated three-path (τ = {0, \frac{1}{3}T, \frac{2}{3}T}) mixed mode Ricean/Rayleigh channels with K=0-20dB (spreading gain=3)

4.1.3 Performance of the EM-based receiver for Rayleigh channels

For L-path Rayleigh channels, the received signal to noise ratio is

\[
\frac{E_b}{N_0} = \left( \sum_{l=1}^{L} 2\sigma_l^2 \right) \frac{\xi_b}{2N_0}
\]

We assume the path gains have equal strength (i.e. \( \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_L^2 \)) in all our simulations except the following one, in which we show performance for 2-path Rayleigh channels where the path strengths are not equal. The simulation procedure here is the same as for mixed mode Ricean/Rayleigh fading channels.
Fig. 4.5 Performance of BPSK signaling over highly correlated four-path ($\tau = \{0, \frac{1}{3}T, T, \frac{2}{3}T\}$) mixed mode channels with $K=0-20$dB (spreading gain=3)

For 2-path Rayleigh channels, the received SNR is given by

$$\frac{E_b}{N_0} = \frac{(2\sigma_1^2 + 2\sigma_2^2) \xi_b}{2N_0} = 2\sigma_1^2(1 + s) \frac{\xi_b}{2N_0}$$

where $s$ is the relative Rayleigh component strength between the first and the second path ($s = \sigma_2^2/\sigma_1^2$). Fig. 4.8 shows the performance for Rayleigh channels with different $s$. It is seen that the best performance is obtained for equal path strength channels (i.e., $s = 1$). The influence of the parameter $s$ for other Rayleigh channels and mixed mode Ricean/Rayleigh channels is similar. So for the other simulations, we will only consider channels where the Rayleigh part of the path gains has equal strength (i.e., $s = 1$).
Fig. 4.6 Performance of BPSK signaling over low correlated three-path ($\tau = \{0, 0.7T, 1.2T\}$) mixed mode channels with K=0-20dB (spreading gain=31)

Fig. 4.9 shows bit error probabilities of BPSK signaling for highly correlated 2-path, 3-path and 4-path Rayleigh channels with low spreading gain (3). The transmitted data structure, the spreading code and the path delays are the same as in Fig. 4.3- Fig. 4.5. As expected, the performance over 2-path, 3-path and 4-path Rayleigh channels are, respectively, close to 2-path, 3-path and 4-path mixed mode channels with K = 0. It is shown that the EM-based receiver gives the highest diversity-like gains over the 4-path Rayleigh channel compared with 2-path and 3-path channels.
Fig. 4.7 Performance of BPSK signaling over highly correlated three-path (\(\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}\)) mixed mode channels with K=0-20dB (spreading gain=31)

4.2 Performance comparison of the EM, ML and K-MMSE approach

In this section, we compare the performance of our receiver with the ML approach to show that using the iterative technique we introduced, close to optimal detection can be achieved by a non-exhaustive approach. The ML estimation can be achieved by exhaustive search in case of short sequences. The error probabilities of known channel minimum mean square error linear equalizers (K-MMSE), ones of the most often used schemes in dealing with band-limited channels that result in ISI, are also presented. The K-MMSE receiver structure (including the KS-MMSE and the KD-
Fig. 4.8 Performance of BPSK signaling over highly correlated two-path \((\tau = \{0, \frac{T}{2}\})\) Rayleigh channels with \(s = 0.1, 0.3, 1\) (spreading gain=3)

MMSE approaches) is illustrated in Fig. 3.3. Here, we show that the EM-based receiver outperforms greatly the KS-MMSE scheme and the performance of the EM-based receiver is close to KD-MMSE scheme even when the channel is known for both K-MMSE approaches while it is not for the EM-based one.

4.2.1 Performance comparison of the EM, ML and K-MMSE approach for mixed mode Ricean/Rayleigh fading channels

Fig. 4.10 presents performance of BPSK signaling of the EM-based receiver, the ML approach and the K-MMSE schemes for highly correlated three-path Ricean/Rayleigh channels when \(K = 5\)dB and \(K = 10\)dB. One transmission includes 6 data symbols
Fig. 4.9 Performance of BPSK signaling over highly correlated two-path (\(\tau = \{0, \frac{1}{3}T\}\)), three-path (\(\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}\)) and four-path (\(\tau = \{0, \frac{1}{3}T, T, \frac{4}{3}T\}\)) Rayleigh channels (spreading gain=3)

framed by pilot symbol blocks and spreading code is \(\{1, -1, -1\}\). It is shown that for the same bit error rate, the difference of the SNR between the EM approach and the ML approach is within 1dB when \(K = 5dB\). At high SNR (with \(K = 5dB\)), the EM receiver gives about 3-dB gains compared to KS-MMSE, and has a performance very close to KD-MMSE. As \(K\) increases, the difference between the ML and EM approaches becomes smaller, and the performance of the EM is very close to the KD-MMSE, and still better than KS-MMSE. We can also see that the KD-MMSE outperforms the KS-MMSE scheme for highly correlated mixed mode channels.

Fig. 4.11 compares performance of 4QAM signaling of the EM-based receiver and the K-MMSE schemes for highly correlated three-path Ricean/Rayleigh channels
Fig. 4.10 Performance comparison of BPSK for EM, ML and K-MMSE over highly correlated three-path ($\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}$) mixed mode channels with K=5dB and K=10dB (spreading gain=3)

when K = 5, 10, 15dB. The inter-path delays and the data structure of the transmitted sequence are the same as the previous example. As we can see, the performance of the EM-based receiver is much better than the KS-MMSE, especially at high SNR’s. And also, the EM-based receiver outperforms the KD-MMSE in this case, which is very obvious at high SNR’s for K = 5dB.

4.2.2 Performance comparison of the EM, ML and K-MMSE approach for Rayleigh channels

Performances of BPSK signaling of the EM-based receiver, the ML approach and the KS-MMSE and KD-MMSE schemes over a highly correlated 3-path Rayleigh channel
are compared in Fig. 4.12. The length of the transmitted sequence is 12 (including 6 pilot symbols) and the spreading gain is 3. As we can see, the performance of the EM receiver is very close to the optimal one (even at high SNR, the difference is within 1dB). Meanwhile, the EM algorithm obtains more than 4dB gains at high SNR compared with the KS-MMSE receiver and outperforms the KD-MMSE slightly.

Fig. 4.13 compares bit error probabilities of BPSK signaling of the EM, K-MMSE approaches over low correlated ($\tau = \{0, 0.7T, 1.2T\}$) and highly correlated ($\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}$) three-path Rayleigh channels in the case of high spreading gain (31). The spreading code is the Gold sequence $\{0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 0\}$. One transmission block includes 46 data sym-
Fig. 4.12 Performance comparison of BPSK for EM, ML and K-MMSE over a highly correlated three-path ($\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}$) Rayleigh channel (spreading gain=3)

bols framed by preamble and postamble. The performances of the EM-based receiver for highly correlated and low correlated Rayleigh channels are, respectively, close to those for highly correlated and low correlated mixed mode channels with $K = 0$ as expected. For the highly correlated Rayleigh channel, the EM-based receiver outperforms the KS-MMSE scheme greatly (about 4dB) at high SNR and is very close to the KD-MMSE. For the low correlated channel, the performances of the KS-MMSE and the KD-MMSE overlap. For neatness, only one performance line is presented in Fig. 4.13. The performance of the EM-based approach is very close to the K-MMSE receivers (Note that the K-MMSE receivers assume knowledge of the channel but the EM algorithm does not). From Fig. 4.13, it is also shown that low correlated Rayleigh
channels outperform highly correlated ones.

![Graph showing performance comparison](image)

**Fig. 4.13** Performance comparison of BPSK for EM and K-MMSE over highly correlated ($\tau = \{0, \frac{1}{3}T, \frac{1}{6}T\}$) and low correlated ($\tau = \{0, 0.7T, 1.2T\}$) three-path Rayleigh channels (spreading gain=31)

### 4.3 Effect of different spreading codes on performance

In DS-CDMA system, for a fixed spreading gain $m$, there are $2^m$ spreading codes available. However, these spreading codes may have different performances in terms of their autocorrelations. Fig. 4.14 illustrates the effect of different spreading codes on performance. Short sequences (6 data symbols together with 3 preamble and 3 postamble) are transmitted for both Group A and Group B. Group A in Fig. 4.14 shows the performance of BPSK signaling with different spreading codes when the
spreading gain is 3 for $K = 0 \text{dB}$ with path delays $\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}$ and Group B is the case that the spreading gain is 4 for $K = 10 \text{dB}$ with $\tau = \{0, 0.5T, 1.5T\}$. It is seen that for $\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}$, when the spreading gain is 3, spreading codes $\{1,1,-1\}$ and $\{1,-1,-1\}$ have better performance than spreading codes $\{1,1,1\}$ and $\{1,-1,1\}$ because they are less correlated at $\tau = \frac{1}{3}T$ as shown in Fig. 4.15 (resulting in lower cross-correlations to the first path component). For last two path components, the autocorrelations of spreading codes all exhibit one due to their integer inter-path delays.). Due to the same reason, $\{1,1,-1,1\}$ and $\{1,-1,-1,-1\}$ have lower error probabilities than $\{1,-1,-1,1\}$ and $\{1,1,1,1\}$ with $\tau = \{0,0.5T,1.5T\}$. Their autocorrelations are shown in Fig. 4.16. Similar results could be observed if the values of $K$ are changed.
Fig. 4.14 Performance of BPSK with different spreading codes over highly correlated three-path mixed mode Ricean/Rayleigh channels ($\tau = \{0, \frac{1}{3}T, \frac{2}{3}T\}$, K=0dB for Group A and $\tau = \{0, 0.5T, 1.5T\}$, K=10dB for Group B)
Fig. 4.15 Autocorrelation of spreading codes with spreading gain 3

Fig. 4.16 Autocorrelation of spreading codes with spreading gain 4
Chapter 5

Conclusions and future work

5.1 Summary

This thesis studies non-coherent sequential detection techniques for mixed mode Ricean/Rayleigh multipath fading channels. The multipath delays are assumed to be known but unresolved. For non-coherent channels, the ML function is highly non-linear involving a Bessel function, thus making the search for the maximum (optimal receiver) not suitable for large sequence length or large multipath spread. A pilot-aided sequential receiver based on the EM algorithm was introduced and its structure was derived for such unresolved channels. The principle of the EM is to iteratively find an estimate of the ML estimate by alternating between a maximization step (M-step) and an expectation step (E-step). The expression of the goal function (function to be maximized at the M-step) has been derived in terms of the sequence to be maximized for an arbitrary modulation. Generally the M-step of the EM algorithm is a joint optimization with respect to all data symbols of the considered sequence. It has been shown however that for linear modulation systems over mixed mode Ricean/Rayleigh fading channels, the sequence maximization reduces to a symbol-by-symbol maximiza-
tion, for which a closed-form solution of the maximum can be found. Therefore, even large blocks of data can be processed by this procedure. Using a decorrelation matrix, the algorithm resolves the received samples at each iteration and thus diversity-like gains can be obtained.

MMSE linear equalization is one of the most often used approach in dealing with band-limited channels that result in ISI, therefore, we also derived the receiver structure of linear MMSE equalizers that assume knowledge of the channel is known (K-MMSE) for mixed mode Ricean/Rayleigh channels. The K-MMSE receivers were realized by a maximal ratio combiner followed by a tapped delay line. Through this structure, diversities can be gained and ISI are mitigated. The tapped delay line could be single-sided or double-sided, which results in the KS-MMSE and KD-MMSE receivers. In other words, estimation of the transmitted data is obtained from only past and current received samples for the KS-MMSE, and from future, current and past received samples for the KD-MMSE. The K-MMSE receivers are optimum in the sense of minimum mean square error when the channel knowledge is available, while the ML approach is optimum in the sense of minimum probability of error.

Performances of the EM-based receiver for two, three and four-path mixed mode channels have been presented for DS-CDMA with BPSK and QAM, and it was shown that the performance becomes better with the increase of K, which indicates the dominance of the Ricean channel and the fact that the channel fluctuations in amplitude depend more on the specular component of the Ricean channel. For instance, when \( K = 0 \)dB, the performance for mixed mode channels is almost the same as Rayleigh channels. On the other hand, when \( K = 20 \)dB, the performance is very close to that of the non-coherent Gaussian channel, which provides a lower bound and the ultimate performance goal.

The EM-based receiver was shown to have close to optimal performance (ML
5 Conclusions and future work

approach) with a non-exhaustive approach. The difference of the SNR between the EM approach and the ML approach is within 1dB for the same bit error rate regardless of the value of the Ricean specular component $K$. It was seen that the EM-based receiver outperforms greatly the KS-MMSE scheme especially for small $K$ even if that scheme assumes that the channel is known. For instance, from Fig. 4.10, the EM receiver gives about 3-dB gains compared to KS-MMSE at high SNR for BPSK and $K = 5$dB. And from Fig. 4.11, we can see that even more gains are achieved by the EM-based receiver for 4QAM. Provided that preamble and postamble pilots are used, the performance of the EM-based receiver is very close to the performance of the KD-MMSE receiver that has complete channel knowledge and even higher at high SNR in some cases (such as highly correlated channels and 4QAM), although the EM-based structure does not know the channel. It is also shown that the KS-MMSE does not work as well as the KD-MMSE over highly correlated multipath fading channels.

The receiver structure based on the EM algorithm was also derived for Rayleigh multipath channels as a special case of mixed mode Ricean/Rayleigh channels. For Rayleigh channels, although the ML function is much simpler than the mixed mode case, the ML-solution is still a non-linear problem unless the Grammian of the transmitted signal is a diagonal matrix (which corresponds to resolved channels). It was shown that our detection technique provides a non-exhaustive approach which achieves close to optimal performance for Rayleigh channels. The superiority of the EM algorithm over the KS-MMSE scheme that assumes knowledge of the channel for Rayleigh channels was also illustrated. And the performance of the EM receiver is close to the KD-MMSE approach (which assumes the channel is known) provided that the EM-based receiver is used with preamble and postamble pilot symbols. For highly correlated Rayleigh channels, the EM may even outperform the KD-MMSE at high SNR.
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Finally, we demonstrated that the performance of the EM-based receiver can be improved by carefully choosing the spreading code, especially for small spreading gains. Better performance is obtained for spreading codes with a low correlation at the channel inter path delays. For Gold codes, the effect of different spreading codes is not obvious since the autocorrelation of the Gold code is low.

5.2 Future work

In this thesis, we have addressed the problem of non-coherent sequential detections for unresolved multipath mixed mode Rayleigh/Ricean and Rayleigh fading channels. Sequential detection for more complicated channels such as unresolved \( M \) Ricean/\( L-M \) Rayleigh channels (i.e., a \( L \) path channel, for which the first \( M \) paths are Ricean and others are Rayleigh) or multipath Ricean channels might deserve to be further investigated, as well as channels subject to other types of fading such as Nakagami. It is expected that the performance for channels with more Ricean components is greater than those achieved for mixed mode or Rayleigh channels due to the increase of the number of the specular components. Another line of research could be to consider detection techniques for Ricean multipath fading channels modeled as tapped delay lines.

We considered DS-CDMA system with uncoded linear modulations such as BPSK and QAM, which make symbol-by-symbol maximization in the M-step possible for the EM-based receiver. It would be interesting to generalize our work with other uncoded or coded modulations, which could lead to other EM-based receiver structures. It would also be interesting to study the performance of higher order modulations such as 16QAM (which are suitable candidates in high SNR channels) since the performance of the EM-based receiver is better than K-MMSE at high SNR’s.
5 Conclusions and future work

The performance study of this thesis assumes that all receivers have perfect knowledge of the channel multipath delays. In practice, these delays are estimated and thus might be subject to estimation errors. The effect of the multipath delay estimation errors on the performance could be taken into considerations in future studies.

Simulations showed that the EM-based receiver has often a performance close to the KD-MMSE. Taking into consideration for the KD-MMSE the complexity of the front-end channel estimator as well, it would be interesting to do a detailed analysis of the complexity of the EM and KD-MMSE schemes in order to provide insight on performance/complexity trade-offs.

One problem of the EM algorithm is its convergence speed. In our work, pilot symbols were employed to improve the convergence speed and more than ten iterations were used in our simulations. Several extended EM algorithms such as $\alpha$-EM algorithm [48] and SAGE algorithm [33] have been proposed in recent papers for accelerating the convergence of the algorithm. Applying these extended EM algorithm to our sequential detection problems would be under the considerations of future work.
Appendix A

Conditional log-likelihood function for mixed mode Ricean/Rayleigh fading channels

Using the same approach as [12], we can obtain the log-likelihood function for L-path mixed mode multipath fading channels. Detailed calculations are as follows.

From (3.15), the conditional log-likelihood function given \( \theta \) can be expressed as

\[
\ln(\Lambda(a_m|\theta)) = \ln\left( \frac{|R_0|}{|R_{r|\theta,a_m}} \right) - \frac{1}{2} r^\dagger(a_m)(R_{r|\theta,a_m}^{-1} - R_0^{-1})r(a_m)
\]

\[
+ \frac{1}{2} r^\dagger(a_m)R_{r|\theta,a_m}^{-1} \mu(a_m) + \frac{1}{2} \mu^\dagger(a_m)R_{r|\theta,a_m}^{-1} r(a_m)
\]

\[
- \frac{1}{2} \mu^\dagger(a_m)R_{r|\theta,a_m}^{-1} \mu(a_m) \tag{A.1}
\]

In (A.1), the first two terms are independent of \( \theta \), the following two terms depend on \( \theta \) since \( \mu(a_m) \) is a function of \( \theta \) (see (3.16)). Using (3.16), the last term \( \mu^\dagger(a_m)R_{r|\theta,a_m}^{-1} \mu(a_m) \) can be split into a term independent of \( \theta \) and a term dependent
of $\theta$ as follows:

$$
\mu^\dagger(a_m) R_r^{-1}_{\theta, a_m} \mu(a_m)
\xi(a_m) q^\dagger(\theta) \left\{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_r^{-1}_{\theta, a_m} Y^*(a_m) \Gamma^*(a_m) \epsilon^{1/2}(a_m) \right\} d q(\theta)
+ 2 \xi(a_m) \Re \left\{ q^\dagger(\theta) \left\{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_r^{-1}_{\theta, a_m} Y^*(a_m) \Gamma^*(a_m) \epsilon^{1/2}(a_m) \right\} q(\theta) \right\}
$$

(A.2)

Since $\{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_r^{-1}_{\theta, a_m} Y^*(a_m) \Gamma^*(a_m) \epsilon^{1/2}(a_m) \}_d$ is a diagonal matrix, the first term in (A.2) is independent of $\theta$ and the second term is a function of $\theta$. Thus, (A.2) is composed of two parts, one with $\theta$ and the other without $\theta$ and is given by

$$
\ln(A(a_m | \theta)) = \left\{ \ln \left( \left| R_r \right| \left| R_r^{-1}_{\theta, a_m} \right| \right) - \frac{1}{2} \epsilon^\dagger(a_m) (R_r^{-1}_{\theta, a_m} - R_0^{-1}) \epsilon(a_m)
\right\}
- \frac{\xi(a_m)}{2} q^\dagger(\theta) \left\{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_r^{-1}_{\theta, a_m} Y^*(a_m) \Gamma^*(a_m) \epsilon^{1/2}(a_m) \right\} d q(\theta)
+ \left\{ \frac{1}{2} \epsilon^\dagger(a_m) R_r^{-1}_{\theta, a_m} \mu(a_m) + \frac{1}{2} \mu^\dagger(a_m) R_r^{-1}_{\theta, a_m} \epsilon(a_m)
\right\}
- \xi(a_m) \Re \left\{ q^\dagger(\theta) \left\{ \epsilon^{1/2}(a_m) \Gamma^*(a_m) Y^T(a_m) R_r^{-1}_{\theta, a_m} Y^*(a_m) \Gamma^*(a_m) \epsilon^{1/2}(a_m) \right\} q(\theta) \right\}
$$

(A.3)
Appendix B

Calculations of $\ln(L(a_m))$ and $g(\theta; a_m)$

This appendix provides the detailed derivations of (3.27) and (3.28). Using (3.18), (3.19) and some matrix manipulation, the first term of (3.27) comes from the second term in (3.21) as follows:

$$- \frac{1}{2} r^t(a_m) (R^{-1}_{\theta,a_m} - R^{-1}_0) r(a_m)$$

$$= - \frac{1}{2} r^t(a_m) \left\{ \frac{1}{N_0} \left( I + \xi(a_m) \frac{D(a_m)}{2N_0} \right)^{-1} - \frac{1}{N_0} I \right\} r(a_m)$$

$$= - \frac{1}{2} r^t(a_m) \left\{ \frac{1}{N_0} \left( I + \xi(a_m) \frac{D(a_m)}{2N_0} \right)^{-1} \left( I + \xi(a_m) \frac{D(a_m)}{2N_0} \right) \right\} r(a_m)$$

$$= - \frac{1}{2} r^t(a_m) \left\{ \frac{1}{N_0} \left( I + \xi(a_m) \frac{D(a_m)}{2N_0} \right)^{-1} \left( I - \xi(a_m) \frac{D(a_m)}{2N_0} \right) \right\} r(a_m)$$

$$= \frac{\xi(a_m)}{4N_0^2} r^t(a_m) \left( D^{-1}(a_m) + \frac{\xi(a_m)}{2N_0} I \right)^{-1} r(a_m)$$
The second term of (3.27) is obtained by substituting (3.18-3.19) into the term \( \ln \left( \frac{|R_0|}{R_i(\theta_m)} \right) \) of (3.21). The last term of (3.27) is obtained from the last term of (3.21) by using (3.25).

Using (3.26), the last term of (3.22) is removed, therefore \( g(\theta; a_m) \) can be calculated as follows

\[
g(\theta; a_m) = \exp \left\{ \frac{1}{2} r_i(a_m) R_i^{-1} \mu(a_m) + \frac{1}{2} \mu^i(a_m) R_i^{-1} r(a_m) \right\}
= \exp \left\{ \Re \mu^i(a_m) R_i^{-1} r(a_m) \right\}
= \exp \left\{ \sqrt{\xi(a_m)} \Re g^i(\theta) e^{1/2}(a_m) \Gamma_0(a_m) Y^T(a_m) R_i^{-1} r(a_m) \right\} \text{ using (3.16)}
= \exp \left\{ \sqrt{\xi(a_m)} \Re g^i(\theta) e^{1/2}(a_m) R_i^{-1} Y^T(a_m) D(a_m) \times \left( N_0 I + \frac{\xi(a_m)}{2} D(a_m) \right)^{-1} r(a_m) \right\} \text{ using (3.8b) and (3.18)}
= \exp \left\{ \frac{1}{\sqrt{\xi(a_m)}} \Re g^i(\theta) e^{-1/2}(a_m) R_i^{-1} Y^T(a_m) Q(a_m) r(a_m) \right\} \text{ using (3.30)}
= \exp \left\{ \Re g^i(\theta) e^{-1/2}(a_m) R_i^{-1} b(a_m) \right\} \text{ using (3.29a)}
= \exp \left\{ \Re \left\{ \frac{-\sigma_1 e^{-j\theta_1}}{\sigma_1 \sqrt{\epsilon_1(a_m)}} b_1(a_m) \right\} \right\} \text{ using (3.29b) and (3.17)}
= \exp \left\{ \left\{ \frac{-\sigma_1 e^{-j\theta_1}}{\sigma_1 \sqrt{\epsilon_1(a_m)}} b_1(a_m) \right\} \cos \left( \arg \left\{ \frac{-\sigma_1 e^{-j\theta_1}}{\sigma_1 \sqrt{\epsilon_1(a_m)}} b_1(a_m) \right\} \right) \right\}
= \exp \left\{ \frac{|\sigma_1|}{\sigma_1 \sqrt{\epsilon_1(a_m)}} b_1(a_m) |\cos(\theta_1 - \varphi_1(a_m)) | \right\}
\]  

(B.1)

where \( \varphi_1(a_m) = \arg \left[ b_1(a_m) \right] - \arg \left[ \sigma_1 \right] \).
Appendix C

Conditional pdf of $x_l(a)$ given $(\alpha, \theta, a)$

Given $(\alpha, \theta, a)$, $x_l(a)$ is a complex Gaussian random vector with conditional pdf given by (3.42). Substituting (3.38)-(3.41) into (3.42), the conditional pdf can be expressed as

$$f(x_l(a)|\alpha, \theta, a)$$

$$= \frac{1}{(2\pi)^L(\beta_l N_0)^L} \exp \left\{ -\frac{1}{2\beta_l N_0} \sum_{i=1}^{L} \left[ |x_{li}(a) - \int_{T_l}^{T_F} \alpha_l e^{j\theta_l s(t - \tau_l; a)} \phi_i^*(t; a) \, dt|^2 \right] \right\}$$

$$= \frac{1}{(2\pi)^L(\beta_l N_0)^L} \exp \left\{ -\frac{1}{2\beta_l N_0} \sum_{i=1}^{L} \left[ \int_{T_l}^{T_F} (x_l(t) - \alpha_l e^{j\theta_l s(t - \tau_l; a)}) \phi_i^*(t; a) \, dt \right] \right\}$$

$$\cdot \left[ x_{li}(a) - \int_{T_l}^{T_F} \alpha_l e^{j\theta_l s(u - \tau_l; a)} \phi_i^*(u; a) \, du \right]^*$$

$$= \frac{1}{(2\pi)^L(\beta_l N_0)^L} \exp \left\{ -\frac{1}{2\beta_l N_0} \int_{T_l}^{T_F} \sum_{i=1}^{L} \left[ x_{li}(a) - \int_{T_l}^{T_F} \alpha_l e^{j\theta_l s(u - \tau_l; a)} \phi_i^*(u; a) \, du \right]^* \right\}$$

$$\cdot \phi_i^*(t; a) (x_l(t) - \alpha_l e^{j\theta_l s(t - \tau_l; a))} \, dt \right\}$$

$$= \frac{1}{(2\pi)^L(\beta_l N_0)^L} \exp \left\{ -\frac{1}{2\beta_l N_0} \int_{T_l}^{T_F} |x_l(t) - \alpha_l e^{j\theta_l s(t - \tau_l; a)}|^2 \, dt \right\}$$
Hence

$$f(x_l(a)|\alpha, \theta, a) = \frac{1}{(2\pi)^L(\beta_lN_0)^L} \exp \left\{-\frac{1}{2\beta_lN_0} \left\| x_l(t) - \alpha_l e^{i \theta_l} s(t - \tau_l; a) \right\|^2 \right\}$$
Appendix D

Calculation of $E[\alpha_l e^{j \theta_l} | \mathbf{r}(a^m), a^m]$ 

When $l = 1$, $E[\alpha_l e^{j \theta_l} | \mathbf{r}(a^m), a^m]$ can be computed as follows:

$$E[\alpha_l e^{j \theta_1} | \mathbf{r}(a^m), a^m] = E[\alpha_l e^{j \theta_1} | \mathbf{r}(a^m), a^m]$$
$$= E\left[ E[\alpha_l e^{j \theta_1} | \mathbf{r}(a^m), a^m, \theta] | \mathbf{r}(a^m), a^m \right]$$
$$= \int_{-\pi}^{\pi} e^{j \theta_1} \left[ E[\alpha | \mathbf{r}(a^m), a^m, \theta] \right] f(\theta_1 | \mathbf{r}(a^m), a^m) d\theta_1 \quad (D.1)$$

where $f(\theta_1 | \mathbf{r}(a^m), a^m)$ is the conditional pdf of $\theta_1$ given $\mathbf{r}(a^m)$ and $a^m$. Note that for mixed mode, $\theta = (\theta_1, 0, \ldots, 0)^T$, therefore, average over $\theta$ is the same as average over $\theta_1$.

When $l \neq 1$,

$$E[\alpha_l e^{j \theta_l} | \mathbf{r}(a^m), a^m] = E[\alpha_l | \mathbf{r}(a^m), a^m]$$
$$= E\left[ E[\alpha_l | \mathbf{r}(a^m), a^m, \theta] | \mathbf{r}(a^m), a^m \right]$$
$$= \int_{-\pi}^{\pi} \left[ E[\alpha | \mathbf{r}(a^m), a^m, \theta] \right] f(\theta_l | \mathbf{r}(a^m), a^m) d\theta_l \quad (D.2)$$

From (D.1) and (D.2), it is seen that $E[\alpha_l e^{j \theta_l} | \mathbf{r}(a^m), a^m]$ can be calculated from the
expressions of \( f(\theta_1 | r(a_m^n), a_m^n) \) and \( E[\alpha|r(a_m^n), a_m^n, \theta] \).

### D.1 Calculation of \( f(\theta_1 | r(a_m^n), a_m^n) \)

Using Bayes’s rule, the conditional pdf \( f(\theta_1 | r(a_m^n), a_m^n) \) can be expressed as

\[
f(\theta_1 | r(a_m^n), a_m^n) = c_f \int f(r(a_m^n) | \theta_1, a_m^n) f(\theta_1 | a_m^n) d\theta_1 = 1
\]

where \( c_f \) is a constant independent of \( \theta_1 \) satisfying

\[
\int_{-\infty}^{\infty} c_f \int f(r(a_m^n) | \theta_1, a_m^n) f(\theta_1 | a_m^n) d\theta_1 = 1
\]

Given \( \theta \) and \( a_m^n \), from (3.14), \( r(a_m^n) \) is a Gaussian random vector with joint pdf given by

\[
f(r(a_m^n) | \theta_1, a_m^n) = \frac{1}{(2\pi)^{\frac{L}{2}} | R_{r\theta,a_m}^{-1} | \exp \left\{ -\frac{1}{2} (r(a_m^n) - \mu(a_m^n))^\top \frac{1}{2} R_{r\theta,a_m}^{-1} (r(a_m^n) - \mu(a_m^n)) \right\}
\]

where from (3.16) and (3.18)

\[
\mu(a_m^n) \triangleq E[r(a_m^n) | \theta, a_m^n] = \sqrt{\xi(a_m^n)} Y^\top(a_m^n) \Gamma^T(a_m^n) \epsilon^{1/2}(a_m^n) \varphi(\theta) \tag{D.4}
\]

\[
R_{r|\theta,a_m} = N_0 I + \frac{\epsilon(a_m^n)^2}{2} D(a_m^n) \tag{D.5}
\]

The conditional pdf function can be written as

\[
f(r(a_m^n) | \theta_1, a_m^n) = J(a_m^n) g(\theta_1; a_m^n) \tag{D.6}
\]

where \( g(\theta_1; a_m^n) \) is the function that includes all factors involving \( \theta_1 \), and \( J(a_m^n) \) is everything left over. Using the same technique as used when calculating the log-
likelihood function in section 3.2, we get

\[
\ln J(\mathbf{a}^m) = \ln \left( \frac{1}{(2\pi)^L |R_r| a^m} \right) - \frac{1}{2} \mathbf{r}^T(\mathbf{a}^m) R_r^{-1} \mathbf{r}(\mathbf{a}^m) - \frac{\xi(\mathbf{a}^m)}{2} |\mathbf{a}^m|^2 [\mathbf{B}(\mathbf{a}^m)]_{11}
\]

\[
g(\theta_1; \mathbf{a}^m) = \exp \left\{ \frac{|\mathbf{a}^m|}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}} |b_1(\mathbf{a}^m)| \cos(\theta_1 - \varphi_1(\mathbf{a}^m)) \right\}
\]

where

\[
\mathbf{B}(\mathbf{a}^m) \triangleq \left\{ \mathbf{e}^{1/2}(\mathbf{a}^m) \mathbf{Y}^*(\mathbf{a}^m) \mathbf{Y}^T(\mathbf{a}^m) R_r^{-1} \mathbf{a}^m \mathbf{Y}^*(\mathbf{a}^m) \mathbf{e}^{1/2}(\mathbf{a}^m) \right\}_d
\]

\[
b(\mathbf{a}^m) \triangleq \frac{1}{\sqrt{\xi(\mathbf{a}^m)}} \mathbf{Y}^T(\mathbf{a}^m) \mathbf{Q}(\mathbf{a}^m) \mathbf{r}(\mathbf{a}^m) = (b_1(\mathbf{a}^m), b_2(\mathbf{a}^m), \ldots, b_L(\mathbf{a}^m))^T
\]

\[
\mathbf{Q}(\mathbf{a}^m) \triangleq \frac{\xi(\mathbf{a}^m)}{2N_0} \left[ \mathbf{D}(\mathbf{a}^m)^{-1} + \frac{\xi(\mathbf{a}^m)}{2N_0} \mathbf{I} \right]^{-1}
\]

and \( \varphi_1(\mathbf{a}^m) = \arg[b_1(\mathbf{a}^m)] - \arg[\mathbf{a}^m] \).

By properties of pdf, \( c_f \) in (D.3) is given by

\[
c_f = \frac{1}{\int_{-\pi}^{\pi} f(\mathbf{r}(\mathbf{a}^m) \mid \theta_1, \mathbf{a}^m) f(\theta_1 \mid \mathbf{a}^m) \, d\theta_1} = \frac{1}{\int_{-\pi}^{\pi} \frac{1}{2\pi} J(\mathbf{a}^m) g(\theta_1; \mathbf{a}^m) \, d\theta_1}
\]

\[
J(\mathbf{a}^m) I_0 \left( \frac{|\mathbf{a}^m| |b_1(\mathbf{a}^m)|}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}} \right)
\]

where \( I_0(\cdot) \) is the zero order modified Bessel function and (D.10) can be obtained from \( [49, \text{p. 310 3.339}] \) and using the fact that \( \cos(\theta_1 - \varphi_1(\mathbf{a}^m)) \) is periodic with period \( 2\pi \) and the integral is over a period. From (D.3), (D.6) and (D.10)

\[
f(\theta_1 \mid \mathbf{r}(\mathbf{a}^m), \mathbf{a}^m) = \frac{g(\theta_1; \mathbf{a}^m)}{2\pi I_0 \left( \frac{|\mathbf{a}^m| |b_1(\mathbf{a}^m)|}{\sigma_1 \sqrt{\epsilon_1(\mathbf{a}^m)}} \right)}
\]
D Calculation of $E[\alpha e^{j\theta}|r(a^m), a^m]$

D.2 Calculation of $E[\alpha|r(a^m), \theta, a^m]$

From (3.14), $(r^T(a), \alpha^T)^T$ is a linear transformation of $(\alpha^T, n^T(a))^T$,

$$
\begin{pmatrix}
    r(a) \\
    \alpha
\end{pmatrix} =
\begin{pmatrix}
    (P(\theta)G(a))^T & I \\
    I & 0
\end{pmatrix}
\begin{pmatrix}
    \alpha \\
    n(a)
\end{pmatrix}
$$

$\alpha$ and $n(a)$ are jointly Gaussian, so $r(a)$ and $\alpha$ are jointly Gaussian as well [50, p. 273]. Thus, the conditional mean can be straightforwardly computed by [51, p. 155]

$$
E[\alpha|r(a^m), \theta, a^m] = E[\alpha|\theta, a^m] + R^{-1}_{ar|\theta, a^m} R_{r|\theta, a^m} (r(a^m) - \mu(a^m))
$$

(D.12)

where $\mu(a^m)$ is given by (D.4). Since $\alpha$ is independent of $\theta$ and $a^m$, $E[\alpha|\theta, a^m] = \overline{\alpha} = (\overline{\alpha}_1, 0, \ldots, 0)^T$. By straightforward calculations using (3.14)-(3.13), we can get

$$
R_{ar|\theta, a^m} \triangleq \frac{1}{2} E \left[ (\alpha - E[\alpha|\theta, a^m]) (r(a^m) - E[r(a^m)|\theta, a^m])^T | \theta, a^m \right]
$$

$$
= R_{\alpha} P^*(\theta) G^*(a^m)
$$

$$
= \sqrt{\xi(a^m)} R_{\alpha} P^*(\theta) \epsilon^{1/2}(a^m) \Gamma^T(a^m) Y^T(a^m)
$$

(D.13)

Note that $R_{\alpha} P^*(\theta)$ is a diagonal matrix with the $i^{th}$ entry $\sigma_i^2 e^{-j\theta_i}$ and $g(\theta) = P(\theta)\overline{\alpha}$. By substituting (D.13), (D.4) and (D.5) into (D.12), $E[\alpha|r(a^m), \theta, a^m]$ can be expressed by

$$
E[\alpha|r(a^m), \theta, a^m] = \overline{\alpha} + R_{\alpha} P^*(\theta) A(a^m) \left( r(a^m) - \sqrt{\xi(a^m)} Y^*(a^m) \Gamma^T(a^m) \epsilon^{1/2}(a^m) g(\theta) \right)
$$

$$
= \overline{\alpha} + R_{\alpha} P^*(\theta) A(a^m) r(a^m) - R_{\alpha} P^*(\theta) M(a^m) P(\theta) \overline{\alpha}
$$

(D.14)
where $A(a^m)$ and $M(a^m)$ are $L \times L$ matrices defined as

$$A(a^m) \triangleq \sqrt{\xi(a^m)} \xi^{1/2}(a^m) \Gamma^T(a^m) Y^T(a^m) \left( N_0 I + \frac{\xi(a^m)}{2} D(a^m) \right)^{-1} \quad (D.15)$$

$$M(a^m) \triangleq \sqrt{\xi(a^m)} A(a^m) Y^*(a^m) \Gamma^*(a^m) \xi^{1/2}(a^m) \quad (D.16)$$

### D.3 Calculation of $E[\alpha_l e^{j\theta_1} | r(a^m), a^m]$ 

Using that $R_\alpha P^*(\theta) = P^*(\theta) R_\alpha$ and $P(\theta) \overline{\alpha} = (\pi_i e^{j\theta_1}, 0 \ldots 0)^T$, from (D.14) the $l^{th}$ element of $E[\alpha | r(a^m), \theta, a^m]$ may be expressed as

$$[E[\alpha | r(a^m), \theta, a^m]]_l = \overline{\alpha}_l + e^{-j\theta_1} c_l(a^m) - \overline{\alpha}_l e^{j\theta_1} e^{-j\theta_1} d_l(a^m) \quad (D.17)$$

where $d_l(a^m)$ and $c_l(a^m)$ are, respectively, given by:

$$d_l(a^m) = [R_\alpha M(a^m)]_{ll} \quad (D.18)$$

$$c_l(a^m) = [R_\alpha A(a^m) r(a^m)]_l \quad (l = 1, 2, \ldots, L) \quad (D.19)$$

Substituting (D.17), (D.11) and (D.7) into (D.1) yields

$$E[\alpha_l e^{j\theta_1} | r(a^m), a^m]$$

$$= \int_{-\pi}^{\pi} \{ c_l(a^m) + \overline{\alpha}_l (1 - d_l(a^m)) e^{j\theta_1} \} \frac{g(\theta_1; a^m)}{2\pi I_0 \left( \frac{|m|}{\sigma^2 \epsilon_1(a^m)} |b_1(a^m)| \right)} d\theta_1$$

$$= c_l(a^m) + \overline{\alpha}_l (1 - d_l(a^m)) \frac{I_1 \left( \frac{\sigma^2 \epsilon_1(a^m)}{|m|} |b_1(a^m)| \right)}{I_0 \left( \frac{\sigma^2 \epsilon_1(a^m)}{|m|} |b_1(a^m)| \right)} e^{j\varphi_1(a^m)} \quad (D.20)$$

where $I_1(\cdot)$ is the first order modified Bessel function and the last equation is obtained
using [52]
\[
I_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{x \cos \theta\} \cos(n\theta) \, d\theta
\]
and the fact that
\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{x \cos \theta\} \sin(n\theta) \, d\theta = 0
\]
We can calculate (D.2) (i.e. \( l \neq 1 \)) in the same way by using \((D.17)\), \((D.11)\) and \((D.7)\)
\[
E[\alpha_1 e^{j\theta_1} | r(a^m), a^m] = c_2(a^m) - \pi_1 d_1(a^m) \frac{I_1 \left( \frac{|\sigma_1|}{\sigma_1 c_1(a^m)} |b_1(a^m)| \right)}{I_0 \left( \frac{|\sigma_1|}{\sigma_1 c_1(a^m)} |b_1(a^m)| \right)} e^{j\varphi_1(a^m)} \tag{D.21}
\]
From (D.20)-(D.21), \( E[\alpha_1 e^{j\theta_1} | r(a^m), a^m] \) can be expressed by
\[
E[\alpha_1 e^{j\theta_1} | r(a^m), a^m] = c_2(a^m) + (\overline{\alpha_1} - \overline{\alpha_1} d_1(a^m)) \frac{I_1 \left( \frac{|\sigma_1|}{\sigma_1 c_1(a^m)} |b_1(a^m)| \right)}{I_0 \left( \frac{|\sigma_1|}{\sigma_1 c_1(a^m)} |b_1(a^m)| \right)} e^{j\varphi_1(a^m)} \tag{D.22}
\]
Appendix E

Calculation of $\hat{\rho}_{li}(a^m)$

We may express $\hat{\rho}_{li}(a^m) = E[\alpha_i^* e^{-j\theta_i} \alpha_l e^{j\theta_l} | r(a^m), a^m]$ separately in terms of the values of the indexes $l$ and $i$

$$\hat{\rho}_{li}(a^m) = \begin{cases} 
E[\alpha_i^* \alpha_l e^{-j\theta_l} | r(a^m), a^m] & l = 1 \text{ and } i \neq 1 \\
E[\alpha_i^* \alpha_i e^{j\theta_i} | r(a^m), a^m] & l \neq 1 \text{ and } i = 1 \\
E[\alpha_i^* \alpha_i | r(a^m), a^m] & \text{otherwise}
\end{cases} \quad (E.1)$$

From (3.49) and (3.50)

$$E[\alpha_i^* \alpha_i | r(a^m), a^m] = \int_{-\pi}^{\pi} E[\alpha_i^* \alpha_i | r(a^m), \theta, a^m] f(\theta_1 | r(a^m), a^m) d\theta_1 \quad (E.2)$$

$$E[\alpha_i^* e^{-j\theta} | r(a^m), a^m] = \int_{-\pi}^{\pi} e^{-j\theta_1} E[\alpha_i^* \alpha_i | r(a^m), \theta, a^m] f(\theta_1 | r(a^m), a^m) d\theta_1 \quad (E.3)$$

From (E.2-E.3), it is seen that $E[\alpha_i^* \alpha_i | r(a^m), a^m]$ and $E[\alpha_i^* e^{-j\theta_1} | r(a^m), a^m]$ can be calculated from the expressions of $f(\theta_1 | r(a^m), a^m)$ and $E[\alpha_i^* \alpha_i | r(a^m), \theta, a^m]$. 
E.1 Calculation of $E[\alpha_i^* \alpha_i | \mathbf{r}(a^m), \theta, a^m]$ 

From the definition of the covariance matrix, $E[\alpha_i^* \alpha_i | \mathbf{r}(a^m), \theta, a^m]$ is given by 

$$
E[\alpha_i^* \alpha_i | \mathbf{r}(a^m), \theta, a^m] = 2[R_{\alpha | \mathbf{r}, \theta, a^m}]_i + E[\alpha_i^* | \mathbf{r}(a^m), \theta, a^m]E[\alpha_i | \mathbf{r}(a^m), \theta, a^m] \quad (E.4)
$$

where the conditional means $E[\alpha_i^* | \mathbf{r}(a^m), \theta, a^m] \overset{\Delta}{=} [E[\alpha | \mathbf{r}(a^m), \theta, a^m]]_i$ and $E[\alpha_i | \mathbf{r}(a^m), \theta, a^m] \overset{\Delta}{=} [E[\alpha | \mathbf{r}(a^m), \theta, a^m]]_i$ can be obtained from (D.17).

As mentioned on page 94, $\alpha$ and $\mathbf{r}(a^m)$ are jointly Gaussian, so the conditional covariance $R_{\alpha | \mathbf{r}, \theta, a^m}$ can be computed by [51, p. 155]

$$
R_{\alpha | \mathbf{r}, \theta, a^m} = R_{\alpha} - R_{\alpha | \mathbf{r}, \theta, a^m} R_{\mathbf{r} | \theta, a^m}^{-1} R_{\mathbf{r} | \theta, a^m}
$$

By using (D.13), (D.5), (D.16) and the fact that $R_{\mathbf{r} | \theta, a^m} = R_{\mathbf{r} | \theta, a^m}^T$, $R_{\alpha | \mathbf{r}, \theta, a^m}$ can be expressed by

$$
R_{\alpha | \mathbf{r}, \theta, a^m} = R_{\alpha} - P^*(\theta) R_{\alpha} M(a^m) R_{\alpha}^T P^T(\theta)
$$

$$
= \begin{pmatrix}
\sigma_1^2 \\
\sigma_2^2 \\
\vdots \\
\sigma_L^2 \\
\end{pmatrix} - \begin{pmatrix}
\sigma_1^2 \sigma_1^2 M_{11}(a^m) & \sigma_1^2 \sigma_2^2 M_{12}(a^m) e^{-j\theta_1} & \ldots & \sigma_1^2 \sigma_L^2 M_{1L}(a^m) e^{-j\theta_1} \\
\sigma_2^2 \sigma_1^2 M_{21}(a^m) e^{j\theta_1} & \sigma_2^2 \sigma_1^2 M_{22}(a^m) & \ldots & \sigma_2^2 \sigma_L^2 M_{2L}(a^m) \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_L^2 \sigma_1^2 M_{L1}(a^m) e^{j\theta_1} & \sigma_L^2 \sigma_2^2 M_{L2}(a^m) & \ldots & \sigma_L^2 \sigma_L^2 M_{LL}(a^m)
\end{pmatrix}
$$

(E.5)

where $M_{ij}(a^m) = [M(a^m)]_{ij}$. 
Substituting (D.17) and (E.5) into (E.4) yields

\[ E[\alpha_i^*\alpha_i | r(a^m), \theta, a^m] \]

\[
= 2[R_{\alpha[\theta, a^m]}]_{11} + [\bar{\alpha}_1(1 - d_1(a^m))] + [\bar{\alpha}_1(1 - d_1(a^m))] + c_1(a^m)e^{-j\theta_1}
\]

\[
= 2\sigma^2 - 2\sigma^4 M_{11}(a^m) + |\bar{\alpha}_1|^2 |1 - d_1(a^m)|^2 + |c_1(a^m)|^2 + \bar{\alpha}_1(1 - d_1(a^m)) c_1(a^m) e^{-j\theta_1}
\]

\[
+ \bar{\alpha}_1(1 - d_1(a^m)) c_1(a^m) e^{j\theta_1}
\]

(E.6)

\[ E[\alpha_i^*\alpha_i | r(a^m), \theta, a^m] \]

\[
= 2[R_{\alpha[\theta, a^m]}]_{22} + [c_1(a^m) - \bar{\alpha}_1 d_i(a^m) e^{j\theta_1}]^* [c_1(a^m) - \bar{\alpha}_1 d_i(a^m) e^{j\theta_1}]
\]

\[
= 2\sigma^2 \delta_{ii} - 2\sigma^4 M_{ii}(a^m) + c_1(a^m) c_i(a^m) + |\bar{\alpha}_1|^2 d_i(a^m) d_i(a^m) - c_i(a^m) \bar{\alpha}_1 d_i(a^m) e^{-j\theta_1}
\]

\[
- c_i(a^m) \bar{\alpha}_1 d_i(a^m) e^{j\theta_1}
\]

(E.7)

E.2 Calculation of \( E[\alpha_i^*\alpha_i | r(a^m), a^m] \) when \( l \neq 1, i \neq 1 \) or \( l = i = 1 \)

Substituting (E.6) and (D.11) into (E.2) when \( l = i = 1 \) and solving the integral yields

\[ E[\alpha_i^*\alpha_i | r(a^m), a^m] \]

\[
= 2\sigma^2 - 2\sigma^4 M_{11}(a^m) + |\bar{\alpha}_1|^2 |1 - d_1(a^m)|^2 + |c_1(a^m)|^2 + |\bar{\alpha}_1|^2 |1 - d_1(a^m)| c_1(a^m) e^{-j\varphi_1(a^m)}
\]

\[
+ \bar{\alpha}_1(1 - d_1(a^m)) c_1(a^m) e^{j\varphi_1(a^m)}
\]

(E.8)
Substituting (E.7) and (D.11) into (E.2) when \( l \neq 1, i \neq 1 \) and solving the integral yields

\[
E[\alpha_i^* \alpha_i | r(a^m), a^m] 
= 2\sigma_i^2 \delta_{ii} - 2\sigma_i^2 \sigma_i^2 M_{ii}(a^m) + c_i(a^m) \phi_i(a^m) + d_i(a^m) d_i^*(a^m) |\alpha_i|^2 
- \left[ \alpha_i \phi_i(a^m) d_i^*(a^m) e^{-j\phi_1(a^m)} + \alpha_i d_i(a^m) \phi_i(a^m) e^{j\phi_1(a^m)} \right] \frac{I_1 \left( \frac{|\alpha_i|}{\sqrt{\varepsilon_i(a^m)}} |b_1(a^m)| \right)}{I_0 \left( \frac{|\alpha_i|}{\sqrt{\varepsilon_i(a^m)}} |b_1(a^m)| \right)}
\]

(E.9)

**E.3 Calculation of** \( E[\alpha_i^* \alpha_i e^{-j\theta_1} | r(a^m), a^m] \) **when** \( i \neq 1 \)

Substituting (E.4) and (D.11) into (E.3) when \( l = 1 \) and solving the integral yields

\[
E[\alpha_i^* \alpha_i e^{-j\theta_1} | r(a^m), a^m] 
= \int_{-\pi}^{\pi} e^{-j\theta_1} \left[ -2\sigma_i^2 \sigma_i^2 M_{ii}(a^m) e^{j\theta_1} + \left| \alpha_i \right|^2 (1 - d_i(a^m)) c_i(a^m) e^{j\theta_1} \right] \cdot f(\theta_1 | r(a^m), a^m) d\theta_1
= -2\sigma_i^2 \sigma_i^2 M_{ii}(a^m) + c_i(a^m) c_i(a^m) - |\alpha_i|^2 (1 - d_i(a^m)) d_i(a^m)
+ \left[ \alpha_i (1 - d_i(a^m)) c_i(a^m) e^{-j\phi_1(a^m)} - \alpha_i c_i(a^m) d_i(a^m) e^{j\phi_1(a^m)} \right] \frac{I_1 \left( \frac{|\alpha_i|}{\sqrt{\varepsilon_i(a^m)}} |b_1(a^m)| \right)}{I_0 \left( \frac{|\alpha_i|}{\sqrt{\varepsilon_i(a^m)}} |b_1(a^m)| \right)}
\]

(E.10)
E.4 Calculation of $E[\alpha_i^* \alpha_1 e^{j \theta_1} | r(\alpha^m), \alpha^m]$ when $l \neq 1$

For $l \neq 1$ and $i = 1$, $E[\alpha_i^* \alpha_1 e^{j \theta_1} | r(\alpha^m), \alpha^m]$ can be obtained from (E.10)

$$E[\alpha_i^* \alpha_1 e^{j \theta_1} | r(\alpha^m), \alpha^m] = \left( E[\alpha_i^* \alpha_1 e^{-j \theta_1} | r(\alpha^m), \alpha^m] \right)^*$$

$$= -2\sigma_i^2 \sigma_i^2 M_{1i}^* (\alpha^m) + c_i(\alpha^m) c_i^*(\alpha^m) - |\alpha_i|^2 (1 - d_i(\alpha^m)) d_i^*(\alpha^m)$$

$$+ \left[ \alpha_i (1 - d_i(\alpha^m)) c_i^*(\alpha^m) e^{j \varphi_1(\alpha^m)} - \alpha_i c_i(\alpha^m) d_i^*(\alpha^m) e^{-j \varphi_1(\alpha^m)} \right] \frac{I_1 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}{I_0 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}$$

$$= -2\sigma_i^2 \sigma_i^2 M_{1i}^* (\alpha^m) + c_i(\alpha^m) c_i^*(\alpha^m) - |\alpha_i|^2 (1 - d_i(\alpha^m)) d_i^*(\alpha^m)$$

$$- \left[ \alpha_i c_i(\alpha^m) d_i^*(\alpha^m) e^{-j \varphi_1(\alpha^m)} - \alpha_i c_i^*(\alpha^m) (1 - d_i(\alpha^m)) e^{j \varphi_1(\alpha^m)} \right] \frac{I_1 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}{I_0 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}$$

(E.11)

since from (D.16), $M^*(\alpha^m) = M^T(\alpha^m)$.

Therefore, combining (E.8)-(E.11) with (E.1), $\hat{\rho}_{il}(\alpha^m)$ can be expressed as

$$\hat{\rho}_{il}(\alpha^m) = 2[R_{\alpha}]_{il} + 2[R_{\alpha} M(\alpha^m) R_{\alpha}]_{il} + c_i(\alpha^m) c_i^*(\alpha^m) + \left( \bar{\alpha}_i - \bar{\alpha}_1 d_i(\alpha^m) \right) \left( \bar{\alpha}_i - \bar{\alpha}_1 d_i(\alpha^m) \right)$$

$$+ \left( \bar{\alpha}_i - \bar{\alpha}_1 d_i^*(\alpha^m) \right) c_i(\alpha^m) e^{-j \varphi_1(\alpha^m)} \frac{I_1 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}{I_0 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}$$

$$+ \left( \bar{\alpha}_i - \bar{\alpha}_1 d_i(\alpha^m) \right) c_i^*(\alpha^m) e^{j \varphi_1(\alpha^m)} \frac{I_1 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}{I_0 \left( \frac{|\alpha_i|}{\sigma_i \sqrt{\epsilon_1(\alpha^m)}} |b_1(\alpha^m)| \right)}$$

(E.12)

where $M(\alpha^m)$, $c_i^*(\alpha^m)$, $d_i^*(\alpha^m)$ and $\varphi_1(\alpha^m)$ are defined by (D.16), (D.19), (D.18) and (3.31).
Appendix F

Calculation of $\hat{\chi}_l(t; a^m)$

Recall from (3.47) that $\hat{\chi}_l(t; a^m)$ is defined as

$$\hat{\chi}_l(t; a^m) \triangleq E \left[ \alpha_i e^{-j\theta_i x_l(t)} | r(a^m), a^m \right]$$  \hspace{1cm} (F.1)

We know that the expectation of a random variable can be viewed as the expectation of the conditional mean given another variable [47, p. 119], so from (F.1) can be expressed as

$$\hat{\chi}_l(t; a^m) = E \left[ \alpha_i e^{-j\theta_i} E \left[ x_l(t) | r(a^m), a^m, \alpha, \theta \right] | r(a^m), a^m \right]$$  \hspace{1cm} (F.2)

Using Karhunen-Loève expansion and (3.38-3.39)

$$E \left[ x_l(t) | r(a^m), a^m, \alpha, \theta \right] = E \left[ \sum_{k=1}^{L} x_{l,k}(a^m) \phi_k(t; a^m) | r(a^m), a^m, \alpha, \theta \right]$$

$$= \sum_{k=1}^{L} \left[ E \left[ x_l(a^m) | r(a^m), a^m, \alpha, \theta \right] \right] \phi_k(t; a^m)$$  \hspace{1cm} (F.3)

From (3.11), given $(a^m, \alpha, \theta)$, $r(a^m)$ is a Gaussian random vector. Similarly,
from (3.33), given \((a^m, \alpha, \theta)\), \(x_l(a^m)\) is a Gaussian random vector related to \(r(a^m)\) by a linear transformation given by \(r(a^m) = \sum_{i=1}^{L} x_l(a^m)\). Hence, given \((a^m, \alpha, \theta)\), \(x_l(a^m)\) and \(r(a^m)\) are jointly Gaussian [50, p. 273] with conditional mean given by [51, p. 155]

\[
E[x_l(a^m)|r(a^m), a^m, \alpha, \theta] = E[x_l(a^m)|a^m, \alpha, \theta] + R_{x_l|a^m, \alpha, \theta} R_{r|a^m, \alpha, \theta}^{-1} (r(a^m) - E[r(a^m)|a^m, \alpha, \theta])
\]

where \(R_{x_l|a^m, \alpha, \theta}\), the covariance of \(r(a^m)\) when \(a^m, \alpha\) and \(\theta\) are held fixed is equal to \(N_0 I\) and \(R_{x_l|a^m, \alpha, \theta}\), the covariance of \(x_l(a^m)\) and \(r(a^m)\) is equal to \(\beta I N_0 I\) since the noise terms \(n_i(t)\) are independent and satisfy (3.34). \(E[x_l(a^m)|a^m, \alpha, \theta]\) is given by (3.40) and \(E[r(a^m)|a^m, \alpha, \theta]\) can be obtained from (3.11) directly. Consequently from (3.10) and (F.3) we get

\[
E[x_l(t)|r(a^m), a^m, \alpha, \theta] = \\
= \sum_{k=1}^{L} \left\{ \int_{T_l}^{T_F} \alpha_i e^{j\theta_i} s(u - \tau_i; a^m) \phi_k^*(u; a^m) du \right. \\
+ \beta t \left[ \int_{T_l}^{T_F} \left[ r(u) - \sum_{i=1}^{L} \alpha_i e^{j\theta_i} s(u - \tau_i; a^m) \right] \phi_k^*(u; a^m) du \right\} \left[ \phi_k(t; a^m) \right]
\]

Substituting (F.4) into (F.2) yields

\[
\hat{x}_l(t; a^m) = E\left[ |\alpha_l e^{j\theta_l}|^2 s(t - \tau_l; a^m) |r(a^m), a^m\right] + E\left[ \beta \alpha_l e^{-j\theta_l} r(t) |r(a^m), a^m\right] \\
- E\left[ \beta L \sum_{i=1}^{L} \alpha_i e^{-j\theta_i} \alpha_i e^{j\theta_i} s(t - \tau_i; a^m) |r(a^m), a^m\right]
\]
\[ \hat{\chi}_i(t; \mathbf{a}_m^m) = \hat{\rho}_i(\mathbf{a}_m^m) s(t - \tau_i; \mathbf{a}_m^m) + \beta_i \left\{ r(t) E[\alpha_i^* e^{-j\theta_i} | \mathbf{r}(\mathbf{a}_m^m), \mathbf{a}_m^m] - \sum_{i=1}^{L} \hat{\rho}_i(\mathbf{a}_m^m) s(t - \tau_i; \mathbf{a}_m^m) \right\} \]

where \( \hat{\rho}_i(\mathbf{a}_m^m) \) defined in (3.46) can be expressed as (E.12).
Appendix G

Verification of the steps of the EM algorithm for Rayleigh fading channels

In section 3.3.2, (3.67) and (3.68) can be transformed as follows

\[
E[\alpha_i e^{j\theta_i}] r(a^m), a^m] = c_t(a^m)
\]

\[
= [R_\alpha A(a^m) r(a^m)]_l
\]

\[
= \left[ R_\alpha \sqrt{\xi(a^m)} e^{1/2} (a^m) \Gamma^T(a^m) Y^T(a^m) \left( N_0 I + \frac{\xi(a^m)}{2} D(a^m) \right)^{-1} r(a^m) \right]_l \quad \text{using (D.15)}
\]

\[
= \left[ \sqrt{\xi(a^m)} e^{1/2} (a^m) R_\alpha \Gamma^T(a^m) Y^T(a^m) D^{-1}(a^m) \left( N_0 D^{-1}(a^m) + \frac{\xi(a^m)}{2} I \right)^{-1} r(a^m) \right]_l
\]

\[
= \left[ \sqrt{\xi(a^m)} e^{1/2} (a^m) R_\alpha \Gamma^T(a^m) Y^T(a^m) D^{-1}(a^m) \right.

\cdot \left( N_0 (Y^T(a^m))^{-1} (2 \xi(a^m) R_\alpha \Gamma^*(a^m))^{-1} Y^T(a^m) + \frac{\xi(a^m)}{2} I \right)^{-1} r(a^m) \right]_l \quad \text{using (3.8b)}
\]
\[
\hat{\rho}_i(a^m) = 2[R_{\alpha}]_{ii} - 2[R_{\alpha} M(a^m) R_{\alpha}]_{ii} + c_i(a^m) c_i^*(a^m)
\]

\[
= 2R_{\alpha} - 2R_{\alpha} \xi(a^m) a^{1/2}(a^m) R_{\alpha} Y^T(a^m) \left( N_0 I + \frac{\xi(a^m)}{2} D(a^m) \right)^{-1}
\]

\[
\cdot Y^*(a^m) R_{\alpha} (a^m) c_i(a^m) c_i^*(a^m) \quad \text{using (D.16)}
\]

\[
= 2R_{\alpha} - 2R_{\alpha} \xi(a^m) a^{1/2}(a^m) R_{\alpha} Y^T(a^m) (2\epsilon(a^m) R_{\alpha} \Gamma^*(a^m) Y^T(a^m))^{-1}
\]

\[
\cdot \left( N_0 (2\epsilon(a^m) R_{\alpha})^{-1} + \frac{\xi(a^m)}{2} \Gamma^*(a^m) \right)^{-1} (Y^*(a^m))^{-1} \Gamma^*(a^m) \epsilon^{1/2}(a^m) R_{\alpha} \\
\quad + c_i(a^m) c_i^*(a^m) \quad \text{using (3.8)}
\]

\[
= 2R_{\alpha} - \frac{1}{N_0} \epsilon^{-1/2}(a^m) \left( \frac{1}{2} (\epsilon(a^m) R_{\alpha})^{-1} + \frac{\xi(a^m)}{2N_0} \Gamma^*(a^m) \right)^{-1} \xi(a^m) \Gamma^*(a^m) \epsilon^{1/2}(a^m) R_{\alpha} \\
\quad + c_i(a^m) c_i^*(a^m)
\]
G Verification of the steps of the EM algorithm for Rayleigh fading channels

\[ \begin{align*}
&= \left[ 2\mathbf{\epsilon}^{-1/2}(\mathbf{a}^m) \left( \mathbf{I} - \frac{1}{2N_0} \left( \frac{1}{2} (\mathbf{\epsilon}(\mathbf{a}^m) \mathbf{R}_\alpha)^{-1} + \frac{\xi(\mathbf{a}^m)}{2N_0} \mathbf{Y}^*(\mathbf{a}^m) \mathbf{\xi}(\mathbf{a}^m) \right) \right)^{-1} \mathbf{\Gamma}^*(\mathbf{a}^m) \mathbf{\xi}(\mathbf{a}^m) \right] \\
&\quad \cdot \mathbf{\epsilon}^{1/2}(\mathbf{a}^m) \mathbf{R}_\alpha \right]_{ii} + c_i(\mathbf{a}^m) c_i^*(\mathbf{a}^m) \\
&= \left[ 2\mathbf{\epsilon}^{-1/2}(\mathbf{a}^m) \left( \frac{1}{2} (\mathbf{\epsilon}(\mathbf{a}^m) \mathbf{R}_\alpha)^{-1} + \frac{\xi(\mathbf{a}^m)}{2N_0} \mathbf{\Gamma}^*(\mathbf{a}^m) \right)^{-1} \\
&\quad \cdot \left( \frac{1}{2} (\mathbf{\epsilon}(\mathbf{a}^m) \mathbf{R}_\alpha)^{-1} + \frac{\xi(\mathbf{a}^m)}{2N_0} \mathbf{\Gamma}^*(\mathbf{a}^m) - \frac{\xi(\mathbf{a}^m)}{2N_0} \mathbf{\Gamma}^*(\mathbf{a}^m) \right) \mathbf{\epsilon}^{1/2}(\mathbf{a}^m) \mathbf{R}_\alpha \right]_{ii} + c_i(\mathbf{a}^m) c_i^*(\mathbf{a}^m) \\
&= \left[ 2\mathbf{\epsilon}^{-1/2}(\mathbf{a}^m) \left( (\mathbf{\epsilon}(\mathbf{a}^m) \mathbf{R}_\alpha)^{-1} + \frac{\xi(\mathbf{a}^m)}{N_0} \mathbf{\Gamma}^*(\mathbf{a}^m) \right)^{-1} \mathbf{\epsilon}^{-1/2}(\mathbf{a}^m) \right]_{ii} + c_i(\mathbf{a}^m) c_i^*(\mathbf{a}^m) \\
\end{align*} \]

(G.2)

Assuming a big observation interval \([T_i, T_F]\) such that all delayed signals \(s(t - \tau; \mathbf{a})\) are contained within \([T_i, T_F]\), \(\mathbf{\epsilon}(\mathbf{a}^m) = \mathbf{I}\) and (G.1) and (G.2) can be reduced to

\[ E[\alpha_1 e^{i\theta} | \mathbf{r}(\mathbf{a}^m), \mathbf{a}^m] = \left[ (N_0 \mathbf{R}_\alpha^{-1} + \xi(\mathbf{a}^m) \mathbf{\Gamma}^*(\mathbf{a}^m))^{-1} \sqrt{\xi(\mathbf{a}^m)} (\mathbf{Y}^*(\mathbf{a}^m))^{-1} \mathbf{r}(\mathbf{a}^m) \right], \]

\[ \hat{\rho}_{ii}(\mathbf{a}^m) = \left[ 2 \left( \mathbf{R}_\alpha^{-1} + \frac{\xi(\mathbf{a}^m)}{N_0} \mathbf{\Gamma}^*(\mathbf{a}^m) \right)^{-1} \right]_{ii} + c_i(\mathbf{a}^m) c_i^*(\mathbf{a}^m) \]

The results here are similar to [16] and corresponding to equation (3) in [16]. The covariance matrix \(\mathbf{\Gamma}\) of \(\mathbf{a}\) and the Grammian \(\mathbf{G}(\mathbf{a})\) of \(s(t; \mathbf{a})\) in the \(L_2\) space in [16] correspond respectively, to \(2\mathbf{R}_\alpha\) and \(\xi(\mathbf{a})\mathbf{\Gamma}^*(\mathbf{a})\) in this thesis. In [16], \(\mathbf{G}(\mathbf{a})\) and \(< s(t; \mathbf{a}), r^H(t) >\) are defined as follows:

\[ \mathbf{G}(\mathbf{a})_{ij} = \int s^*(t - \tau_i; \mathbf{a}) s(t - \tau_j; \mathbf{a}) \, dt \]

\[ < s(t; \mathbf{a}), r^H(t) > = \int s^*(t; \mathbf{a}) r(t) \, dt \]
Appendix H

Expression of the average received SNR per bit

Supposing $s(t; a)$ is transmitted over an $L$-path mixed mode Ricean/Rayleigh channel (a channel with $L$ independent paths for which the first path is Ricean distributed and the rest $L - 1$ paths are Rayleigh distributed), the complex envelope of the received signal is given by (3.1). Assuming only one symbol transmission (3.1) reduces to

$$r(t) = \sum_{l=1}^{L} \alpha_l e^{j\theta_l} s(t - \tau_l; a_1) + n(t)$$

where $s(t; a_1)$ is the transmitted signal associated with the first symbol $a_1$. Assuming a long observation interval such that it contains all the signal energies of all paths, the average received signal energy per symbol is given by

$$E_s = E \left[ \int_{-\infty}^{\infty} \left| \sum_{l=1}^{L} \alpha_l e^{j\theta_l} s(t - \tau_l; a_1) \right|^2 dt \right]$$

$$= \int_{-\infty}^{\infty} \sum_{l=1}^{L} \sum_{k=1}^{L} E \left[ |\alpha_l \alpha_k^* e^{j(\theta_l - \theta_k)}| s(t - \tau_l; a_1) |^2 \right] dt$$
\[\begin{align*}
&= \int_{-\infty}^{\infty} \sum_{l=1}^{L} \sum_{k=1}^{L} E \left[ \alpha_l \alpha_k e^{j(\theta_l - \theta_k)} \right] |s(t - \tau_l; a_1)|^2 dt \\
&= \int_{-\infty}^{\infty} \sum_{l=1}^{L} E \left[ E \left[ \alpha_l \alpha_k e^{j(\theta_l - \theta_k)} |\theta| \right] \right] |s(t - \tau_l; a_1)|^2 dt \\
&= \int_{-\infty}^{\infty} \sum_{l=1}^{L} E \left[ |\alpha_l|^2 \right] |s(t - \tau_l; a_1)|^2 dt \\
&= \int_{-\infty}^{\infty} \sum_{l=1}^{L} E \left[ |\alpha_l|^2 \right] |s(t - \tau_l; a_1)|^2 dt = \sum_{l=1}^{L} E \left[ |\alpha_l|^2 \right] \int_{-\infty}^{\infty} |s(t; a_1)|^2 dt
\end{align*}\]

The average received signal energy per bit is then

\[E_b = \frac{E_s}{N_b} = \sum_{l=1}^{L} \left( E \left[ |\alpha_l - \bar{\alpha_l}|^2 \right] + |\bar{\alpha_l}|^2 \right) \xi_b\]

\[= \sum_{l=1}^{L} \left( 2\sigma_l^2 + |\bar{\alpha_l}|^2 \right) \xi_b \quad \text{(H.1)}\]

where \(N_b\) is the number of bit per symbol and \(\xi_b = \frac{1}{N_b} \int_{-\infty}^{\infty} |s(t; a_1)|^2 dt\) is the energy per bit of the lowpass equivalent signal (assumed to be independent of the transmitted data).
References


References


