Inverse Problems & Applications

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Inverse Problems ... Plato's cave
Plato’s cave ... Shadows on the wall

Source: iamcriselleeee.files.wordpress.com/2013/11/cave-2.jpg
Inverse Problems

Forward Problem:  \( Forms \Rightarrow Shadows \)
Inverse Problems

Forward Problem:  \( \text{Forms} \Rightarrow \text{Shadows} \)
Inverse Problem:  \( \text{Shadows} \Rightarrow \text{Forms} \)

• Ill-conditioned
  Sensitivity to some movements is low

• Ill-posed
  Some movements don't change shadows

• Noisy
  Flickering light
Inverse Problems

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Inverse Problems

Techniques: to calculate stable & meaningful parameters in the presence of inversion difficulties

Examples

- Image deblurring / restoration
- Medical imaging
- Geophysical imaging
- Model parameter fitting
- Reconstruction with incomplete/noisy data
Electrical Impedance Tomography

10-day old healthy baby with EIT electrodes

Source: eidors3d.sf.net/data_contrib/if-neonate-spontaneous
Electronics – Block Diagram
Current Propagation

Healthy Adult Male
CT slide at heart

Source: ei-dors3d.sf.net/tutorial/netgen/extrusion
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Finite Element Modelling
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Simulated Voltages

Voxel Currents
Thorax Propagation

CT Slice with simulated current streamlines and voltage equipotentials
Thorax Propagation

CT Slice with simulated current streamlines and voltage equipotentials
Changing Conductivity

Heart receives blood (diastole) and is more conductive
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Application: Breathing

Chest images of tidal breathing in healthy adult
Application: Heart

EIT Signal in ROI around heart (and ECG)
Mechanical Ventilation

Mechanical Ventilator with EIT monitor

Source: Swisstom.com
Acute Respiratory Distress Syndrome (ARDS)
EIT + Lung State
EIT + Lung State

[Graphical representation with time sequence and pressure data, labeled A to G]

A B C D E F G
EIT + Lung State

[Diagram and images with labels A to G]

A B C D E F G

[Graph showing ventilatory pressure over time with annotations]

ΔP = 15 cm H₂O
Tv = 6 mL/kg
EIT + Lung State

![Graph showing ventilaory pressure over time with labeled stages A to G.]

- **A**: Baseline
- **B**: Sustained inflation
- **C**: Pflex + 2
- **D**: PC
- **E**: 15 cm H2O
- **F**: TV = 6 mL/kg
- **G**: "P380"
EIT for Brain Imaging

Applications:
- Epileptic foci
- Stroke (Ischaemic vs. Haemoragic)
- Fast Neural Imaging

Source: Holder, www.ucl.ac.uk/medphys/research/eit/pubs/brain_EIT_over
EIT for Cancer Imaging: Breast/Prostate

- Cancerous tissue has different electrical properties
- Image tissue
- Image increased vascularization

Source: Khan, Mahara, Halter et al, Conf. EIT, 2014
Non-medical applications

- Flow in pipes
- Mixing tanks
- Imaging metallic ores
- Hydro-geology

Figure shows resistivity in a cross-section of La Soufrière de Guadaloupe volcano.
Source: N. Lesparre et al, Conf. EIT, 2014
Data Quality
Depth Sounder – with analog and digital gauges
What’s the problem?

With strong priors and complex algorithms, algorithms give us pretty pictures, even when they are irrelevant.

Question:
- how can we know when to trust a pretty picture?
- how can we know when the data are junk?
Data Quality Measure: Concept

- **Concept**: High Quality Data is Consistent

- **Idea**: Use IP to predict each data point from all others

\[
\begin{align*}
\text{Original} & \quad \Rightarrow \quad \text{Remove} \ i & \quad \Rightarrow \quad \text{Solve} \ \hat{m}^{(i)} & \quad \Rightarrow \quad \text{Predict} \ \hat{d}^{(i)} \\
\ d & \quad \Rightarrow \quad \ d^{(i)} & \\
\end{align*}
\]

- **Calculate error**

\[
\epsilon_i = d_i - \hat{d}_i^{(i)}
\]
Clinical data and data quality metric for each stage of the protocol (R1–R4 — recruitment: PEEP↑, T1–T4 — titration: PEEP↓).

A: EIT images  B: Calculated data quality.
Perspectives

- Data analysis is hard
- powerful algorithms are useful
- we live in a world of big data
- complex systems fail in complex ways
- users like pretty pictures

So ... the situation will get worse
Solutions?

Thus, we need:

• Open Data
• Open source analysis
Solutions?

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For EIT . . .
For EIT ...
For EIT...

Contributed EIT Data:

**Authors:** S. Heinrich, H. Schiffmann, A. Frerichs, A. Klockgether-Radke, I. Frerichs

**Date:** 2006

**Brief Description:** 10-day old spontaneously breathing neonate lying in the prone position with the head turned to the left. Data were published in S. Heinrich, H. Schiffmann, A. Frerichs, A. Klockgether-Radke, I. Frerichs, *Body and head position effects on regional lung ventilation in infants: an electrical impedance tomography study*. Intensive Care Med., 32:1392-1398, 2006.

**License:** Creative Commons Artistic License (with Attribution)

**Attribution Requirement:** Use or presentation of these data must acknowledge Inez Frerichs, and reference this publication.

**Format:** EIT data were acquired with the Göttingen Geo-MF II device, 220 frames, 13 frames/s. Data are in the .GET file format.

**Methods:** Neonate in prone position, with electrode #1 at the front of the chest, electrode #5 on the left side of the chest, electrode #9 on the back and electrode #13 on the right side of the chest.

**Data:** Data (zip format)

Image of Experimental Configuration:
Traffic jam on the way to Carleton