Receivers Structures and Performance for Unresolved Ricean/Rayleigh Multipath Fading Channels

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Abstract
This paper considers receiver structures and performance for unresolved mixed mode Ricean/Rayleigh multipath channels. A fading channel is said to be mixed mode if the first path gain is Ricean distributed and the other path gains are Rayleigh. The optimal receiver for this case consists of a decorrelation stage combined with a quadratic form, as well as a nonlinear term related to the Ricean specular component. Replacing the nonlinear term by a quadratic form, yields a novel family of quadratic sub-optimal receivers, Quadratic Decorrelation Receivers (QDR), that could be more easily implemented. These receivers exploit the decorrelation performed on the signal samples similar to the optimal receiver. Single pulse performance of the QDR and other sub-optimal structures is studied for binary FSK and DPSK. It is shown that SNR gains can be obtained by the use of receivers that exploit the knowledge of the specular component. Furthermore, we demonstrate the importance of the decorrelation operation in eliminating the error floors when the multipath is unresolved.

1. Introduction

The growing interest in Personal Communications that we witness [1], [2], creates a significant interest in detection techniques for fading multipath channels [3]. The well known RAKE receiver [4], [5] is a fundamental structure that exploits the inherent time diversity of a multipath fading channel. However, the RAKE is based on the assumption that the multipath is resolved. Therefore, over the years, RAKE receivers have been used with wide-band signals, such as those found in Spread Spectrum systems, ensuring multipath resolvability. It is interesting to explore the counter-part of the RAKE receiver for narrow-band systems, when the multipath is not entirely resolved.

The optimal receiver for unresolved multipath Rayleigh channels has been considered by Aiken [6]. Performance of widely orthogonal or uniformly orthogonal signals, that resolve the multipath, is considered in [7]. Based on a matched filter performance bound, Mazo [8] showed that diversity-like gains can be achieved at high SNR over completely known two-path Rayleigh fading channels even if the multipath is not resolved. Alles & Pasupathy [9], [10] considered receivers structures for two-path Rayleigh with different levels of channel knowledge. They addressed specific modulation formats, envelope orthogonal Frequency-Shift Keying and variants of chirp or linear frequency sweep modulation, and showed that two fold diversity-like effects exist in this case.

In our work we present the optimal receiver structure without any constraint on the modulation format, over a channel with an arbitrary number of paths that are not resolved by the information conveying signal. This could correspond to a system operating in the indoor radio environment, where the inter-path delays are relatively small. We assume that the multipath delays are known. They could have been estimated by super-resolution techniques [11], or by sounding the channel with a wide-band pulse. Furthermore, we assume a mixed mode channel, where the first path gain is Ricean distributed and the other path gains are Rayleigh. The Ricean component models a line of sight between the transmitter and receiver. Based on the insight provided by the optimal receiver, we introduce sub-optimal structures that are more suitable for implementation. The paper is organized as follows. Section 2 considers receiver structures, optimal as well as sub-optimal. Section 3 presents the performance of binary FSK and DPSK for mixed mode two-path Rayleigh/Ricean fading channels. Section 4 presents the conclusions.

2. Receiver structures

One of M possible bandpass signals of finite energy is transmitted over a fading multipath channel. For convenience the M possible transmitted signals are represented by their complex envelopes \( \tilde{s}_m(t) \). Under the hypothesis \( H_m \) (i.e., \( \tilde{s}_m(t) \) was transmitted), the complex envelope of the received signal \( r(t) \) is given by

\[
\tilde{r}(t) = \sum_{i=0}^{L-1} a_i e^{j\theta_i} \tilde{s}_m(t - \tau_i) + \tilde{n}(t) \quad m=1,2,...,M
\]

where \( a_i \) are independent circularly complex Gaussian random variables with mean \( \alpha \) and variance \( E[|a_i - \alpha|^2] = \sigma_i^2 \) [12]. The receiver lack of reference phase information is represented by \( \theta_i \), the multipath component phase shifts, that are independent uniformly distributed random variables between \(-\pi\) and \(\pi\). The multipath delays \( \tau_i \) are assumed to be known and \( \tau_i \neq \tau_j \) if \( i \neq j \). The channel is also corrupted by an additive zero mean circularly complex Gaussian process \( \tilde{n}(t) \) satisfying \( E[\tilde{n}(t)\tilde{n}^*(u)/2] = \sigma_n^2 \delta(t - u) \). For all \( m \), conditioned on \( \theta_0, \theta_1, ... , \theta_{L-1} \), the signal process \( \tilde{v}_m(t) = \sum_{i=0}^{L-1} a_i e^{j\theta_i} \tilde{s}_m(t - \tau_i) \) is Gaussian, has a finite mean-square value on the observation interval \([0,T]\) and is statistically independent of \( \tilde{n}(t) \). The multipath is resolved when the delayed signals are orthogonal, i.e.

\[
\int_0^T \tilde{s}_m(t - \tau_i) \tilde{s}_m(t - \tau_j) dt = 0, \quad i \neq j.
\]

This assumption is central in the derivation of the well known RAKE receiver [4], [5]. In our work we don’t assume this!

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stead, we assume that the observation interval $T_o$ is much longer than the multipath delays and the delayed signals $\delta_m(t - \tau_0), \delta_m(t - \tau_1), \ldots, \delta_m(t - \tau_{l-1})$ are linearly independent and contained in the observation interval. It can be shown that any square integrable waveform with standard Fourier transform will satisfy the linear independence condition. Thus linear independence is a very mild condition compared to the orthogonality constraint required by multipath resolvability.

We use the following notation: Bold capital letters denote matrices whereas bold lowercase letters denote vectors. The symbols $^T$ and $^*$ respectively the transposition, complex conjugation and Hermitian conjugation of a matrix or vector. The diagonal matrix composed of the main diagonal entries of $M$ is denoted by $[M]_d$. The $ij^{th}$ entry of a matrix $M$ is denoted as $[M]_{ij}$ and the $i^{th}$ entry of a vector $v$ is denoted as $[v]_i$. Since $T_o \gg \tau_i$ for all $i$, the energies of the signals $\delta_m(t - \tau_i)$ over the observation interval will be identical and denoted by $E_m$. We define the correlation matrix of the signal under $H_m, \Gamma_m$, as $[\Gamma_m]_{ij} = \frac{1}{E_m} \int_0^{T_o} \delta_m^*(t - \tau_i) \delta_m^*(t - \tau_j) dt$. The covariance matrix of the channel $C$, defined as $[C]_{ij} = E[(a_i - \mu_a)(a_j - \mu_a)^*]$, is a diagonal positive definite matrix with $i^{th}$ entry $2\alpha_i^2$.

The minimum probability of error(log-likelihood ratio) receiver for Ricean channels has been introduced in [13]. It can be shown that the log-likelihood ratio is the sum of a biased quadratic form of the input samples and the logarithm of a series of products of Bessel functions. In this paper a biased quadratic form refers to the sum of a quadratic form and a bias term. Here we consider a channel of practical interest, the mixed mode Ricean/Rayleigh channel. A multipath fading channel is said to be mixed mode Ricean/Rayleigh if the first path gain is Ricean distributed and the other path gains are Rayleigh distributed, i.e. $\alpha \triangleq [\alpha_0, \alpha_1, \ldots, \alpha_{l-1}]^T = [\alpha_0, 0, \ldots, 0]^T$. This channel may represent transmission with a line of sight. For such a channel, the log-likelihood ratio is given by

$$\ln[\Lambda_m(\hat{r}(t))] = \ln(J_m) + \ln [I_0(|d_{0m}|)]$$

(1)

where

$$J_m = \exp\left\{\gamma_m r_m^T Q_m r_m - \alpha^T \{X_m^* C^{-1} Q_m X_m^* \}_d C^{-1} \alpha \right\}$$

(2)

$$|d_{0m}| = 2\sqrt{r_m^T Q_m X_m^* C^{-1} \alpha C^{-1} X_m Q_m r_m}$$

(3)

$$\gamma_m = \frac{E_m}{N_0}$$

denotes the signal-to-noise ratio(SNR), $D_m$ is a diagonal matrix with entries equal to the eigenvalues of the matrix $C\Gamma_m^*, Q = \gamma_m [D_m^* + \gamma_m I]^{-1}$ and $X_m$ is the matrix that satisfies the equations $C\Gamma_m^* X_m^T = X_m^T D_m$ and

$$X_m \Gamma_m X_m^T = I$$

(4)

$r_m$ is the vector $[r_m, r_{m1}, \ldots, r_{m,l-1}]^T$, where

$$r_m = \frac{1}{\sqrt{2 \pi} E_m} \int_0^{T_o} \hat{r}(t) \phi(t) \phi(t)^* dt$$

and $\phi(t) \phi(t)^*$ are the eigenfunctions associated with the covariance of the signal process $\hat{u}_m(t)$, given by

$$\phi_m(t) = \frac{1}{\sqrt{E_m}} \sum_{s=0}^{l-1} [X_m]^s \delta_m(t - \tau_s).$$

Therefore

$$r_m = X_m^* u_m$$

(5)

where

$$|u_m|^2 = \frac{1}{E_m} \int_0^{T_o} \hat{r}(t)^2 dt.$$ Using the relationship between $D_m$ and $X_m$, we have

$$Q_m = \gamma_m \Gamma_m^* \{C^{-1} + \gamma_m \Gamma_m^* \}^{-1} \{C^{-1} + \gamma_m \Gamma_m^* \}^{-1}$$

(6)

It has been shown in [13] that the optimum receiver for unresolved Rayleigh multipath channels consists of an orthogonolization(or decorrelation stage), $r_m = X_m u_m$, and a resolved multipath optimum decision rule(RAKE structure) for the transformed signals. The log-likelihood ratio for unresolved Rayleigh multipath channels is given by

$$\gamma_m r_m^T Q_m r_m - \ln \det(I + \gamma_m D_m).$$

This quantity appears in the term $\ln(J_m)$ of (1). Thus from (1) we see that the optimum receiver for mixed mode Ricean/Rayleigh channels performs a decorrelation on the input samples as well as non linear operations related to the Ricean specular term.

Note that for low $\gamma_m$, from (5), (4) and (6) regardless of $\alpha$, we have

$$\gamma_m r_m^T Q_m r_m = \gamma_m u_m^T \{C^{-1} + \gamma_m \Gamma_m^* \}^{-1} u_m \approx \gamma_m u_m^T C u_m$$

(7)

Therefore (2) can be approximated by

$$\ln(J_m) \approx \gamma_m u_m^T C u_m - \gamma_m \alpha^T \alpha$$

Similarly from (5) and (6) we have

$$X_m^* Q_m r_m = \gamma_m \{C^{-1} + \gamma_m \Gamma_m^* \}^{-1} u_m \approx \gamma_m C u_m$$

Thus (3) can be approximated by

$$|d_{0m}| \approx 2 \gamma_m |u_m|^2 \alpha^T \alpha$$

(8)

Therefore we see that for the mixed mode Ricean/Rayleigh channel optimal receiver, at low $\gamma_m$, the decorrelation operation on the input signal vanishes. This is to be expected since a side effect of the decorrelation operation is to enhance the white background channel noise.

From (3), $|d_{0m}| = |\alpha_0| |d_{0m}|$, where $|d_{0m}|$ is independent of $|\alpha_0|$. When $|\alpha_0|$ is small, $|d_{0m}|$ is small and using the first terms of the Taylor’s series expansion of the modified Bessel function of the first kind, we have

$$\ln[\Lambda_m(\hat{r}(t))] \approx \ln(J_m) + \left[1 + \frac{|d_{0m}|^2}{4}\right]$$

(9)

$$\approx \ln(J_m) + \frac{|d_{0m}|^2}{4} \approx \ln(J_m)$$

When $|\alpha_0|$ is large, $|d_{0m}|$ is large and using the asymptotic expansion of the modified Bessel function of the first kind, we have $\ln(I_0(|d_{0m}|)) \approx |d_{0m}|$. If $|\alpha_0|$ is small then $|d_{0m}| \gg \ln(J_m)$ and the log-likelihood ratio can be approximated by $|d_{0m}|$. An equivalent decision rule when $|\alpha_0|$ is small can be based on

$$\Lambda_m(\hat{r}(t)) = |d_{0m}|^2$$

(10)
If \( \gamma_m \) is large then \( \ln(J_m) \gg |d_0m| \) and the log-likelihood ratio can be approximated by
\[
\ln|A_m(f(t))| = \ln(J_m) \tag{9}
\]

Summarizing, we can see that for extreme values of \( |\alpha_0| \), the decision variables (7)-(9) can be viewed as linear combinations of \( \ln(J_m) \) and \( |d_0m|^2 \). Therefore we propose a sub-optimal receiver based on the functional
\[
\chi_m(f(t)) = \ln(J_m) + d|d_0m|^2 \tag{10}
\]
where \( d \) is a constant to be determined. For large values of \( |\alpha_0| \), the constant \( d \) should vanish as \( \gamma_m \) increases since in that case the log-likelihood ratio can be approximated by \( \ln(J_m) \). The simplest function achieving this goal is \( d = \gamma_m^{-\beta} \) where \( \beta \) is a parameter. With this choice, the decision variable \( \chi_m(f(t)) \) tends to the true log-likelihood ratio for extreme values of \( |\alpha_0| \). Thus we obtain a family of receivers of the form given by (10) called Quadratic Decorrelation Receivers(QDR) whose decision variables are
\[
\chi_m(f(t)) = \ln(J_m) + \frac{4}{\gamma_m} r_m Q_m X_m^{\ast} C^{-1} \alpha \alpha^\dagger C^{-1} X_m^{T} Q_m r_m \tag{11}
\]
and illustrated in Fig. 1.

\[\text{Figure 1: QDR for a mixed mode Ricean/Rayleigh channel(Block diagram for the } m^{th} \text{ hypothesis)}\]

These receivers exploit the decorrelation performed on the input samples similar to the optimal receiver. Furthermore, similarly to the optimal receiver, the decorrelation operation vanishes at low \( \gamma_m \) for the QDR as explained in the following. The QDR decision variable depends only on \( \ln(J_m) \) and \( |d_0m|^2 \). At low \( \gamma_m \), these two terms do not depend on \( r_m \) and \( X_m \) implying that the QDR does not perform any decorrelation. For extreme values of \( |\alpha_0| \), the QDR tends to the optimum receiver at low and high SNR. Finally, for a fixed value of \( |\alpha_0| \), the term \( \ln(J_m) \) dominates at high SNR in (11), thus the QDR and the optimal receiver decision rules are equivalent. As far as the Ricean component is concerned, both receivers require knowledge of \( |\alpha_0| \) only. What differentiates the QDR from the optimal receiver is that the nonlinear term due to the specular component is replaced by a quadratic form.

It is to be noted that the QDR reduces to Aiken’s receiver(denoted here R OPT) [6] when the path magnitudes are Rayleigh distributed. Comparing bit error probability of R OPT and QDR will indicate performance improvement due to the knowledge of the magnitude of the Ricean specular component.

In order to assess the performance improvement due to the decorrelation operation, we consider also receivers very similar to QDR except that they do not employ decorrelation. Therefore we consider also simple Quadratic Receivers(QR) that are a limiting form of the QDR (11) when the multipath is resolved(i.e. when \( X_m = \Gamma_m = I \)). The decision variable for the QR is then
\[
\gamma_m u_m^\dagger Q_m^\ast u_m - \alpha \alpha^\dagger Q_m^\ast C^{-1} \alpha 
- \ln[\det(I + \gamma_m C)] + \frac{4}{\gamma_m} u_m^\dagger Q_m^\ast X_m^{T} Q_m r_m 
\]
where \( Q_m' = \gamma_m(C^{-1} + \gamma_m I)^{-1} \). The QR can be considered as sub-optimum receivers with respect to Turin’s resolved multipath optimum receiver [4].

3. Performance
Following [8] we consider single-pulse performance of the QDR, R OPT and QR schemes for binary Frequency Shift Keying with frequency deviation equal to 1/2T (FSK(1/2)) with
\[
\begin{align*}
\text{Figure 1: QDR for a mixed mode Ricean/Rayleigh channel(Block diagram for the } m^{th} \text{ hypothesis)}
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These receivers exploit the decorrelation performed on the input samples similar to the optimal receiver. Furthermore, similarly to the optimal receiver, the decorrelation operation vanishes at low \( \gamma_m \) for the QDR as explained in the following. The QDR decision variable depends only on \( \ln(J_m) \) and \( |d_0m|^2 \). At low \( \gamma_m \), these two terms do not depend on \( r_m \) and \( X_m \) implying that the QDR does not perform any decorrelation. For extreme values of \( |\alpha_0| \), the QDR tends to the optimum receiver at low and high SNR. Finally, for a fixed value of \( |\alpha_0| \), the term \( \ln(J_m) \) dominates at high SNR in (11), thus the QDR and the optimal receiver decision rules are equivalent. As far as the Ricean component is concerned, both receivers require knowledge of \( |\alpha_0| \) only. What differentiates the QDR from the optimal receiver is that the nonlinear term due to the specular component is replaced by a quadratic form.

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\[
\gamma_m u_m^\dagger Q_m^\ast u_m - \alpha \alpha^\dagger Q_m^\ast C^{-1} \alpha 
- \ln[\det(I + \gamma_m C)] + \frac{4}{\gamma_m} u_m^\dagger Q_m^\ast X_m^{T} Q_m r_m 
\]
where \( Q_m' = \gamma_m(C^{-1} + \gamma_m I)^{-1} \). The QR can be considered as sub-optimum receivers with respect to Turin’s resolved multipath optimum receiver [4].

3. Performance
Following [8] we consider single-pulse performance of the QDR, R OPT and QR schemes for binary Frequency Shift Keying with frequency deviation equal to 1/2T (FSK(1/2))
interval needs to be equal to $2T$. Therefore for DPSK, $E_b = E_r/4$, $m=1,2$ which takes into account the energy gain due to the longer observation interval. The relative delay between the first path and the second path, $\tau = \tau_1 - \tau_0$ is expressed as a percentage of $T$. The two-path mixed mode Ricean/Rayleigh fading channel is then characterized by the parameters $s$, $K$ and $\tau$. For convenience $K$ is expressed in dB.

Fig. 2 and Fig. 3 present the probability of error for FSK(1/2) and DPSK with QDR and QR for several values of the parameter $\beta$. The probability of error with R OPT is also included as a reference. Fig. 2 and Fig. 3 illustrate the importance of the choice of the parameter $\beta$ on the performance of the QDR and QR scheme for FSK(1/2) and DPSK. From these figures it is seen that there is no value of $\beta$ which gives the best performance over the entire range of SNR. However Fig. 2 shows that $\beta$ should not be chosen too small (i.e. less than 1) since in that case the R OPT scheme which does not use the knowledge of the specular component outperforms the QDR. In fact a proper choice for $\beta$ seems around unity. Assuming a fixed value of $\beta$, the best we could find for the QDR scheme is $\beta = 1.3$. This value gives a low probability of error at high as well as low SNR for both FSK(1/2) and DPSK. Furthermore it gives SNR gains with respect to R OPT. Studies of other modulations such as orthogonal FSK indicates that the best value of $\beta$ is quite independent of the modulation index. Fig. 2 and Fig. 3 show that the performance of the QR scheme improves as $\beta$ decreases thus the best we could find for the QR scheme is $\beta = 0$.

**Figure 2: The effect of the parameter $\beta$ for FSK(1/2)**

Fig. 4 and Fig. 5 present the probability of error for FSK(1/2) and DPSK with QDR($\beta = 1.3$), R OPT and QR($\beta = 0$). From Fig. 4 it is seen that the QDR scheme with FSK(1/2) gives up to 1dB gain with respect to the R OPT scheme at an error probability of $3 \times 10^{-5}$ for $K = 13$dB and at an error probability of $10^{-7}$ for $K = 15$dB. Fig. 5 shows that the QDR scheme with DPSK gives up to 1 dB gain with respect to R OPT for $K = 13$dB in the error probability range of $10^{-5} - 10^{-8}$ and up to 1.5dB for $K = 15$dB at an error probability of $10^{-8}$. This shows that SNR gains can be obtained by the use of receivers which exploit the knowledge of the specular component magnitude. Note that these SNR gains are obtained only with a proper choice of the parameter $\beta$.

**Figure 3: The effect of the parameter $\beta$ for DPSK**

The curves for $K = 13$dB, in Fig. 4 and Fig. 5, show a slight twist. This may be explained first by the fact that for this intermediate value of $K$, the “sub-optimality” of the QDR is the highest compared to the optimum receiver. Secondly, for a fixed value of $|\gamma|$, at low SNR, all three receivers perform the same and their performance are expected to be close to that of the optimum receiver. As the SNR increases, the QDR becomes more sub-optimum, thus the probability of error decreases less rapidly. However at high SNR, the QDR and the optimum receiver have similar decision rules. This explains why at higher SNR the probability of error decreases again more rapidly.

**Figure 4: Performance of FSK(1/2) over a two-path Ricean/Rayleigh fading channel ($K$ is expressed in dB)**

Fig. 4 and Fig. 5 show that with FSK(1/2) and DPSK the QR scheme yields error floors which are eliminated
by the QDR due to its decorrelation operation. Hence the QDR scheme outperforms significantly the QR at high SNR. At low SNR, the QR scheme performs the same or better than the QDR. However the performance degradation of the QDR is in general small compared with the QR gains which it provides with respect to R OPT or QR at high SNR. The QDR has 0.36dB loss for $K = 13$dB at an error probability of $5 \cdot 10^{-3}$ for FSK(1/2), and 0.3dB loss for $K = 20$dB for DPSK at an error probability of $10^{-8}$. Note that when $K = 20$dB, the QR scheme with FSK(1/2) outperforms the QDR scheme over the entire range of error probabilities considered in this paper by at most 0.8dB. This may be explained by the fact that the QR does not employ decorrelation similarly to the optimal receiver at low SNR (the decorrelation vanishes at low SNR for the optimal receiver). However overall the QDR offers an attractive choice for mixed mode Ricean/Rayleigh channels since it eliminates the error floors.

![Figure 5: Performance of DPSK over a two-path Ricean/Rayleigh fading channel (K is expressed in dB)](image)

4. Conclusions

This paper considers detection techniques for mixed mode Ricean/Rayleigh multipath channels. The multipath delays are assumed to be known but unresolved. For these channels, the optimum decision rule involves a quadratic form including a decorrelation on the signal samples, and nonlinear operations related to the specular component. Replacing the nonlinear term by another quadratic form more suitable for implementation, yields a family of sub-optimal receivers, the Quadratic Decorrelation Receivers(QDR), which depend on a parameter $\beta$. These receivers tend to optimal detection structures for extreme values of the specular term magnitude. Similar to the optimal receiver, the proposed structures include the decorrelation on the input samples, that vanishes at low SNR. We also consider simple Quadratic Receivers(QR) that are similar to QDR except that they do not employ decorrelation. Single-pulse performance of the QDR and QR schemes for FSK(1/2) and DPSK over two-path mixed mode Ricean/Rayleigh channels is studied for several values of $\beta$. With a proper choice of $\beta$ (for example $\beta = 1.3$ for QDR and $\beta = 0$ for QR), it is shown that SNR gains can be obtained by the use of receivers which exploit the knowledge of the specular component magnitude. Furthermore we illustrate the importance of the decorrelation operation in eliminating the error floors. The absence of error floors in its performance makes the QDR receivers attractive for unresolved multipath channels.

References