Asymptotic Multiuser Efficiency of a Decorrelator Based Successive Interference Cancellation DS-CDMA Multiuser Receiver

Bin Yang
Dept. of Systems and Computer Engineering
Carleton University, Ottawa, ON, Canada.
Email: byang@sce.carleton.ca

Florence Danilo-Lemoine
Dept. of Systems and Computer Engineering
Carleton University, Ottawa, ON, Canada.
Email: fdanilo@sce.carleton.ca

ABSTRACT

This paper presents asymptotic multiuser efficiency (AME) performance analysis of the decorrelator based successive interference cancellation (DBSIC) DS-CDMA multiuser receiver. DBSIC is similar to the conventional SIC receiver except that a decorrelator is used at each detection stage prior to cancellation. AMEs of DBSIC are compared with those of single-user matched filter, optimum, decorrelating and conventional SIC receivers over both additive white Gaussian noise (AWGN) and Rayleigh fading channels. It is shown that over AWGN channels, DBSIC AME is lower than the optimum receiver AME when the interfering user is weaker than the user of interest but approaches the unity optimum receiver AME as the interferer becomes stronger than the user of interest. For all spreading codes cross-correlation considered, DBSIC has a similar AME curve shape as the conventional SIC receiver but has a higher AME which remains positive for all user energy ratios, illustrating its near-far resistance over AWGN channels. The AME of DBSIC is better than that of the decorrelating detector when the interferer is much weaker or slightly stronger than the user of interest and has worse performance when the interferer is not much weaker. It is shown that DBSIC is completely near-far resistant over Rayleigh fading channels with the same AME as the decorrelating scheme for user 1 and as the optimum scheme for user 2, both AMEs being independent of the user energy ratio. This shows that DBSIC is preferable compared to the decorrelating detector, while still being less complex than the optimum scheme over Rayleigh fading channels.

I. INTRODUCTION

Multiuser detection techniques such as the decorrelating, interference cancellation (IC) or minimum mean-square-error (MMSE) detectors combat the limiting effects of multiple access interference (MAI) and inter-symbol interference (ISI) on CDMA systems, and improve the signal-to-interference-plus-noise ratio (SINR) and system capacity [1]. Bit error rate (BER) is one commonly used performance measure for comparing multiuser schemes. Asymptotic multiuser efficiency (AME) is another performance analysis tool that is also used to evaluate the performance of wireless CDMA systems. [2] investigates the impact of imperfect channel estimation on the AME of a two-stage multiuser detector and finds that channel mismatch leads to AME degradation over AWGN channels. The AME for an M-stage parallel interference cancellation receiver is presented in [3]. It is found that the M-stage cancellation receiver always outperforms the conventional receiver in terms of AME. As the number of detection stages goes to infinity, the AME over AWGN channels has a value larger than zero for all user energy meaning that the detector is near-far resistant. AME upper and lower bounds are derived in [4] for decision-feedback (DFB) multiuser detectors. The results show that DFB detection with decorrelated tentative decision achieves better AME over AWGN channels than that with zero-forcing feed-forward equalizer. [5] investigates further the AME for multi-stage decision-directed multiuser detectors and derives a closed form expression for a two-user system with soft-decision. AME performance of variable-spreading-length (VSL) and multi-code (MC) multirate CDMA systems is evaluated for optimum [6], [7], decorrelating and MMSE detectors [7] over AWGN channels showing that VSL outperforms MC in case of equicorrelated correlation matrix but has similar performance for a correlation matrix obtained from random spreading codes. [8] shows that VSL has better AME compared to MC over 2-path Rayleigh channels with optimal detection. Upper and lower bounds of AME are given in [9] for a dual-rate decorrelating decision feedback (DDFB) detector. It shows that DDFB always outperforms the high-rate decorrelator (HRD). While DDFB achieves better performance than the low-rate decorrelator (LRD) in the scenario of strong interference, it has worse performance than LRD when the interference is weak. [10] presents an improved MMSE-based receiver and compares its AME performance with the conventional MMSE. In this paper the conventional AME definition was modified by considering the mismatched delay error channel. [11] studies AME for a generalized linear SIC detector for a DS-CDMA system over AWGN channels.

The decorrelator based SIC receiver (DBSIC) is presented in [12] for a multi-rate CDMA system. Unlike the conventional SIC scheme, at each stage of the DBSIC receiver, a decorrelator on top of the conventional Matched Filter (MF) detector is employed to detect the user’s signal to be used at the next stage. Using simulations and analytical analysis, [12] illustrates the performance gains in BER of DBSIC over the conventional single-user MF, decorrelating and conventional SIC detectors under both perfect and imperfect channel side information, but does not consider AME performance.
This paper extends previous work done in [12] by analyzing AME performance of DBSIC over AWGN and Rayleigh fading channels and comparing its performance with classical multiuser schemes such as optimum, conventional single-user MF, SIC and decorrelating detectors.

The paper is organized as follows. Section II presents the CDMA system model and describes the DBSIC scheme and the other commonly-used multiuser receivers considered in this paper. AME performance analysis of DBSIC is developed and compared with those of classical multiuser schemes in section III. Conclusions are given in section IV.

II. SYSTEM MODEL AND RECEIVER STRUCTURES

Considering a synchronous single-rate BPSK DS-CDMA system with $K$ users, the received signal is

$$r(t) = \sum_{k=1}^{K} \sqrt{E_k} b_k \alpha_k S_k(t) + n(t)$$

where $b_k \in \{-1, 1\}$, $E_k$, $S_k(t)$ and $\alpha_k$ are respectively the transmitted bit, signal energy, signature waveform and path gain (modeled as a zero-mean circularly complex Gaussian random variable) of the $k^{th}$ physical user. The noise $n(t)$ is a zero mean complex Gaussian random process with power spectral density $\sigma^2$.

Let $C_k(t) = \alpha_k S_k(t)$ be the effective signature sequence for the $k^{th}$ user. The received signal can be re-written as:

$$r(t) = \sum_{k=1}^{K} \sqrt{E_k} b_k C_k(t) + n(t)$$

Assuming that the channel gains are known to the receiver, the sampled outputs of $K$ matched filters are

$$y_k = \int_{0}^{T_s} r(t) C_k^*(t) \, dt \quad k = 1, \ldots, K$$

where $T_s$ is the symbol duration. Let $\vec{y} = [y_1, \ldots, y_K]^T$. The output vector $\vec{y}$ is

$$\vec{y} = \Re \vec{b} + \vec{n}$$

where $\Re$ is a correlation matrix of dimension $K \times K$ with $m^n$ th element:

$$\rho_{nm} = \int_{0}^{T_s} C_n(t) C^*_m(t) \, dt$$

$$\mathbf{E} = \text{diag}(\sqrt{E_1}, \ldots, \sqrt{E_K}), \quad \vec{b} = [b_1, \ldots, b_K]^T, \quad \vec{n} = [n_1, \ldots, n_K]^T, \quad n_k = \int_{0}^{T_s} n(t) C^*_k(t) \, dt$$

1. Conventional single-user matched filter (MF) detector:

$$\hat{b}_{\text{MF}} = \text{sgn} (\Re \{ \vec{y} \})$$

where $\Re \{ \vec{y} \}$ represents the vector formed by the real parts of the elements of the vector $\vec{y}$.

2. Decorrelating detector:

$$\hat{b}_{\text{DEC}} = \text{sgn} (\Re \{ [\mathbf{E}]^{-1} \vec{y} \})$$

For the conventional SIC and DBSIC, the received signal is passed through the bank of matched filters once and the SIC cancellation/detecting order is determined based on these outputs for all the symbols of the sequence. Let $\vec{y}_{\text{new}}$ be the vector obtained by reordering $\vec{y}$ so that its elements $y_{i\text{new}}^\text{new}$ are listed in decreasing order. Then

$$\vec{y}_{\text{new}} = \Re \vec{y}_{\text{new}} \vec{b}_{\text{new}}^\text{new} + \vec{n}_{\text{new}} = Z \vec{b}_{\text{new}} + \vec{n}_{\text{new}}$$

$$y_{i\text{new}}^\text{new} = \sum_{j=1}^{K} z_{ij} y_{j\text{new}}^\text{new} + n_{i\text{new}}^\text{new} \quad i = 1, \ldots, K$$

with $Z = \Re \vec{y}_{\text{new}} \vec{b}_{\text{new}}$ and $y_{i\text{new}}^\text{new}$ is the $i^{th}$ element of $\vec{y}_{\text{new}}$.

3. Conventional SIC detector:

$$\hat{y}_{i,\text{SIC}}^\text{new} = \text{sgn}(\Re \{ y_{i\text{new}}^\text{new} \})$$

$$\hat{y}_{i,\text{SIC}}^\text{new} = \text{sgn} \left( \Re \left\{ y_{i\text{new}}^\text{new} - \sum_{j=1}^{k-1} z_{ij} y_{j,\text{SIC}}^\text{new} \right\} \right) \quad k = 2, \ldots, K$$

4. DBSIC detector: The first symbol of the strongest user is determined using

$$\hat{b}_{i,\text{DBSIC}}^\text{new} = \text{sgn} (\Re \{ [Z^{-1}] \vec{y}_{i\text{new}} \})$$

but unlike the decorrelating detector that detects all users’ signals, only the first signal is determined at this stage. The other $y_{i\text{new}}^\text{new}$’s are determined as follows. Define $\vec{y}_{(k)}^\text{new} = [y_{k\text{new}}, \ldots, y_{K\text{new}}]^T$, $\vec{n}_{(k)}^\text{new} = [n_{k\text{new}}, \ldots, n_{K\text{new}}]^T$, $\vec{b}_{(k)}^\text{new} = [b_{k\text{new}}, \ldots, b_{K\text{new}}]^T$, where $y_{i(1)} = y_{i\text{new}}$ for $i = k, \ldots, K$, $k \geq 2$

$$y_{i\text{new}}^\text{new} = y_{i(k-1)}^\text{new} - z_{i(k-1)} \hat{b}_{(k-1)}^\text{new} - \sum_{j=k}^{K} z_{ij} \hat{y}_{j,\text{SIC}}^\text{new} + n_{i\text{new}}^\text{new} \quad (1)$$

$$n_{i\text{new}}^\text{new} = n_{i\text{new}} + \sum_{j=1}^{k-1} z_{ij} (\hat{y}_{j\text{new}} - \hat{b}_{j\text{new}}^\text{new})$$

$$\Rightarrow \vec{y}_{(k)}^\text{new} = Z^{(k)} \vec{b}_{(k)}^\text{new} + \vec{n}_{(k)}^\text{new}$$

where the matrix $Z^{(k)} = (K-k+1) \times (K-k+1)$ is given by

$$
\begin{pmatrix}
z_{kk} & z_{kk+1} & \cdots & z_{kK} 
z_{k+1,k} & z_{k+1,k+1} & \cdots & z_{k+1,K} 
\vdots & \vdots & \ddots & \vdots 
z_{KK-k} & z_{KK-k+1} & \cdots & z_{KK}
\end{pmatrix}
$$
The estimate of $\hat{b}_{k,\text{DBSC}}$ is obtained using
\[
\hat{b}_{k,\text{DBSC}} = \text{sgn} \left( \Re \left\{ \left[ \{ Z(k)^{-1} \} y_{\text{new}} \right]_{1} \right\} \right)
\]
(2)

III. PERFORMANCE ANALYSIS

A. AME Performance over AWGN channels

In the case of AWGN channels, the AME is defined as [1]
\[
\eta_k = \lim_{\sigma \to 0} \frac{e_k(\sigma)}{E_k}
\]
or equivalently using
\[
\eta_k = \sup_{r \in [0,1]} \left\{ \lim_{\sigma \to 0} \left( \frac{P_{e,k}(\sigma)}{Q\left( \sqrt{\frac{r E_k}{\sigma}} \right)} \right) = 0 \right\}
\]
(3)
\[
= 2 \lim_{\sigma \to 0} \sigma^2 \log \left( \frac{1}{P_{e,k}(\sigma)} \right)
\]
(4)
where $P_{e,k}(\sigma)$, $E_k$ and $e_k(\sigma)$ are respectively the BER, actual energy and effective energy of the $k^{th}$ user, and $\sigma^2 = N_0/2$ is the power spectral density of the additive white Gaussian noise. The effective energy is the energy required by user $k$ such that it achieves the same $P_{e,k}(\sigma)$ in the same AWGN channel but without the MAI.

A two-user synchronous system over AWGN and Rayleigh fading channels is considered to evaluate the AME performance.

For the AWGN channel, the matched filter (MF) outputs simplifies to
\[
\begin{align*}
y_1 &= \sqrt{E_1} b_1 + \sqrt{E_2} \rho b_2 + n_1 \\
y_2 &= \sqrt{E_1} \rho b_1 + \sqrt{E_2} b_2 + n_2
\end{align*}
\]
(5)
where $\rho = \int_{0}^{T} s_1(t) s_2(t) dt$ is the cross-correlation between user 1 and user 2.

Referring to [1], the AMEs for the conventional single-user MF, optimum, decorrelating and SIC receivers are given as follows:

1. Conventional MF receiver:
\[
\eta_1 = \left( \max \left\{ 0, 1 - \sqrt{\frac{E_2}{E_1}} |\rho| \right\} \right)^2
\]
\[
\eta_2 = \left( \max \left\{ 0, 1 - \sqrt{\frac{E_1}{E_2}} |\rho| \right\} \right)^2
\]
2. Optimum receiver:
\[
\eta_1 = \min \left\{ 1, 1 + \frac{E_2}{E_1} - 2\sqrt{\frac{E_2}{E_1}} |\rho| \right\}
\]
\[
\eta_2 = \min \left\{ 1, 1 + \frac{E_1}{E_2} - 2\sqrt{\frac{E_1}{E_2}} |\rho| \right\}
\]

3. Decorrelating receiver:
\[
\eta_1 = \sqrt{\frac{E_1}{E_2}} |\rho| + \frac{E_1}{E_2} \frac{1 - \rho^2}{1 - |\rho|^2}
\]
\[
\eta_2 = \min \left\{ 1, 1 + \frac{E_2}{E_1} - 2\sqrt{\frac{E_2}{E_1}} |\rho| \right\}
\]

4. SIC receiver:
\[
\eta_1 = \left( \max \left\{ 0, 1 - \sqrt{\frac{E_2}{E_1}} |\rho| \right\} \right)^2
\]
\[
\eta_2 = \left\{ \begin{array}{ll}
1 - 4\sqrt{\frac{E_1}{E_2}} |\rho| + \frac{E_1}{E_2} \frac{1 - \rho^2}{1 - |\rho|^2}, & |\rho| \leq \frac{E_1}{E_2} \leq 1 + |\rho|, |\rho| \geq \frac{1}{2} \\
1, & 1 + |\rho| < \sqrt{\frac{E_1}{E_2}} \\
1 - 4\sqrt{\frac{E_1}{E_2}} |\rho| + \frac{E_1}{E_2} \frac{1 - \rho^2}{1 - |\rho|^2}, & \sqrt{\frac{E_1}{E_2}} < 4 |\rho| (1 - \rho^2), |\rho| < \frac{1}{2}
\end{array} \right.
\]

AME Performance of DBSC for AWGN channels:

For simplicity we assume that user 1 is the first user to be canceled (no reordering done, $\hat{y}_{\text{new}} = \hat{y}$, $\hat{b}_{\text{new}} = \hat{b}$). Since the estimate of the first bit using DBSC is the same as the one that would be obtained with the decorrelating detector, we obtain
\[
\eta_1 = \eta_{1,\text{dec}} = 1 - \rho^2
\]

To obtain $\eta_2$, we compute the BER $P_{e,2}(N_0)$ for the second user. From (1) and (5)
\[
y_2^{(2)} = y_2^{(1)} - \sqrt{E_1} \rho \hat{b}_1 = y_2 - \sqrt{E_1} \rho \hat{b}_1
\]
\[
y_2^{(2)} = \sqrt{E_2} \rho \hat{b}_2 + n_2 + \sqrt{E_1} \rho (b_1 - \hat{b}_1)
\]
\[
\hat{b}_2 = \text{sgn} \left( \Re \left\{ \left[ \{ Z^{(2)} \}^{-1} \hat{y}^{(2)} \} \right] \right\} \right)
\]
\[
= \text{sgn} \left( b_2 + \frac{1}{\sqrt{E_2}} \rho (b_1 - \hat{b}_1) \right)
\]
(6)

since $z_{21} = \sqrt{E_1} \rho$ and $Z^{(2)} = [z_{22}] = [\sqrt{E_2}]$. Then assuming equally likely transmitted bits, the BER of user 2 is
\[
P_{e,2}(N_0) = P(\hat{b}_2 \neq b_2)
\]
\[
= P(\hat{b}_2 = 0 | b_2 = -1) + P(\hat{b}_2 = 1 | b_2 = 1)
\]
\[
= \frac{1}{2} P(\hat{b}_2 = 0 | (b_2 = -1, b_1 = 1))
\]
\[
+ \frac{1}{2} P(\hat{b}_2 = 1 | (b_2 = -1, b_1 = 1))
\]
where $P(\hat{b}_2 = 1 | (b_2 = -1, b_1 = 1))$ and $P(\hat{b}_2 = 1 | (b_2 = 1)$
\[ P(\hat{b}_2 = 1 | b_2 = -1, b_1 = 1) = \sum_{i = \pm 1} P(\hat{b}_2 = 1 | b_2 = -1, b_1 = 1, \hat{b}_1 = i) \] \[ \cdot P(\hat{b}_1 = i | b_2 = -1, b_1 = 1) \]
\[ P(\hat{b}_2 = 1 | b_2 = -1, b_1 = 1) = Q \left( \sqrt{\frac{2E_2}{N_0}} \right) \left( 1 - Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0}} \right) \right) \]
\[ + Q \left( \sqrt{\frac{2E_2 - 2\rho \sqrt{2E_1}}{N_0}} \right) Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0}} \right) \]
where we have used (6) for \( P(\hat{b}_2 = 1 | b_2 = -1, b_1 = 1, \hat{b}_1 = i) \) and
\[ P(\hat{b}_1 = -1 | b_2 = -1, b_1 = 1) = P(\hat{b}_1 = -1 | b_1 = 1) = P_{\epsilon_1}(N_0) = Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0}} \right) \]
\[ \] since \( \hat{b}_1 = \text{sgn} \left( \Re \left\{ (R\mathbf{E})^{-1} \hat{y}_1 \right\} \right) \) or equivalently \( \hat{b}_1 = \text{sgn} \left( \Re \left\{ \hat{b} + (R\mathbf{E})^{-1} \eta_1 \right\} \right) \). Therefore,
\[ P_{\epsilon_2}(N_0) = P_{\epsilon_2}(N_0) + P_{\epsilon_2}(N_0) \]
where
\[ P_{\epsilon_2}(N_0) = Q \left( \sqrt{\frac{2E_2}{N_0}} \right) \left( 1 - Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0}} \right) \right) \]
\[ \] and
\[ P_{\epsilon_2}(N_0) = \frac{1}{2} Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0}} \right) \]
\[ \cdot \left[ Q \left( \sqrt{\frac{2E_2 - 2\rho \sqrt{2E_1}}{N_0}} \right) + Q \left( \sqrt{\frac{2E_2 + 2\rho \sqrt{2E_1}}{N_0}} \right) \right] \]
It can be shown that \( \eta_2 \) can be obtained as
\[ \eta_2 = \min \{ \eta_2^a, \eta_2^b \} \]
where \( \eta_2^a, \eta_2^b \) correspond to \( P_{\epsilon_2}(N_0), P_{\epsilon_2}(N_0) \) respectively according to (3-4). Referring to equations 3.41 and 3.43 in [1], for all \( r \in [0, 1] \), we have \( 0 \leq \sqrt{2rE_2} < \sqrt{2E_2} \) and
\[ \lim_{N_0 \to 0} Q \left( \sqrt{\frac{2E_2}{N_0}} \right) = 0 \to \lim_{N_0 \to 0} \frac{P_{\epsilon_2}(N_0)}{Q \left( \sqrt{\frac{2E_2}{N_0}} \right)} = 0 \]
therefore,
\[ \eta_2^a = 1 \] (8)
From (4)
\[ \eta_2^b = \frac{1}{E_2} \lim_{N_0 \to 0} N_0 \log \frac{1}{P_{\epsilon_2}(N_0)} = \eta_2^{(b,1)} + \eta_2^{(b,2)} \] (9)
where
\[ \eta_2^{(b,1)} = \frac{1}{E_2} \lim_{N_0 \to 0} N_0 \log \frac{1}{Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0}} \right)} \]
\[ = \frac{1}{E_2} \left( \sqrt{E_1(1 - \rho^2)} \right)^2 = \frac{E_1}{E_2} (1 - \rho^2) \] (10)
since \( 2 \lim_{x \to 0} \sigma^2 \log Q(x/\sigma) = -[(x)^+]^2 \), where \( [x]^+ = \max\{0, x\} \).
\[ \eta_2^{(b,2)} = \frac{1}{E_2} \lim_{N_0 \to 0} N_0 \log \frac{1}{Q \left( \sqrt{\frac{2E_2 - 2\rho \sqrt{2E_1}}{N_0}} \right) + Q \left( \sqrt{\frac{2E_2 + 2\rho \sqrt{2E_1}}{N_0}} \right)} \]
\[ = \sup_{\rho \in [0,1]} \left\{ \frac{1}{2} \left[ Q \left( \sqrt{\frac{2E_2 - 2\rho \sqrt{2E_1}}{N_0}} \right) + Q \left( \sqrt{\frac{2E_2 + 2\rho \sqrt{2E_1}}{N_0}} \right) \right] \right\} = 0 \]
It is observed that if \( \sqrt{E_2} \leq 2 |\rho| \sqrt{E_1} \), the term
\[ \left[ \frac{1}{2} Q \left( \sqrt{\frac{2E_2 - 2\rho \sqrt{2E_1}}{N_0}} \right) + \frac{1}{2} Q \left( \sqrt{2E_2 + 2\rho \sqrt{2E_1}} \right) \right] \]
does not approach to zero as \( N_0 \) goes to zero which concludes that \( \eta_2^{(b,2)} \) is 0. If \( \sqrt{E_2} > 2 |\rho| \sqrt{E_1} \) then
\[ \lim_{N_0 \to 0} \left[ \frac{1}{2} Q \left( \sqrt{\frac{2E_2 - 2\rho \sqrt{2E_1}}{N_0}} \right) + \frac{1}{2} Q \left( \sqrt{\frac{2E_2 + 2\rho \sqrt{2E_1}}{N_0}} \right) \right] = 0 \]
when \( r < \left( 1 - 2 |\rho| \sqrt{\frac{E_1}{E_2}} \right)^2 \)
which implies that \( \eta_2^{(b,2)} = \left( 1 - 2 |\rho| \sqrt{\frac{E_1}{E_2}} \right)^2 \), therefore,
\[ \eta_2^{(b,2)} = \left( \max \left\{ \frac{1}{2} - 2 |\rho| \sqrt{\frac{E_1}{E_2}} \right\} \right)^2 \] (11)
Combining (7), (8), (9), (10) and (11), it is obtained that

$$\eta_2 = \min \left\{ 1, \frac{E_1}{E_2}(1 - \rho^2) + \left( \max \left\{ 0, 1 - 2|\rho| \sqrt{\frac{E_1}{E_2}} \right\} \right)^2 \right\}$$

B. AME Performance over Rayleigh fading channels

In the case of Rayleigh fading channels, the AME can be obtained as [1]

$$\eta_k = \lim_{\sigma \to 0} \frac{\sigma^2}{4E_k P_{e,k}(\sigma)}$$

(12)

Considering a two-user system over a single-path Rayleigh fading channel, the MF outputs can be written as:

$$y_1 = |\alpha_1|^2 \sqrt{E_1} b_1 + \alpha_2 \alpha_1 \sqrt{E_2} \rho b_2 + n_1$$
$$y_2 = \alpha_2 \alpha_1 \sqrt{E_1} \rho b_1 + |\alpha_2|^2 \sqrt{E_2} b_2 + n_2$$

(13)

For a Rayleigh fading channel, \( R = |\alpha_k| \) has a Rayleigh probability distribution:

$$P_R(r) = re^{-r^2/2}, \quad r \geq 0$$

From [1], the AMEs for the conventional single-user MF, optimum, decorrelating and SIC receivers over a single-path Rayleigh fading channel are given as follows:

1. Conventional MF receiver:

$$\eta_1 = \eta_2 = 0$$

2. Optimum receiver:

$$\eta_1 = \eta_2 = 1$$

3. Decorrelating receiver:

$$\eta_1 = \eta_2 = 1 - \rho^2$$

AME Performance of DBSIC for Rayleigh fading channels:

Using similar techniques as for the AWGN channels, it can be shown that

$$P_{e,1|\alpha_1}(N_0) = Q \left( \frac{2E_1(1 - \rho^2)}{N_0} |\alpha_1| \right)$$

$$P_{e,1}(N_0) = E \left[ P_{e,1|\alpha_1}(N_0) \right] = \int_0^{+\infty} P_R(r)P_{e,1|\alpha_1}(N_0) \, dr$$

$$= \int_0^{+\infty} re^{-r^2/2} Q \left( \frac{2E_1(1 - \rho^2)}{N_0} r \right) \, dr$$

$$= \frac{1}{2} \left( 1 - \frac{\frac{N_0}{2}}{\frac{E_1}{(1 - \rho^2)}} \right)$$

From (12)

$$\eta_1 = \lim_{N_0 \to 0} \frac{\frac{N_0}{2}}{4E_1 P_{e,1}(N_0)} = 1 - \rho^2$$

Using (1) and (13), we have

$$y_2^{(2)} = y_2^{(1)} - z_2 \hat{b}_1 = y_2 - \alpha_1 \alpha_2^* \rho \sqrt{E_1} \hat{b}_1$$

$$= |\alpha_2|^2 \sqrt{E_2} b_2 + n_2 + \alpha_1 \alpha_2^* \sqrt{E_1} \rho (b_1 - \hat{b}_1)$$

and from (2) it can be shown that

$$P_{e,2|\alpha_1,\alpha_2}(N_0) = Q \left( \sqrt{\frac{2E_2}{N_0} |\alpha_2|} \right) \cdot \left[ 1 - Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0} |\alpha_1|} \right) \right]$$

$$+ \frac{1}{2} Q \left( \sqrt{\frac{2E_1(1 - \rho^2)}{N_0} |\alpha_1|} \right)$$

$$\cdot Q \left( \frac{|\alpha_2| \sqrt{2E_2} - 2R \{ \alpha_1 \exp(-j \text{phase}(\alpha_2)) \} \rho \sqrt{2E_1}}{\sqrt{N_0}} \right)$$

$$+ Q \left( \frac{|\alpha_2| \sqrt{2E_2} + 2R \{ \alpha_1 \exp(-j \text{phase}(\alpha_2)) \} \rho \sqrt{2E_1}}{\sqrt{N_0}} \right)$$

Integrating with respect to \( |\alpha_2| \), \( |\alpha_1| \), \( \text{phase}(\alpha_1) \) and \( \text{phase}(\alpha_2) \), it can be shown that

$$P_{e,2}(N_0) = E \left[ P_{e,2|\alpha_1,\alpha_2}(N_0) \right]$$

$$= \frac{1}{4} \left[ 1 - \frac{1}{\sqrt{1 + \frac{N_0}{2E_2}}} \right] \left[ 1 + \frac{1}{\sqrt{1 - \frac{N_0}{2E_1(1 - \rho^2)}}} \right]$$

$$+ \frac{1}{4} \left[ 1 - \frac{1}{\sqrt{1 + \frac{N_0}{2E_1(1 - \rho^2)}}} \right] - \frac{1}{4} \sqrt{1 - \gamma}$$

$$+ \frac{1}{2\pi} (1 + \gamma) \left[ 1 + \frac{\sqrt{1 - \gamma}}{\sqrt{1 + \mu \rho N_0}} \right] \left[ 1 + \nu N_0 + \zeta N_0^2 \right]^{1/2}$$

$$\cdot \Pi \left( \frac{\pi}{\sqrt{1 + \mu \rho N_0}}, \sqrt{\frac{N_0\mu}{1 + \nu N_0 + \zeta N_0^2}} \right)$$

(14)

where

$$\gamma = \frac{4\rho^2 E_1 / E_2}{1 + 4\rho^2 E_1 / E_2}$$

$$\delta = \frac{1}{2E_2 (1 + 4\rho^2 E_1 / E_2)}$$

$$\nu = \frac{4\rho^2 + (1 - \rho^2) + E_2 / E_1}{(1 - \rho^2) 2E_2}$$

$$\zeta = \frac{1}{(1 - \rho^2) 2E_2}$$

$$\mu = \frac{4\rho^2}{(1 - \rho^2) 2E_2}$$

and \( \Pi \left( \frac{\pi}{\sqrt{n}}, n, k \right) \) is the complete elliptic integral of the third kind [13]. From (12), (14) and using Taylor series expansion,
it can be shown that
\[
\eta_2 = \lim_{N_0 \to 0} \frac{N_0/2}{4E_2 P_{e_2}(N_0)} = 1
\]

C. AME performance comparison

Fig. 1 shows the AME results for DBSIC, optimum, conventional SIC, and decorrelating receivers over the AWGN channel when \( \rho = 0.5 \) corresponding to an intermediate value of the users spreading sequences cross-correlation. It is observed that DBSIC has a lower AME than the optimum receiver when the interfering user is weaker than the user of interest and achieves the unity optimum receiver AME as the interferer becomes stronger than the user of interest. The decorrelating receiver has a constant AME due to the fact its performance does not depend on the received energy of all users, i.e., it is completely near-far resistant. When the interferer is much weaker or slightly stronger than the user of interest, DBSIC AME is better than that of the decorrelating detector but is worse when the interferer is not much weaker.

It is also observed that DBSIC shows a similar AME curve shape as the conventional SIC receiver but with higher AME performance for all user energy ratios. Such trends are also seen for other values of the spreading codes cross-correlation as illustrated in Fig. 2 (highly correlated scenario, \( \rho = 0.8 \)) and Fig. 3 (low correlated scenario, \( \rho = 0.2 \)). Combining both the decorrelating and SIC schemes, DBSIC implements a decorrelating process at each stage of the conventional SIC so that smaller decision errors and hence less error propagation from one stage to the next are obtained yielding improved performance. In a highly correlated scenario such as in Fig. 2 where \( \rho \) equals to 0.8, the AME performance of all receivers degrades and the inverse bell shape for optimum, DBSIC and SIC becomes deeper as the minimum shifts lower. In a low correlated scenario where \( \rho \) is equal to 0.2, illustrated in Fig. 3, all detectors except MF have a significant AME performance improvement and approach the unity optimum receiver AME as expected. Figs. 1-3 also show that DBSIC is near-far resistant over AWGN channels as its AME remains positive for all user energy ratios.

The AME of user 1 for the Rayleigh fading channel case is presented in Fig. 4 for various values of \( \rho \). It is seen that both DBSIC and the decorrelating detectors have the same AME, which is lower than the optimum scheme one. When \( \rho \) becomes smaller, their AME performance approaches close to that of the optimum detector which is unity. Fig. 5 illustrates user 2 AME and shows that DBSIC and the optimum detector have the same AME value, which is independent of the value of \( \rho \), and have higher AME than the decorrelating detector. Finally, it is seen that DBSIC is completely near-far resistant for both users over Rayleigh fading channels, since its AME is independent of the user energy ratio.

IV. CONCLUSIONS

This paper investigates AME for the DBSIC scheme over both AWGN and Rayleigh fading channels and compares with commonly used DS-CDMA receivers such as the conventional single-user MF scheme, and the optimum, decorrelating and conventional SIC multiuser detectors. Unlike the conventional SIC, DBSIC implements a decorrelator on top of the conventional matched filtering to detect the user's signal that will be used at the next interference cancellation stage. It is shown that for AWGN channels, DBSIC has a lower AME than the optimum receiver when the interfering user is weaker than the user of interest but approaches the unity optimum receiver AME as the interferer becomes
stronger than the user of interest. For all values of the spreading codes correlation considered, DBSIC shows a similar AME curve shape as the conventional SIC receiver but has a higher AME which remains positive for all user energy ratios, showing that DBSIC is near-far resistant and confirming the superiority of DBSIC over the conventional SIC previously seen in terms of BER [12]. The AME of DBSIC is better than that of the decorrelating detector when the interferer is much weaker or slightly stronger than the user of interest and is worse when the interferer is not much weaker. The range over which the decorrelating detector AME is better than DBSIC depends on the spreading codes correlation. It is shown that DBSIC is completely near-far resistant over Rayleigh fading channels with the same AME as the decorrelating scheme for user 1 and as the optimum scheme for user 2, both AMEs being independent of the user energy ratio. This shows that DBSIC is preferable compared to the decorrelating detector, while still being less complex than the optimum scheme.

REFERENCES