Appendix for

Achieving MILP Feasibility Quickly Using General Disjunctions

Hanan Mahmoud (hamahmou@connect.carleton.ca)
John W. Chinneck (chinneck@sce.carleton.ca)

Systems and Computer Engineering
Carleton University
Ottawa, Ontario K1S 5B6
Canada

November 30, 2012

Appendix A: Test Sets

A.1 MIPLIB 2010 Subset of Equality-Containing MIP Models

The table below lists the 100 randomly-chosen easy models used in training experiments requiring equality-containing models. Statistics on the numbers and types of variables and constraints in each model are listed. Average values are given at the end of the table.

MIP Model	Number of Constraints	Number of Variables	Number of Binary Vars.	Number of Integer Vars.	Number of Real Vars.
janos-us-DDM.mps	760	2184	0	84	2100
b2c1s1.mps	3904	3872	288	0	3584
g200x740i.mps	940	1480	740	0	740
neos-1396125.mps	1494	1161	129	0	1032
neos-506428.mps	129925	42981	42981	0	0
neos-693347.mps	3192	1576	1405	0	171
neos-799711.mps	59218	41998	910	0	41088
beasleyC3.mps	1750	2500	1250	0	1250
dg012142.mps	6310	2080	640	0	1440
neos-826841.mps	2354	5516	3488	0	2028
neos-1112782.mps	2115	4140	2070	0	2070
neos-777800.mps	479	6400	6400	0	0
dano3mip.mps	3202	13873	552	0	13321
k16x240.mps	256	480	240	0	240
lectsched-4-obj.mps	14163	7901	7665	236	0
neos-	3467	4173	3633	0	540

1605075.mps					
neos-935769.mps	6741	9799	7020	0	2779
acc-tight5.mps	3052	1339	1339	0	0
app1-2.mps	53467	26871	13300	0	13571
gmu-35-40.mps	424	1205	1200	0	5
neos-941262.mps	6703	9480	6710	0	2770
neos-916792.mps	1909	1474	717	0	757
berlin_5_8_0.mps	1532	1083	794	0	289
bienst2.mps	576	505	35	0	470
map18.mps	328818	164547	146	0	164401
ger50_17_trans.mps	499	22414	0	18062	4352
d20200.mps	1502	4000	3181	819	0
mcsched.mps	2107	1747	1731	14	2
neos-506422.mps	6811	2527	63	0	2464
danoint.mps	664	521	56	0	465
neos-826812.mps	6844	15864	10350	0	5514
50v-10.mps	233	2013	1464	183	366
neos-847302.mps	609	737	729	0	8
neos-824661.mps	18804	45390	15640	0	29750
neos-911880.mps	83	888	840	0	48
co-100.mps	2187	48417	48417	0	0
gmut-75-50.mps	2565	68865	68859	0	6
dolom1.mps	1803	11612	9720	0	1892
map20.mps	328818	164547	146	0	164401
dfn-gwin-UUM.mps	158	938	0	90	848
eilA101.2.mps	100	65832	65832	0	0
neos-1109824.mps	28979	1520	1520	0	0
buildingenergy.mps	277594	154978	0	26287	128691
n3700.mps	5150	10000	5000	0	5000
go19.mps	441	441	441	0	0
neos-555424.mps	2676	3815	3800	15	0
neos-1337307.mps	5687	2840	2840	0	0
nag.mps	5840	2884	1350	35	1499
acc-tight4.mps	3285	1620	1620	0	0
csched010.mps	351	1758	1457	0	301
30n20b8.mps	576	18380	11036	7344	0
neos-476283.mps	10015	11915	5588	0	6327
bg512142.mps	1307	792	240	0	552
csched008.mps	351	1536	1284	0	252

lotsize.mps	1920	2985	1195	0	1790
acc-tight6.mps	3047	1335	1335	0	0
neos-1620770.mps	9296	792	792	0	0
map14.mps	328818	164547	146	0	164401
neos-824695.mps	9576	23970	8500	0	15470
neos-1225589.mps	675	1300	650	0	650
n4-3.mps	1236	3596	0	174	3422
neos-941313.mps	13189	167910	167910	0	0
germany50-DBM.mps	2526	8189	0	88	8101
neos-942830.mps	803	882	834	0	48
neos-520729.mps	31178	91149	30708	0	60441
map06.mps	328818	164547	146	0	164401
n9-3.mps	2364	7644	0	252	7392
glass4.mps	396	322	302	0	20
msc98-ip.mps	15850	21143	20237	53	853
markshare_5_0.mps	5	45	40	0	5
binkar10_1.mps	1026	2298	170	0	2128
mc11.mps	1920	3040	1520	0	1520
gmu-35-50.mps	435	1919	1914	0	5
n3-3.mps	2425	9028	0	366	8662
mkc.mps	3411	5325	5323	0	2
blp-ar98.mps	1128	16021	15806	0	215
blp-ic97.mps	923	9845	9753	0	92
neos-826694.mps	6904	16410	16290	0	120
neos-1112787.mps	1680	3280	1640	0	1640
neos-1605061.mps	3474	4111	3570	0	541
leo1.mps	593	6731	6730	0	1
n370a.mps	5150	10000	5000	0	5000
dc1c.mps	1649	10039	8380	0	1659
eilB101.mps	100	2818	2818	0	0
d10200.mps	947	2000	733	1267	0
biella1.mps	1203	7328	6110	0	1218
dc1l.mps	1653	37297	35638	0	1659
atlanta-ip.mps	21732	48738	46667	106	1965
mzzv11.mps	9499	10240	9989	251	0
map10.mps	328818	164547	146	0	164401
leo2.mps	593	11100	11099	0	1
maxgasflow.mps	7160	7437	2456	0	4981

gmut-77-40.mps	2554	24338	24332	0	6
eil33.2.mps	32	4516	4516	0	0
n3705.mps	5150	10000	5000	0	5000
csched007.mps	351	1758	1457	0	301
harp2.mps	112	2993	2993	0	0
neos-1440225.mps	330	1285	1285	0	0
air04.mps	823	8904	8904	0	0
neos-1601936.mps	3131	4446	3906	0	540
AVERAGE	**25173.93**	**21355.57**	**8437.96**	**557.26**	**12360.35**

A.2 MIPLIB 2010 Subset of Inequality-Containing MIP Models

The table below lists the 100 randomly-chosen easy models used in training experiments requiring inequality-containing models. Statistics on the numbers and types of variables and constraints in each model are listed. Average values are given at the end of the table. The subset of 72 models in which only parallel general disjunctions are triggered by all branching-direction selection algorithms are *italicized*.

MIP Model	Number of Constraints	Number of Variables	Number of Binary Vars.	Number of Integer Vars.	Number of Real Vars.
biella1.mps	1203	7328	6110	0	1218
neos-1616732.mps	1999	200	200	0	0
cov1075.mps	637	120	120	0	0
dc1l.mps	1653	37297	35638	0	1659
berlin_5_8_0.mps	1532	1083	794	0	289
mzzv11.mps	9499	10240	9989	251	0
germany50-DBM.mps	2526	8189	0	88	8101
gmut-77-40.mps	2554	24338	24332	0	6
gmu-35-50.mps	435	1919	1914	0	5
neos-1171737.mps	4179	2340	1170	0	1170
ic97_potential.mps	1046	728	450	73	205
binkar10_1.mps	1026	2298	170	0	2128
maxgasflow.mps	7160	7437	2456	0	4981
macrophage.mps	3164	2260	2260	0	0
danoint.mps	664	521	56	0	465
d20200.mps	1502	4000	3181	819	0
acc-tight6.mps	3047	1335	1335	0	0
liu.mps	2178	1156	1089	0	67
map14.mps	328818	164547	146	0	164401
neos-1442119.mps	1524	728	364	0	364
n3700.mps	5150	10000	5000	0	5000
neos-1605061.mps	3474	4111	3570	0	541
harp2.mps	112	2993	2993	0	0
dc1c.mps	1649	10039	8380	0	1659
blp-ar98.mps	1128	16021	15806	0	215
bg512142.mps	1307	792	240	0	552
map06.mps	328818	164547	146	0	164401
blp-ic97.mps	923	9845	9753	0	92

bienst2.mps	576	505	35	0	470
30n20b8.mps	576	18380	11036	7344	0
glass4.mps	396	322	302	0	20
mik.250-1-100.1.mps	151	251	100	150	1
csched008.mps	351	1536	1284	0	252
leo2.mps	593	11100	11099	0	1
buildingenergy.mps	277594	154978	0	26287	128691
n3705.mps	5150	10000	5000	0	5000
mc11.mps	1920	3040	1520	0	1520
lectsched-4-obj.mps	14163	7901	7665	236	0
neos-1337307.mps	5687	2840	2840	0	0
mine-166-5.mps	8429	830	830	0	0
n370a.mps	5150	10000	5000	0	5000
neos-1442657.mps	1310	624	312	0	312
dano3mip.mps	3202	13873	552	0	13321
n3seq24.mps	6044	119856	119856	0	0
neos-1396125.mps	1494	1161	129	0	1032
neos-1109824.mps	28979	1520	1520	0	0
iis-pima-cov.mps	7201	768	768	0	0
neos-1112782.mps	2115	4140	2070	0	2070
iis-bupa-cov.mps	4803	345	345	0	0
dfn-gwin-UUM.mps	158	938	0	90	848
lotsize.mps	1920	2985	1195	0	1790
n3-3.mps	2425	9028	0	366	8662
b2c1s1.mps	3904	3872	288	0	3584
co-100.mps	2187	48417	48417	0	0
n9-3.mps	2364	7644	0	252	7392
ex1010-pi.mps	1468	25200	25200	0	0
neos-1426635.mps	796	520	260	0	260
acc-tight5.mps	3052	1339	1339	0	0
map20.mps	328818	164547	146	0	164401
acc-tight4.mps	3285	1620	1620	0	0
beasleyC3.mps	1750	2500	1250	0	1250
app1-2.mps	53467	26871	13300	0	13571
mcsched.mps	2107	1747	1731	14	2

mine-90-10.mps	6270	900	900	0	0
n3div36.mps	4484	22120	22120	0	0
neos-1436709.mps	1417	676	338	0	338
neos-1440460.mps	989	468	234	0	234
map10.mps	328818	164547	146	0	164401
gmut-75-50.mps	2565	68865	68859	0	6
mkc.mps	3411	5325	5323	0	2
dg012142.mps	6310	2080	640	0	1440
atlanta-ip.mps	21732	48738	46667	106	1965
neos-1605075.mps	3467	4173	3633	0	540
neos-1601936.mps	3131	4446	3906	0	540
dolom1.mps	1803	11612	9720	0	1892
iis-100-0-cov.mps	3831	100	100	0	0
janos-us-DDM.mps	760	2184	0	84	2100
leo1.mps	593	6731	6730	0	1
g200x740i.mps	940	1480	740	0	740
k16x240.mps	256	480	240	0	240
neos-1112787.mps	1680	3280	1640	0	1640
m100n500k4r1.mps	100	500	500	0	0
core2536-691.mps	2539	15293	15284	0	9
gmu-35-40.mps	424	1205	1200	0	5
nag.mps	5840	2884	1350	35	1499
csched010.mps	351	1758	1457	0	301
csched007.mps	351	1758	1457	0	301
go19.mps	441	441	441	0	0
msc98-ip.mps	15850	21143	20237	53	853
30_70_4.5_0.95_100.mps	12526	10976	10975	0	1
methanosarcina.mps	14604	7930	7930	0	0
neos-1426662.mps	1914	832	416	0	416
neos-1171692.mps	4239	1638	819	0	819
ger50_17_trans.mps	499	22414	0	18062	4352
n4-3.mps	1236	3596	0	174	3422
50v-10.mps	233	2013	1464	183	366
d10200.mps	947	2000	733	1267	0

neos-1311124.mps	1643	1092	546	0	546
map18.mps	328818	164547	146	0	164401
neos-1225589.mps	675	1300	650	0	650
AVERAGE	**22881.79**	**17691.35**	**6422.12**	**559.34**	**10709.89**

A.3 MIPLIB 2010 Subset of Equality and Inequality-Containing MIP Models

The table below lists the 100 randomly-chosen easy models used in training experiments requiring equality and inequality-containing models. Statistics on the numbers and types of variables and constraints in each model are listed. Average values are given at the end of the table.

MIP Model	Number of Constraints	Number of Variables	Number of Binary Vars.	Number of Integer Vars.	Number of Real Vars.
neos-1440225.mps	330	1285	1285	0	0
buildingenergy.mps	277594	154978	0	26287	128691
neos-1112782.mps	2115	4140	2070	0	2070
csched008.mps	351	1536	1284	0	252
n370a.mps	5150	10000	5000	0	5000
n3seq24.mps	6044	119856	119856	0	0
gmu-35-40.mps	424	1205	1200	0	5
map06.mps	328818	164547	146	0	164401
cov1075.mps	637	120	120	0	0
csched010.mps	351	1758	1457	0	301
neos-1311124.mps	1643	1092	546	0	546
neos-1109824.mps	28979	1520	1520	0	0
core2536-691.mps	2539	15293	15284	0	9
germany50-DBM.mps	2526	8189	0	88	8101
maxgasflow.mps	7160	7437	2456	0	4981
dfn-gwin-UUM.mps	158	938	0	90	848
app1-2.mps	53467	26871	13300	0	13571
acc-tight4.mps	3285	1620	1620	0	0
glass4.mps	396	322	302	0	20
bienst2.mps	576	505	35	0	470
map20.mps	328818	164547	146	0	164401
dg012142.mps	6310	2080	640	0	1440
biella1.mps	1203	7328	6110	0	1218
dc1c.mps	1649	10039	8380	0	1659
30n20b8.mps	576	18380	11036	7344	0
neos-1396125.mps	1494	1161	129	0	1032
acc-tight5.mps	3052	1339	1339	0	0
map10.mps	328818	164547	146	0	164401
gmut-75-50.mps	2565	68865	68859	0	6

d20200.mps	1502	4000	3181	819	0
leo1.mps	593	6731	6730	0	1
mik.250-1-100.1.mps	151	251	100	150	1
neos-1112787.mps	1680	3280	1640	0	1640
gmut-77-40.mps	2554	24338	24332	0	6
n9-3.mps	2364	7644	0	252	7392
n3705.mps	5150	10000	5000	0	5000
n3div36.mps	4484	22120	22120	0	0
neos-1171737.mps	4179	2340	1170	0	1170
mkc.mps	3411	5325	5323	0	2
msc98-ip.mps	15850	21143	20237	53	853
neos-1426662.mps	1914	832	416	0	416
acc-tight6.mps	3047	1335	1335	0	0
30_70_4.5_0.95_100.mps	12526	10976	10975	0	1
iis-pima-cov.mps	7201	768	768	0	0
atlanta-ip.mps	21732	48738	46667	106	1965
danoint.mps	664	521	56	0	465
b2c1s1.mps	3904	3872	288	0	3584
eilA101.2.mps	100	65832	65832	0	0
leo2.mps	593	11100	11099	0	1
dc1l.mps	1653	37297	35638	0	1659
50v-10.mps	233	2013	1464	183	366
mcsched.mps	2107	1747	1731	14	2
macrophage.mps	3164	2260	2260	0	0
gmu-35-50.mps	435	1919	1914	0	5
ger50_17_trans.mps	499	22414	0	18062	4352
dano3mip.mps	3202	13873	552	0	13321
eilB101.mps	100	2818	2818	0	0
blp-ic97.mps	923	9845	9753	0	92
m100n500k4r1.mps	100	500	500	0	0
harp2.mps	112	2993	2993	0	0
neos-1171692.mps	4239	1638	819	0	819
n3-3.mps	2425	9028	0	366	8662
methanosarcina.mps	14604	7930	7930	0	0
d10200.mps	947	2000	733	1267	0
iis-bupa-cov.mps	4803	345	345	0	0
mc11.mps	1920	3040	1520	0	1520

go19.mps	441	441	441	0	0
ic97_potential.mps	1046	728	450	73	205
dolom1.mps	1803	11612	9720	0	1892
neos-1426635.mps	796	520	260	0	260
bg512142.mps	1307	792	240	0	552
n3700.mps	5150	10000	5000	0	5000
mzzv11.mps	9499	10240	9989	251	0
neos-1225589.mps	675	1300	650	0	650
mine-166-5.mps	8429	830	830	0	0
mine-90-10.mps	6270	900	900	0	0
air04.mps	823	8904	8904	0	0
csched007.mps	351	1758	1457	0	301
neos-1440460.mps	989	468	234	0	234
ex1010-pi.mps	1468	25200	25200	0	0
neos-1337307.mps	5687	2840	2840	0	0
janos-us-DDM.mps	760	2184	0	84	2100
map18.mps	328818	164547	146	0	164401
markshare_5_0.mps	5	45	40	0	5
binkar10_1.mps	1026	2298	170	0	2128
co-100.mps	2187	48417	48417	0	0
neos-1436709.mps	1417	676	338	0	338
beasleyC3.mps	1750	2500	1250	0	1250
iis-100-0-cov.mps	3831	100	100	0	0
lotsize.mps	1920	2985	1195	0	1790
nag.mps	5840	2884	1350	35	1499
blp-ar98.mps	1128	16021	15806	0	215
lectsched-4-obj.mps	14163	7901	7665	236	0
liu.mps	2178	1156	1089	0	67
g200x740i.mps	940	1480	740	0	740
map14.mps	328818	164547	146	0	164401
n4-3.mps	1236	3596	0	174	3422
berlin_5_8_0.mps	1532	1083	794	0	289
eil33.2.mps	32	4516	4516	0	0
k16x240.mps	256	480	240	0	240
AVERAGE	**22746.64**	**18382.53**	**7136.22**	**559.34**	**10686.97**

A.4 MIPLIB 2010 Subset of Easy MIP Models Used for Testing

The table below lists the 112 easy models (i.e. solved by *Baseline_Cplex* in less than an hour) used in testing experiments requiring equality and inequality-containing models. Statistics on the numbers and types of variables and constraints in each model are listed. Average values are given at the end of the table. This is the *easy1* set.

MIP Model	Number of Constraints	Number of Variables	Number of Binary Vars.	Number of Integer Vars.	Number of Real Vars.
ash608gpia-3col.mps	24748	3651	3651	0	0
enlight9.mps	81	162	81	81	0
enlight13.mps	169	338	169	169	0
neos13.mps	20852	1827	1815	0	12
neos15.mps	552	792	160	0	632
neos16.mps	1018	377	336	41	0
neos18.mps	11402	3312	3312	0	0
neos6.mps	1036	8786	8340	0	446
neos788725.mps	433	352	352	0	0
neos-885086.mps	11574	4860	2430	0	2430
neos-933638.mps	13658	32417	28637	0	3780
neos-933966.mps	12047	31762	27982	0	3780
neos-934278.mps	11495	23123	19955	0	3168
neos-948126.mps	7271	9551	6965	0	2586
neos-957389.mps	5115	6036	6036	0	0
newdano.mps	576	505	56	0	449
nobel-eu-DBE.mps	879	3771	1639	0	2132
noswot.mps	182	128	75	25	28
npmv07.mps	76342	220686	1880	0	218806
ns1111636.mps	13895	360822	13200	0	347622
ns1158817.mps	68455	1804022	66022	0	1738000
ns1606230.mps	3503	4173	3633	0	540
ns1686196.mps	4055	2738	2738	0	0
ns1688347.mps	4191	2685	2685	0	0
ns1702808.mps	1474	804	666	0	138
ns1766074.mps	182	100	0	90	10
ns1830653.mps	2932	1629	1458	0	171
ns1856153.mps	35407	11998	11956	0	42
ns2017839.mps	54510	55224	12	0	55212
ns2081729.mps	1190	661	600	0	61
ns2124243.mps	139280	156083	16447	0	139636
ns4-pr3.mps	2210	8601	0	61	8540
ns4-pr9.mps	2220	7350	0	42	7308

ns894244.mps	12129	21856	21856	0	0
ns930473.mps	23240	11328	11176	0	152
nu120-pr3.mps	2210	8601	8540	61	0
nu60-pr9.mps	2220	7350	7308	42	0
ofi.mps	422587	420434	18632	11073	390729
opm2-z7-s2.mps	31798	2023	2023	0	0
p100x588b.mps	688	1176	588	0	588
p6b.mps	5852	462	462	0	0
p80x400b.mps	480	800	400	0	400
pg.mps	125	2700	100	0	2600
pg5_34.mps	225	2600	100	0	2500
pigeon-10.mps	931	490	400	0	90
pigeon-11.mps	1123	572	473	0	99
pigeon-12.mps	1333	660	552	0	108
pigeon-13.mps	1561	754	637	0	117
pigeon-19.mps	3307	1444	1273	0	171
protfold.mps	2112	1835	1835	0	0
pw-myciel4.mps	8164	1059	1058	1	0
qiu.mps	1192	840	48	0	792
queens-30.mps	960	900	900	0	0
r80x800.mps	880	1600	800	0	800
rail507.mps	509	63019	63009	0	10
ramos3.mps	2187	2187	2187	0	0
ran14x18.disj-8.mps	447	504	252	0	252
ran14x18.mps	284	504	252	0	252
ran16x16.mps	288	512	256	0	256
reblock166.mps	17024	1660	1660	0	0
reblock354.mps	19906	3540	3540	0	0
reblock420.mps	62800	4200	4200	0	0
reblock67.mps	2523	670	670	0	0
rmatr100-p10.mps	7260	7359	100	0	7259
rmatr100-p5.mps	8685	8784	100	0	8684
rmatr200-p10.mps	35055	35254	200	0	35054
rmatr200-p20.mps	29406	29605	200	0	29405
rmatr200-p5.mps	37617	37816	200	0	37616
rmine6.mps	7078	1096	1096	0	0
rocII-4-11.mps	21738	9234	9086	0	148
rococoB10-011000.mps	1667	4456	4320	136	0
rococoC10-	1293	3117	2993	124	0

001000.mps					
rococoC11-011100.mps	2367	6491	6325	166	0
roll3000.mps	2295	1166	246	492	428
rvb-sub.mps	225	33765	33763	0	2
satellites1-25.mps	5996	9013	8509	0	504
sct1.mps	12154	22886	9044	1268	12574
sct5.mps	13304	37265	20702	2302	14261
set3-10.mps	3747	4019	1424	0	2595
set3-15.mps	3747	4019	1424	0	2595
set3-20.mps	3747	4019	1424	0	2595
seymour.disj-10.mps	5108	1209	1209	0	0
seymour.mps	4944	1372	1372	0	0
siena1.mps	2220	13741	11775	0	1966
sing2.mps	28891	31630	23377	0	8253
sing245.mps	143161	235146	220692	0	14454
sp97ar.mps	1761	14101	14101	0	0
sp98ic.mps	825	10894	10894	0	0
sp98ir.mps	1531	1680	871	809	0
sts405.mps	27270	405	405	0	0
sts729.mps	88452	729	729	0	0
swath.mps	884	6805	6724	0	81
t1717.mps	551	73885	73885	0	0
t1722.mps	338	36630	36630	0	0
tanglegram1.mps	68342	34759	34759	0	0
tanglegram2.mps	8980	4714	4714	0	0
timtab1.mps	171	397	64	107	226
toll-like.mps	4408	2883	2883	0	0
transportmoment.mps	9616	9685	2456	0	7229
triptim1.mps	15706	30055	20451	9597	7
tw-myciel4.mps	8146	760	759	1	0
uc-case11.mps	51438	34134	3898	302	29934
uct-subprob.mps	1973	2256	379	0	1877
umts.mps	4465	2947	2802	72	73
unitcal_7.mps	48939	25755	2856	0	22899
usAbbrv.8.25_70.mps	3291	2312	1681	0	631
van.mps	27331	12481	192	0	12289
vpphard.mps	47280	51471	51471	0	0
vpphard2.mps	198450	199999	199999	0	0
wachplan.mps	1553	3361	3360	1	0

wnq-n100-mw99-14.mps	656900	10000	10000	0	0
zib54-UUE.mps	1809	5150	81	0	5069
AVERAGE	**25215.48**	**39788.54**	**11027.68**	**241.63**	**28519.22**

A.5 MIPLIB 2010 Subset of Hard MIP Models Used for Testing

The table below lists the 111 hard models (i.e. solved by *Baseline_Cplex* in more than an hour) used in testing experiments requiring equality and inequality-containing models. Statistics on the numbers and types of variables and constraints in each model are listed. Average values are given at the end of the table. Models that are solved by at least one algorithm within the time limit are listed in *italics*. This constitutes the *hard1* set.

MIP Model	Number of Constraints	Number of Variables	Number of Binary Vars.	Number of Integer Vars.	Number of Real Vars.
a1c1s1.mps	3312	3648	192	0	3456
aflow40b.mps	1442	2728	1364	0	1364
atm20-100.mps	4380	6480	2220	0	4260
bab1.mps	60680	61152	61152	0	0
bab3.mps	23069	393800	393800	0	0
bab5.mps	4964	21600	21600	0	0
bley_xl1.mps	175620	5831	5831	0	0
bnatt350.mps	4923	3150	3150	0	0
bnatt400.mps	5614	3600	3600	0	0
cdma.mps	9095	7891	4235	0	3656
circ10-3.mps	42620	2700	2700	0	0
core4872-1529.mps	4875	24656	24645	0	11
datt256.mps	11077	262144	262144	0	0
ds-big.mps	1042	174997	174997	0	0
enlight14.mps	196	392	196	196	0
enlight15.mps	225	450	225	225	0
enlight16.mps	256	512	256	256	0
ex10.mps	69608	17680	17680	0	0
ex9.mps	40962	10404	10404	0	0
f2000.mps	10500	4000	4000	0	0
germanrr.mps	10779	10813	5288	5286	239
hanoi5.mps	16399	3862	3862	0	0
ivu52.mps	2116	157591	157591	0	0
lectsched-1-obj.mps	50108	28718	28236	482	0
lectsched-1.mps	50108	28718	28236	482	0
lectsched-2.mps	30738	17656	17287	369	0
lectsched-3.mps	45262	25776	25319	457	0
lrsa120.mps	14521	3839	120	119	3600
mining.mps	661133	348921	348920	0	1
momentum1.mps	42680	5174	2349	0	2825
momentum2.mps	24237	3732	1808	1	1923

momentum3.mps	56822	13532	6598	1	6933
n15-3.mps	29494	153140	0	780	152360
nb10tb.mps	150495	73340	14124	2756	56460
neos-1140050.mps	3795	40320	38640	0	1680
neos-1224597.mps	3276	3395	3150	245	0
neos-1429212.mps	58726	416040	54756	0	361284
neos-631710.mps	169576	167056	167056	0	0
neos-686190.mps	3664	3660	3600	60	0
neos-738098.mps	25849	9093	8946	0	147
neos-785912.mps	1714	1380	1380	0	0
neos-807456.mps	840	1635	1635	0	0
neos-820146.mps	830	600	600	0	0
neos-820157.mps	1015	1200	1200	0	0
neos-826650.mps	2414	5912	5792	0	120
neos-849702.mps	1041	1737	1737	0	0
neos-859770.mps	2065	2504	2504	0	0
neos-885524.mps	65	91670	91670	0	0
neos-932816.mps	30823	21007	20566	0	441
neos-935627.mps	7859	10301	7522	0	2779
neos-937511.mps	8158	11332	8562	0	2770
neos-937815.mps	9251	11646	8876	0	2770
neos-952987.mps	354	31329	31329	0	0
neos-984165.mps	6962	8883	6478	0	2405
neos808444.mps	18329	19846	19846	0	0
neos858960.mps	132	160	160	0	0
net12.mps	14021	14115	1603	0	12512
netdiversion.mps	119589	129180	129180	0	0
ns1116954.mps	131991	12648	7482	0	5166
ns1208400.mps	4289	2883	2880	0	3
ns1456591.mps	1997	8399	8000	19	380
ns1631475.mps	24496	22696	22470	211	15
ns1644855.mps	40698	30200	10000	0	20200
ns1685374.mps	44121	10000	10000	0	0
ns1696083.mps	11063	7982	7982	0	0
ns1745726.mps	4687	3208	3208	0	0
ns1758913.mps	624166	17956	17822	0	134
ns1769397.mps	5527	3772	3772	0	0
ns1778858.mps	10666	4720	4720	0	0
ns1853823.mps	224526	213440	213440	0	0
ns1854840.mps	143616	135754	135280	474	0

ns1904248.mps	149437	38458	38416	0	42
ns1905797.mps	51884	18192	17676	4	512
ns1905800.mps	8289	3228	3030	3	195
ns1952667.mps	41	13264	0	13264	0
ns2118727.mps	163354	167440	159514	0	7926
ns2122603.mps	24754	19300	7588	0	11712
ns2137859.mps	206726	103361	103041	0	320
ns894236.mps	8218	9666	9666	0	0
ns894786.mps	16794	27278	27278	0	0
ns894788.mps	2279	3463	3463	0	0
ns903616.mps	18052	21582	21582	0	0
nsr8k.mps	6284	38356	32040	0	6316
opm2-z10-s2.mps	160633	6250	6250	0	0
opm2-z11-s8.mps	223082	8019	8019	0	0
opm2-z12-s14.mps	319508	10800	10800	0	0
opm2-z12-s7.mps	319508	10800	10800	0	0
p2m2p1m1p0n100.mps	1	100	100	0	0
probportfolio.mps	302	320	300	0	20
rail01.mps	46843	117527	117527	0	0
rail02.mps	95791	270869	270869	0	0
rail03.mps	253905	758775	758775	0	0
rmine10.mps	65274	8439	8439	0	0
rmine14.mps	268535	32205	32205	0	0
rocII-7-11.mps	37215	16101	15851	0	250
rocII-9-11.mps	47533	20679	20361	0	318
rococoC12-111000.mps	10776	8619	8432	187	0
satellites2-60-fs.mps	16516	35378	34324	0	1054
satellites2-60.mps	20916	35378	34324	0	1054
satellites3-40-fs.mps	35553	81681	79961	0	1720
satellites3-40.mps	44804	81681	79961	0	1720
sct32.mps	5440	9767	6396	1332	2039
shipsched.mps	45554	13594	10549	0	3045
shs1023.mps	133944	444625	1296	440899	2430
sing161.mps	455631	770102	733244	0	36858
sing359.mps	437116	713762	674643	0	39119
stockholm.mps	57346	20644	962	0	19682
stp3d.mps	159488	204880	204880	0	0
triptim2.mps	14427	27326	20771	6548	7
triptim3.mps	14939	28440	21621	6812	7

uc-case3.mps	52003	37749	11256	0	26493
AVERAGE	**66848.99**	**68453.91**	**56794.44**	**4337.55**	**7321.92**

A.6 MIPLIB 2010 Subset of Equality-Containing MIP Models Using Perpendicular General Disjunctions

The table below lists the subset of 43 equality-containing models in which the default general algorithm triggers the use of at least one perpendicular general disjunction. Models in which only perpendicular general disjunctions are triggered by the least-impact-variable-selection and branching-direction-selection methods are indicated in the columns.

MIP Model	Least-impact-variable-selection algorithms trigger only perpendicular disjunctions?	Branching-direction-selection algorithms trigger only perpendicular disjunctions?
neos-1396125.mps	no	no
neos-506428.mps	no	no
neos-799711.mps	yes	yes
neos-826841.mps	yes	no
neos-777800.mps	yes	yes
dano3mip.mps	yes	yes
neos-935769.mps	no	yes
acc-tight5.mps	no	no
gmu-35-40.mps	no	no
neos-941262.mps	no	no
bienst2.mps	yes	yes
mcsched.mps	yes	yes
neos-506422.mps	yes	yes
danoint.mps	yes	yes
neos-911880.mps	yes	yes
co-100.mps	yes	yes
eilA101.2.mps	yes	yes
neos-1109824.mps	no	no
neos-555424.mps	yes	no
nag.mps	yes	yes
csched010.mps	yes	yes
30n20b8.mps	no	no
neos-476283.mps	no	no
csched008.mps	yes	yes
neos-1620770.mps	yes	no
neos-941313.mps	yes	yes
neos-520729.mps	yes	yes
glass4.mps	yes	yes
markshare_5_0.mps	no	yes
gmu-35-50.mps	no	no
mkc.mps	no	no

blp-ar98.mps	no	yes
blp-ic97.mps	yes	yes
leo1.mps	no	no
dc1c.mps	no	no
eilB101.mps	yes	yes
biella1.mps	no	no
leo2.mps	yes	yes
eil33.2.mps	yes	yes
csched007.mps	yes	yes
harp2.mps	yes	yes
neos-1440225.mps	yes	yes
air04.mps	yes	yes

A.7 MIPLIB 2010 Subset of Easy and Hard MIP Models Used for Testing

The table below lists the 179 models from the testing set in which at least one of the algorithms from Experiment 2 completes within the time limit. Models that were solved by *Default_Cplex* in more than an hour (i.e. categorized as "Hard") are listed in italics.

MIP Models			
ash608gpia-3col.mps	pigeon-19.mps	t1722.mps	neos-932816.mps
enlight9.mps	protfold.mps	tanglegram1.mps	neos-935627.mps
enlight13.mps	pw-myciel4.mps	tanglegram2.mps	neos-937511.mps
neos13.mps	qiu.mps	timtab1.mps	neos-937815.mps
neos15.mps	queens-30.mps	toll-like.mps	neos-984165.mps
neos16.mps	r80x800.mps	transportmoment.mps	neos808444.mps
neos18.mps	rail507.mps	triptim1.mps	*neos858960.mps*
neos6.mps	ramos3.mps	tw-myciel4.mps	*net12.mps*
neos788725.mps	ran14x18.disj-8.mps	uc-case11.mps	*ns1208400.mps*
neos-885086.mps	ran14x18.mps	uct-subprob.mps	*ns1456591.mps*
neos-933638.mps	ran16x16.mps	umts.mps	*ns1631475.mps*
neos-933966.mps	reblock166.mps	unitcal_7.mps	ns1644855.mps
neos-934278.mps	reblock354.mps	usAbbrv.8.25_70.mps	*ns1685374.mps*
neos-948126.mps	reblock420.mps	van.mps	*ns1696083.mps*
neos-957389.mps	reblock67.mps	vpphard.mps	ns1758913.mps
newdano.mps	rmatr100-p10.mps	vpphard2.mps	ns1854840.mps
nobel-eu-DBE.mps	rmatr100-p5.mps	wachplan.mps	ns1952667.mps
noswot.mps	rmatr200-p10.mps	wnq-n100-mw99-14.mps	*ns2118727.mps*
npmv07.mps	rmatr200-p20.mps	zib54-UUE.mps	ns2122603.mps
ns1111636.mps	rmatr200-p5.mps	a1c1s1.mps	*ns903616.mps*
ns1158817.mps	rmine6.mps	aflow40b.mps	nsr8k.mps
ns1606230.mps	rocII-4-11.mps	bab1.mps	opm2-z10-s2.mps
ns1688347.mps	rococoB10-011000.mps	bab5.mps	opm2-z11-s8.mps
ns1702808.mps	rococoC10-001000.mps	*cdma.mps*	opm2-z12-s14.mps
ns1766074.mps	rococoC11-011100.mps	*circ10-3.mps*	opm2-z12-s7.mps
ns1830653.mps	roll3000.mps	core4872-1529.mps	*probportfolio.mps*
ns2017839.mps	rvb-sub.mps	ds-big.mps	rail01.mps
ns2081729.mps	satellites1-25.mps	*enlight14.mps*	*rail02.mps*
ns2124243.mps	sct1.mps	enlight15.mps	*rail03.mps*
ns4-pr3.mps	sct5.mps	germanrr.mps	rmine10.mps
ns4-pr9.mps	set3-10.mps	ivu52.mps	rmine14.mps

ns930473.mps	set3-15.mps	mining.mps	rocII-7-11.mps
nu120-pr3.mps	set3-20.mps	momentum1.mps	rococoC12-111000.mps
nu60-pr9.mps	seymour.disj-10.mps	*momentum2.mps*	satellites2-60-fs.mps
ofi.mps	seymour.mps	momentum3.mps	*satellites2-60.mps*
opm2-z7-s2.mps	siena1.mps	n15-3.mps	*satellites3-40-fs.mps*
p100x588b.mps	sing2.mps	neos-1224597.mps	sct32.mps
p6b.mps	sing245.mps	neos-631710.mps	shs1023.mps
p80x400b.mps	sp97ar.mps	neos-686190.mps	sing161.mps
pg.mps	sp98ic.mps	neos-738098.mps	sing359.mps
pg5_34.mps	sp98ir.mps	*neos-785912.mps*	stockholm.mps
pigeon-10.mps	sts405.mps	*neos-820157.mps*	*triptim2.mps*
pigeon-11.mps	sts729.mps	neos-826650.mps	triptim3.mps
pigeon-12.mps	swath.mps	*neos-859770.mps*	uc-case3.mps
pigeon-13.mps	t1717.mps	neos-885524.mps	

Appendix B: Algorithm Tuning Experiments

A number of different choices are available for each element of the algorithm for constructing and using a general disjunction. We tune the selections of the elements in two stages. In stage 1 we use a base algorithm in which one element at a time is varied while all others are held constant. This approach is used to rank the choices for each element. In stage 2 the top few choices for each element are combined to create algorithm variants that are compared. The base algorithm for stage 1 consists of the default choices for each element that are shown in Table B.1. Details are given as the results for each element are presented.

The 127 models used for these tuning experiments are those that are solved by *Baseline_Cplex* in less than 1 hour. Different subsets of 100 of these 127 models are chosen randomly to constitute the tuning sets listed in Appendices A.1 (models having at least one equality constraint), A.2 (models that have at least one inequality constraint), and A.3 (models having a variety of properties).

Stage 1: Ranking Algorithm Elements

Table B.1: Default method used for each decision element of the base general disjunction algorithm.

	Decision	Default Choice
1	Use a general disjunction at the root node?	Yes
2	Triggering insertion of a general disjunction.	Use *cand-var-ratio-progress* and *infeas-var-ratio-progress* .There is no lower limit on the number of candidate variables.
3	Choosing the foundation constraint.	Use *sumCandCoeffs _feasGapSum.* The highest sum of candidate-variable absolute coefficients is normalized by the largest absolute candidate-variable coefficient.
4	Constructing a 45-degree general disjunction that is approximately perpendicular to an equality constraint.	Switch sign of non-zero coefficients on odd counter. If an odd number of non-zeros exist, then set the first non-zero coefficient in the branching disjunction to zero.
5	Branching direction to use for parallel general disjunctions.	Branch to decrease the value of the disjunction function.
6	Branching direction to use for perpendicular general disjunctions.	Branch to increase the value of the disjunction function.

The choices for selecting the foundation constraint are listed in Table B.2, and their experimental evaluation is summarized in Figure B.1.

Table B.2: Foundation constraint selection algorithms.

1	***candVarSum_feasGapSum* (*CVS-FGS*)** Choose the active constraint with the largest number of candidate variables. Break ties using the highest feasibility-gap sum.
2	***intVarSum_sumIntCoeffs* (*IVS-SIC*)** Choose the active constraint with the largest number of integer variables. Break ties using the highest sum of integer-variable absolute coefficients.
3	***sumCandCoeffs_feasGapSum* (*SCC-FGS*)** Choose the active constraint with the highest sum of candidate-variable absolute coefficients. Break ties using the highest feasibility-gap sum.
4	***candVarSum_sumCandCoeffs* (*CVS-SCC*)** Choose the active constraint with the largest number of candidate variables. Break ties using the highest sum of candidate-variable absolute coefficients.

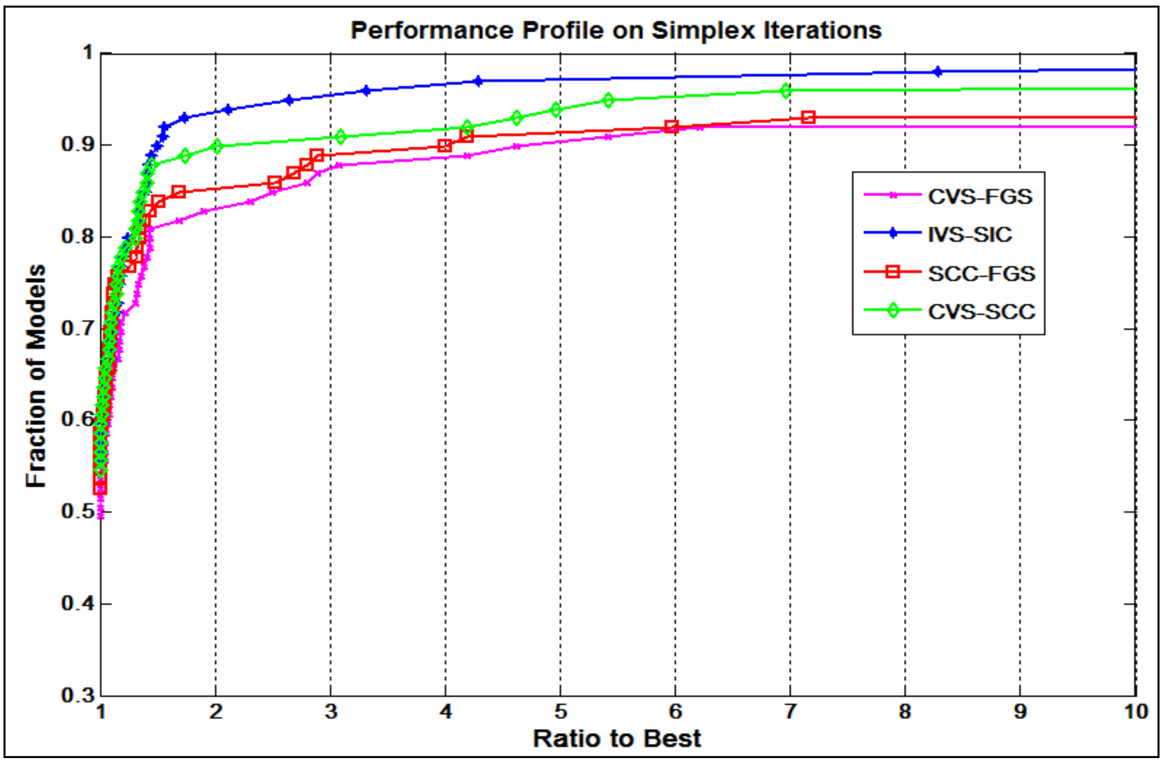

Figure B.1: Experimental evaluation of methods for choosing the foundation constraint.

Methods for triggering the inclusion of a general disjunction are described next.

The candidate-variable count measure (*cand-var-count*) is incremented by 1 whenever the number of candidate variables at a child node increases over the number at the parent node. It is decremented by 1 whenever the number of candidate variables at a child node decreases or stays the same as the number at the parent node. The same concept applies to the infeasibility-sum count measure (*infeas-sum-count*). The candidate-variable ratio measure (*cand-var-ratio*) captures the ratio of the numbers of candidate variables at the parent and child nodes. The candidate variable ratio progress measure (*cand-var-ratio-progress*) then captures the cumulative change in the value of *cand-var-ratio*. The infeasibility sum ratio (*infeas-sum-ratio*) and infeasibility sum ratio progress (*infeas-sum-ratio-progress*) measures are used in a similar manner to measure the change in ratio of the infeasibility sum between parent and child nodes. An example is given in **Error! Not a valid bookmark self-reference.**, assuming that all measures are initialized to 0.

Table B.3: Example of computing the count and ratio measures.

	No. of Cand. variables	Infeas-ibility Sum	*cand-var-count*	*infeas-sum-count*	*cand-var-ratio-progress*	*infeas-sum-ratio-progress*
Parent Node	83	232.5	--	--	--	--
Child 1 Node	70	202	0 + (-1) = -1	0 + (-1) = -1	0+(70-83)/83 = -0.16	0+(202-232.5)/232.5 = -0.13
Child 2 Node	95	257.5	0 + 1 = 1	0 + 1 = 1	0+(95-83)/83 = 0.14	0+(257.5-232.5)/232.5 = 0.11

The methods for triggering the inclusion of a general disjunction are listed in Table B.4.

Table B.4: Methods for triggering the inclusion of a general disjunction.

	Method Name	**Condition Triggering the Inclusion of a General Disjunction**
1	*(CVC+ISC)>0*	(*cand-var-count* + *infeas-sum-count*) > 0
2	*(CVC+ISC)>5*	(*cand-var-count* + *infeas-sum-count*) > 5
3	*(CVC+ISC)>10*	(*cand-var-count* + *infeas-sum-count*) > 10
4	*(CVC-And-ISC)>3*	(*cand-var-count* > 3) and (*infeas-sum-count* > 3)
5	*(CVC-Or-ISC)>3*	(*cand-var-count* > 3) or (*infeas-sum-count* > 3)
6	*(CVC-Or-ISC)>5*	(*cand-var-count* > 3) or (*infeas-sum-count* > 3)
7	*(CVC-Or-ISC)>10*	(*cand-var-count* > 3) or (*infeas-sum-count* > 3)
8	*(CVRP-And-ISRP)>0*	(*cand-var-ratio-progress* > 0) and (*infeas-sum-ratio-progress* > 0)
9	*(CVC-And-ISC)>0*	(*cand-var-count* > 0) and (*infeas-sum-count* > 0)

Experimental results for these methods are shown in Figure B.2.

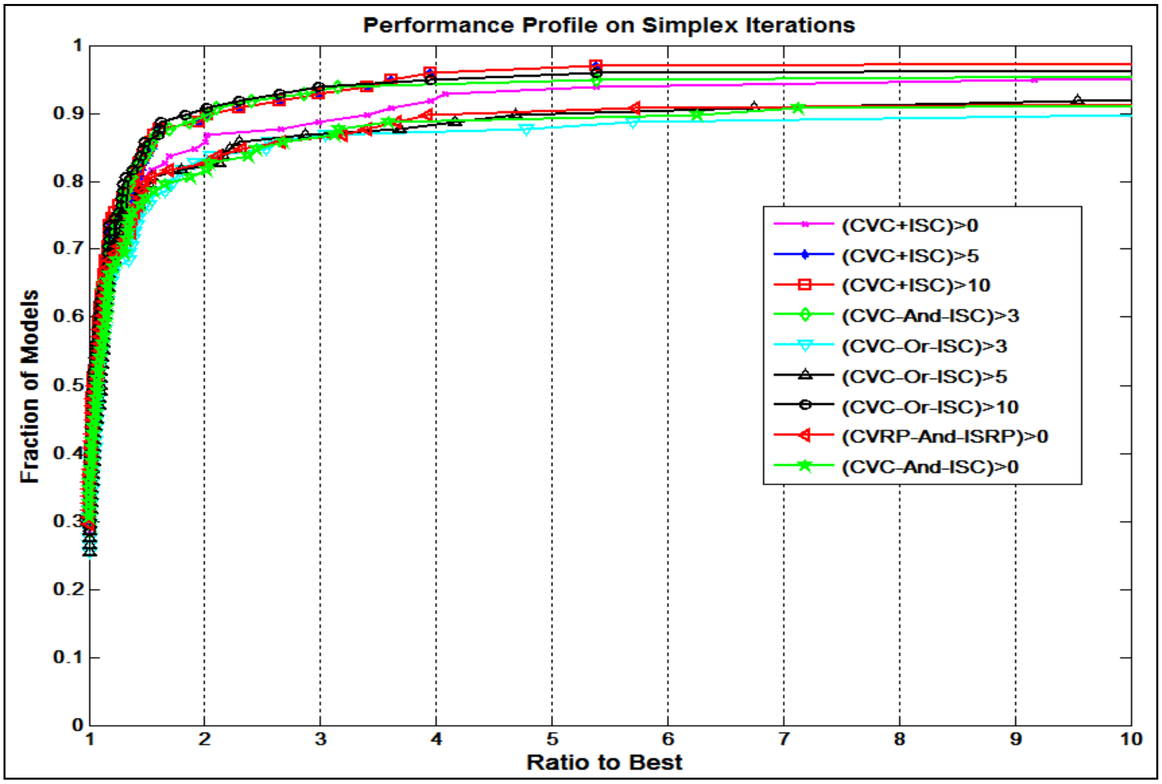

Figure B.2: Experimental evaluation of methods for triggering the inclusion of a general disjunction.

We experimented with three different values for the minimum number of candidate variables required before triggering the inclusion of a general disjunction: 20, 40, and 60. Figure 3 compares the performance of *(CVC-Or-ISC)>10* with the different limits in place, and shows that performance is better at higher limits.

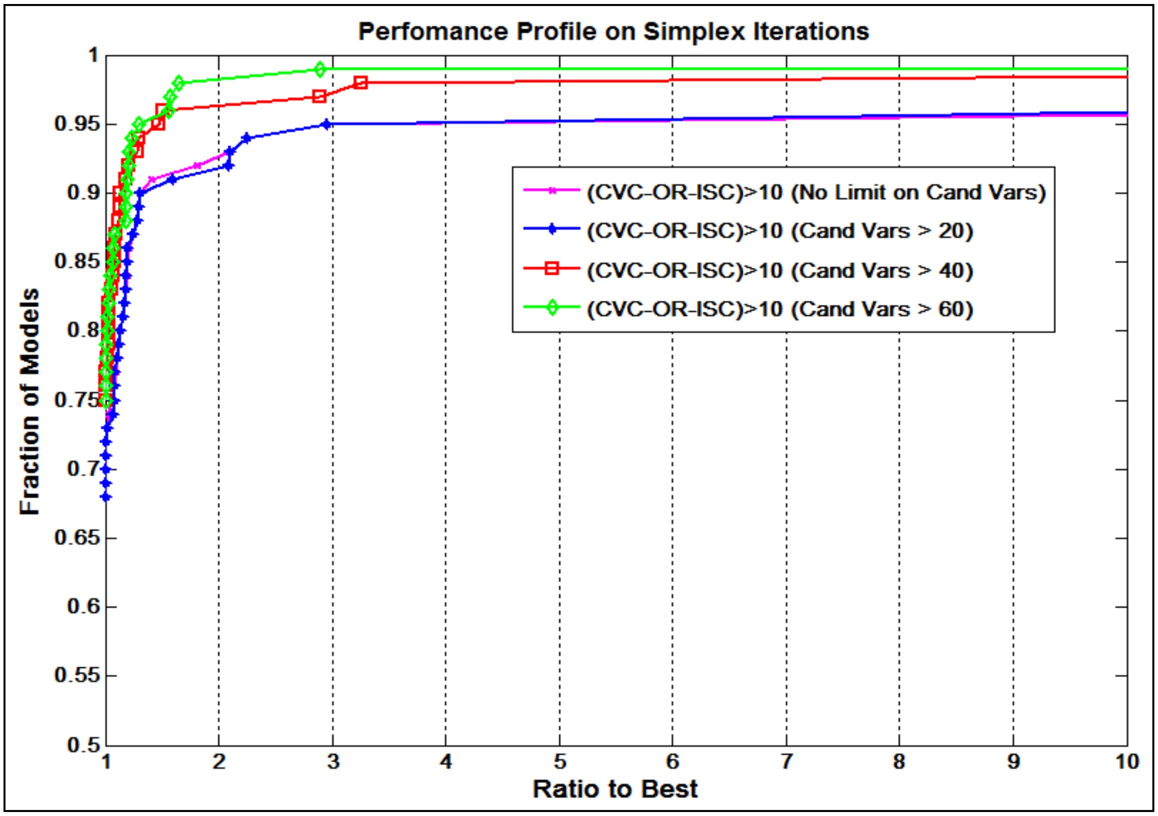

Figure B.3: Experimental evaluation of limits on the minimum number of candidate variables for *(CVC-Or-ISC)>10*.

Methods for selecting the least-impact-variable for an equality foundation constraint are listed in Table B.5.

Table B.1: Summary of least-impact-variable-selection methods.

	Method Name	Description
1	*SII-intVar*	For all variables with non-zero coefficients in the foundation constraint, choose the first integer variable with the smallest integer infeasibility. An integer infeasibility of zero is possible for integer variables that have an integer value.
2	*SC-var*	For all variables with non-zero coefficients in the foundation constraint, choose the variable with the smallest coefficient.
3	*SC(realVar-NCintVar-candVar)*	For all variables with non-zero coefficients in the foundation constraint, choose the real variable with the smallest coefficient. If no real variables are in the constraint, then choose the non-candidate integer variable with the smallest coefficient. If constraint consists only of candidate variables then choose the one with the smallest coefficient.
4	*SC(realVar-NCintVar)-LA(candVar)*	For all variables with non-zero coefficients in the foundation constraint, choose the real variable with the smallest coefficient. If none, then choose the non-candidate integer variable with the smallest coefficient. If constraint consists only of candidate variables then choose the one that is present in the fewest active constraints.

Experimental results are summarized in Figure B.4.

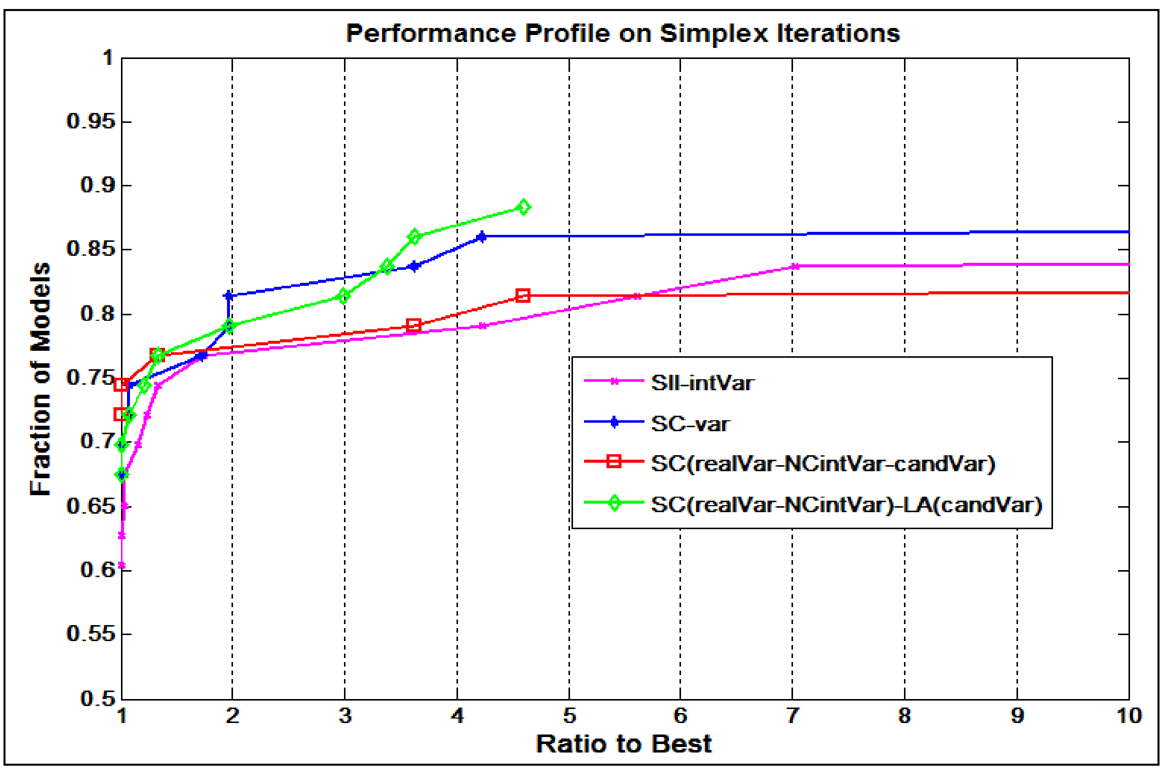

Figure B.4: Experimental evaluation of methods for choosing the variable with the least impact on the MILP solution.

We consider whether to alternate signs on odd or even counter when constructing an approximately perpendicular disjunction equation. The two options are compared in Figure in conjunction with *SC-var*, over the 43 models that use at least one perpendicular general disjunction. Both methods give similar overall results, though switching signs on even counter is fastest on more models.

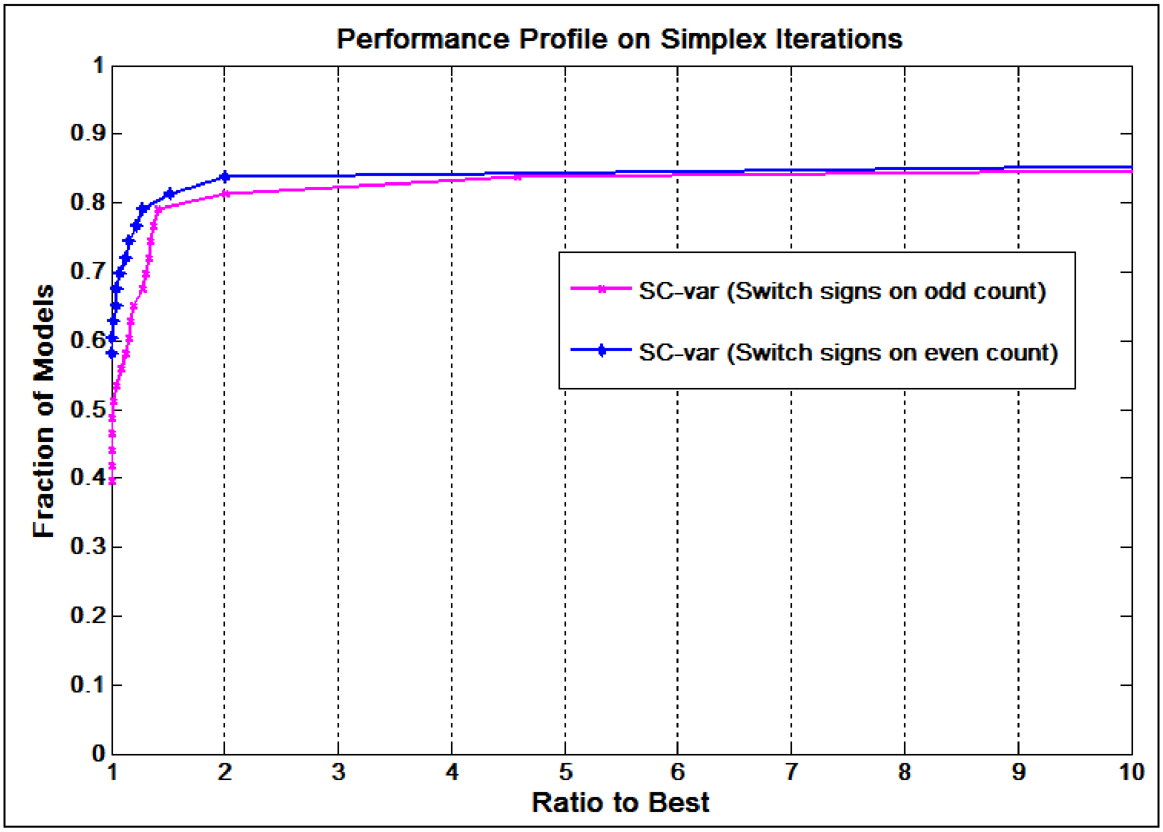

Figure B.5: Performance profile on simplex iterations for creating a 45-degree perpendicular disjunction using *SC-var* by switching signs on odd or even counts.

Methods for selecting the branching direction in approximately parallel general disjunctions are summarized in Table B.5 and experimental results are shown in Figure B.6.

Table B.5: Summary of branching-direction selection methods.

	Method Name	Description
1	*C-LP*	Closer to the LP-relaxation optimum point.
2	*F-LP*	Farther from the LP-relaxation optimum point.
3	*Dec-DF*	Decrease the value of the disjunction function.
4	*Inc-DF*	Increase the value of the disjunction function.
5	*Vio-BC*	Violating direction of the branching constraint.
6	*Sat-BC*	Satisfying direction of the branching constraint.

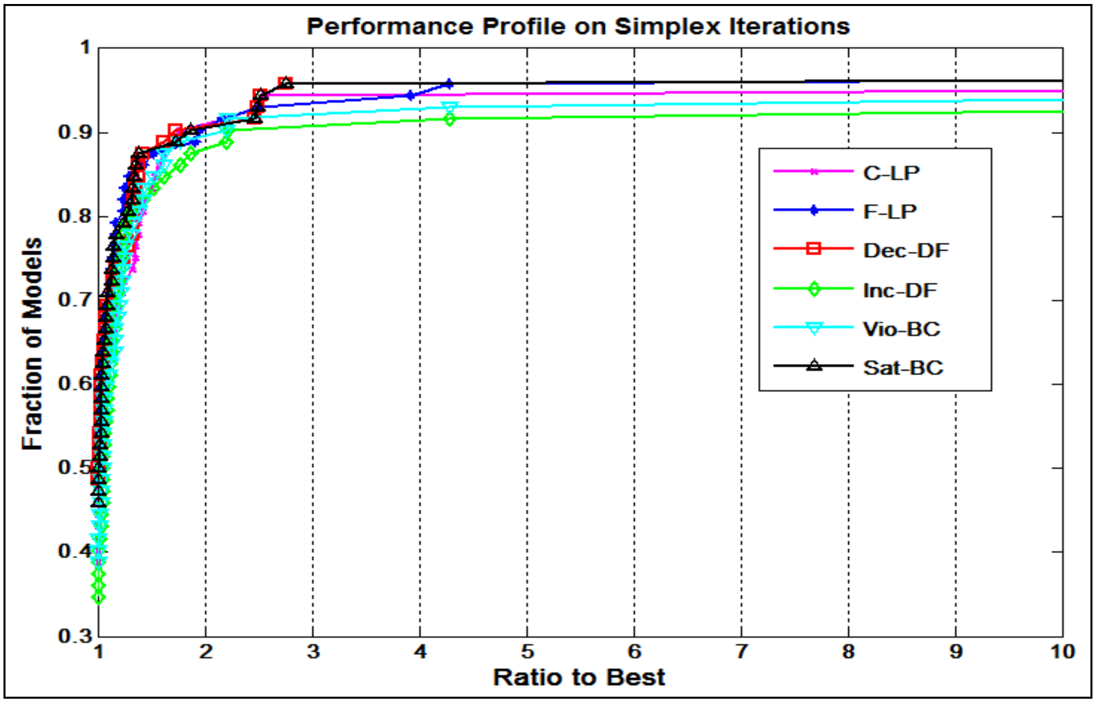

Figure B.6: Performance profile on simplex iterations for selecting the branching direction for parallel general disjunctions.

The branching-direction selection schemes for perpendicular general disjunctions are compared in Figure B.7, which includes the first four methods listed in Table , which are valid for perpendicular general disjunctions, on the subset of 27 models from Appendix A.6 in which only perpendicular general disjunctions are used.

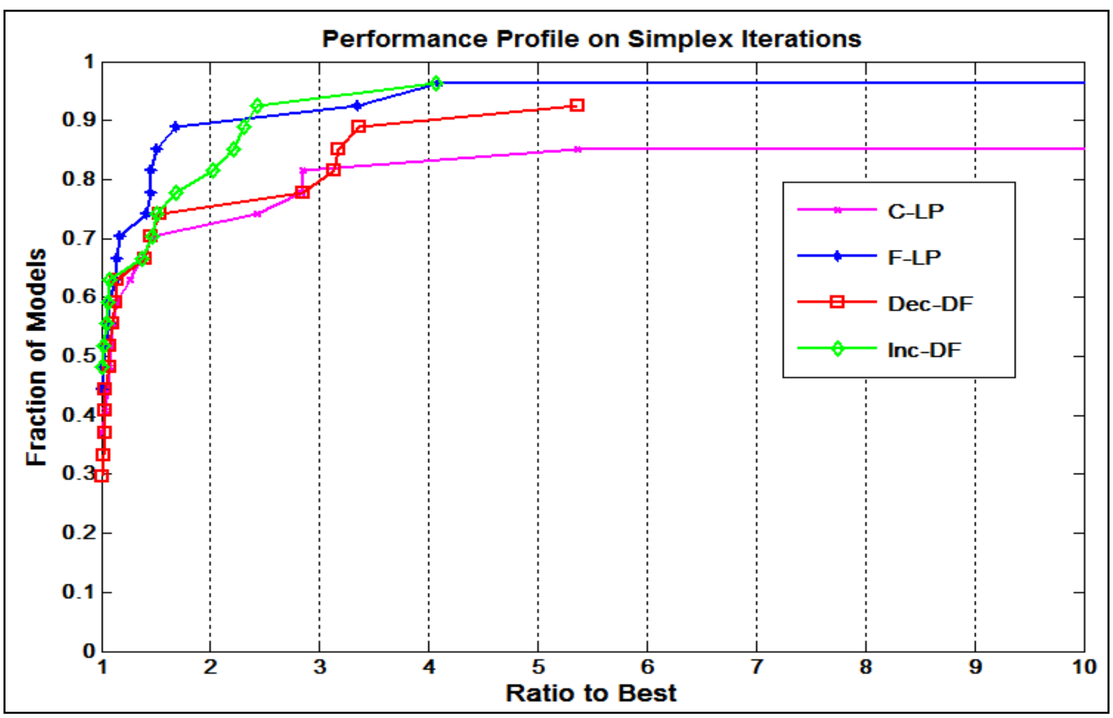

Figure B.7: Experimental evaluation of methods for selecting the branching direction for perpendicular general disjunctions.

Stage 2: Comparing Complete Algorithms

The final experiment combines the best few choices for each element into complete algorithm variants. Table B.6 summarizes the best choice for each element. 8 complete algorithm variants can be created from these choices.

Table B.6: Summary of top methods for making each decision in a complete algorithm.

	Decision	Method Used
1	Which type of disjunction to use at the root node?	A general disjunction is generated if the condition on the minimum number of candidate variables is satisfied.
2	When to perform a general disjunction?	2.A: *(CVC-And-ISC)>3(Cand Vars > 60)*
		2.B: *(CVC-Or-ISC)>10(Cand Vars > 60)*
3	Which active constraint to choose as the foundation constraint?	3.A: *IVS-SIC*
		3.B: *CVS-SCC*
4	How to construct a 45-degree general disjunction that is approximately perpendicular to an equality constraint?	Switch sign of non-zero coefficients on even counter. If an odd number of non-zeros exist, then use *SC-var* to set one of the non-zero coefficients to zero in the disjunction function
5	Which branching direction to use for parallel general disjunctions?	5.A: *Sat-BC* (with offset) 5.B: *F-LP* (without offset)
6	Which branching direction to use for perpendicular general disjunctions?	*F-LP*

Figure B.8 shows the experimental evaluation of these variants. Variants 2A-3B-5A and 2B-3B-5A and 2B-3A-5A are among the most robust; the first two are chosen for comparison to a state of the art commercial solver in the main paper.

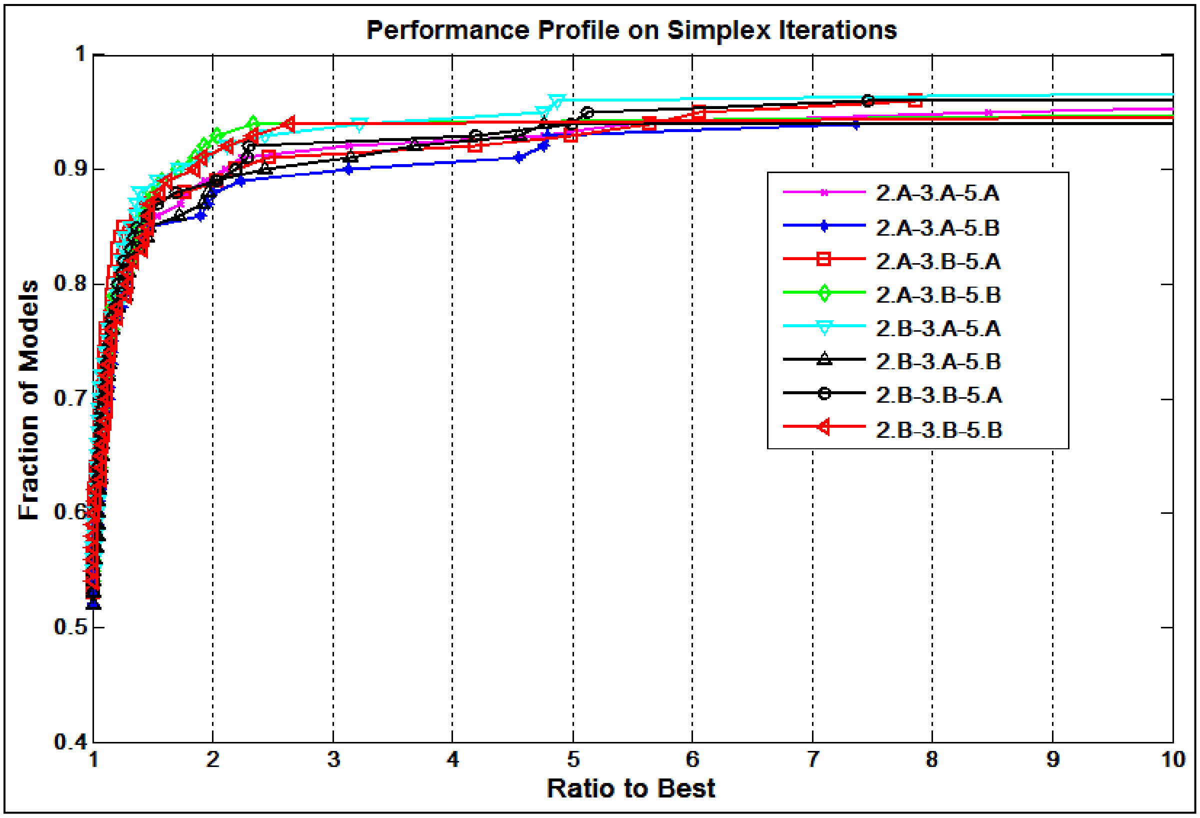

Figure B.8: Experimental evaluation of the best algorithm variants.

All eight variants solve nearly identical fractions of the test models within the time limit, with fractions ranging from 96 to 98 percent.