Edge Preserving Image Reconstruction: Experimental Applications to Thoracic EIT

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- Introduction to Electrical Impedance Tomography (EIT)
- Motivation
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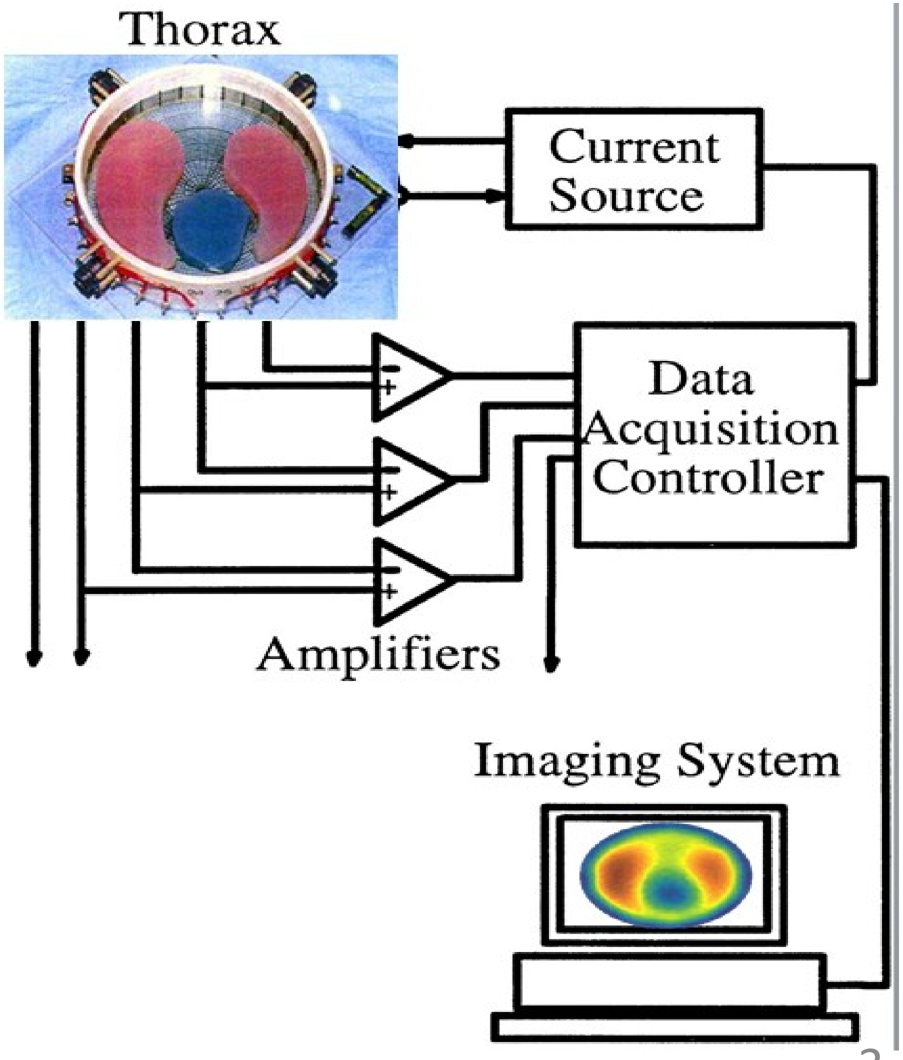
Electrical Impedance Tomography (EIT)

- Medical imaging modality in which an image of the internal conductivity/ permittivity distribution of the body is reconstructed from boundary electrical measurements.

- One pair of electrodes injects low frequency current to the medium and the other pairs of the electrodes collect the difference voltage on the surface.



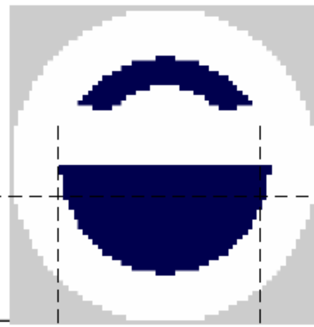
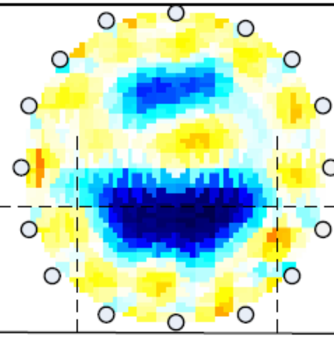
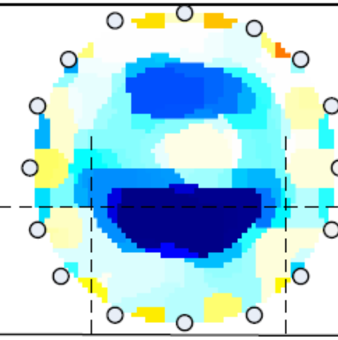
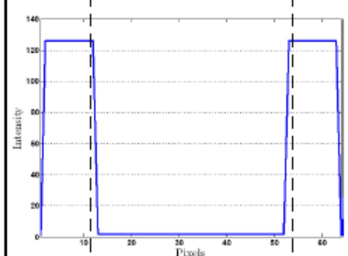
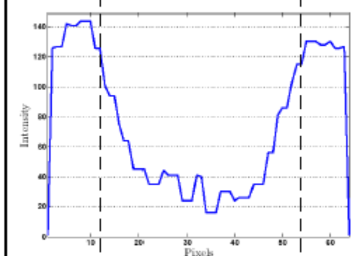
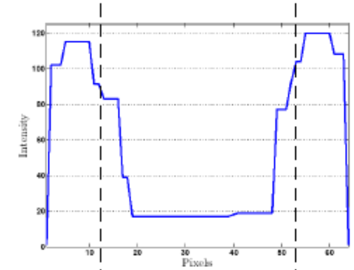
Thesis Motivation using an Example

- A typical image reconstruction problem can be formulated as

$$x = argmin_x \{ \underbrace{D(h(x)-d_{real})}_{\text{Data Term}} + \underbrace{P(x-x_{prior})}_{\text{Image Term}} \}$$

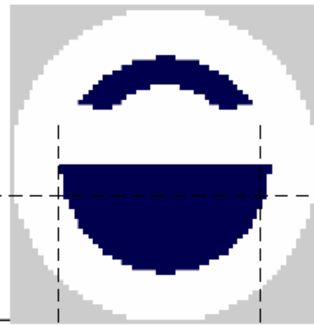
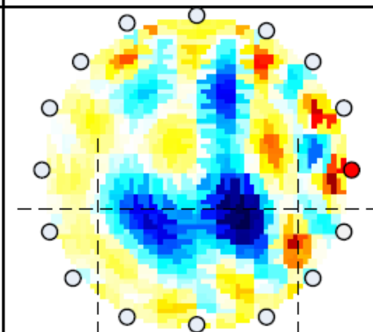
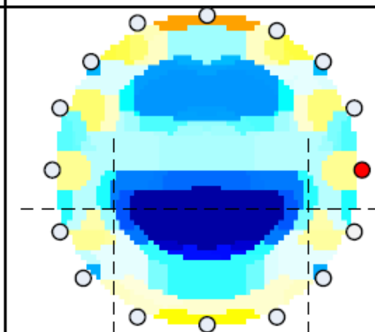
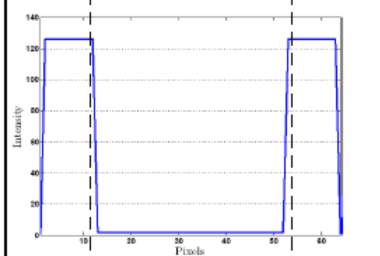
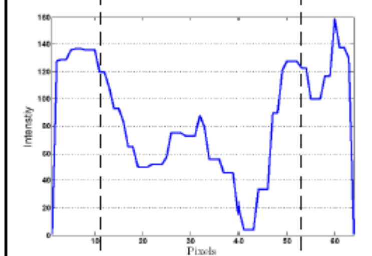
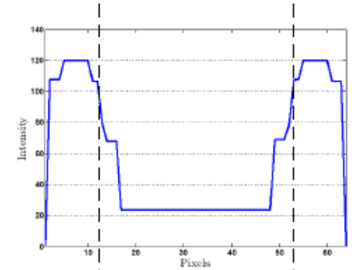
- Where;
  - x is the reconstructed image (inverse solution)
  - $D(h(x)-d_{real})$ is referred to as a data term and increases as the forward model h(x) is less able to predict the real data d_{real}
  - $P(x-x_{prior})$ is an image term (or a regularization term) which increases as the inverse solution x is less likely, given the prior understanding of the model parameters x_{prior}.

Thesis Motivation using an Example

	Actual Image	Reconstruction Method: Traditional Image Reconstruction Method	Reconstruction Method: Edge Preserving Image Reconstruction Method
EIT Simulated Data			
Cross Section Profile			
Inverse Problem (Image Term)	—	$\|\|(m - m_{real})\|\|_2$	$\|(m - m_{real})\|_1$

14 dB zero mean Gaussian noise is added to the EIT simulated data!

Thesis Motivation using an Example

	Actual Image	Reconstruction Method	
		Traditional Image Reconstruction Method	Edge Preserving Image Reconstruction Method
EIT Simulated Data			
Cross Section Profile			
Inverse Problem (Data Term)	——	$\lVert(d - d_{real})\rVert_2$	$\lvert(d - d_{real})\rvert_1$

The 5th electrode causes a data loss (data outliers)!

Thesis Contributions

Edge Preserving Image Reconstruction Method

Shape based Reconstruction Method (Level Set)

Image Reconstruction with the Sum of the Absolutes based Penalty Terms

Main Contributions of this thesis

- Level Set based Reconstruction Algorithm for EIT Lung Images: First Clinical Results

- Level Set Technique for High Contrast Image Reconstruction using the sum of absolutes (L1 norms)

- A Generalized Inverse Problem using Weighted L1 and L2 Norms on Data and Regularization Terms

Contribution #1: Level Set-based Reconstruction Algorithm using difference solver

- Problem:

Many applications deal with the reconstruction and optimization of geometries (shapes, topologies).

However, there is no natural a-priori information on shapes or topological structures of the solution.

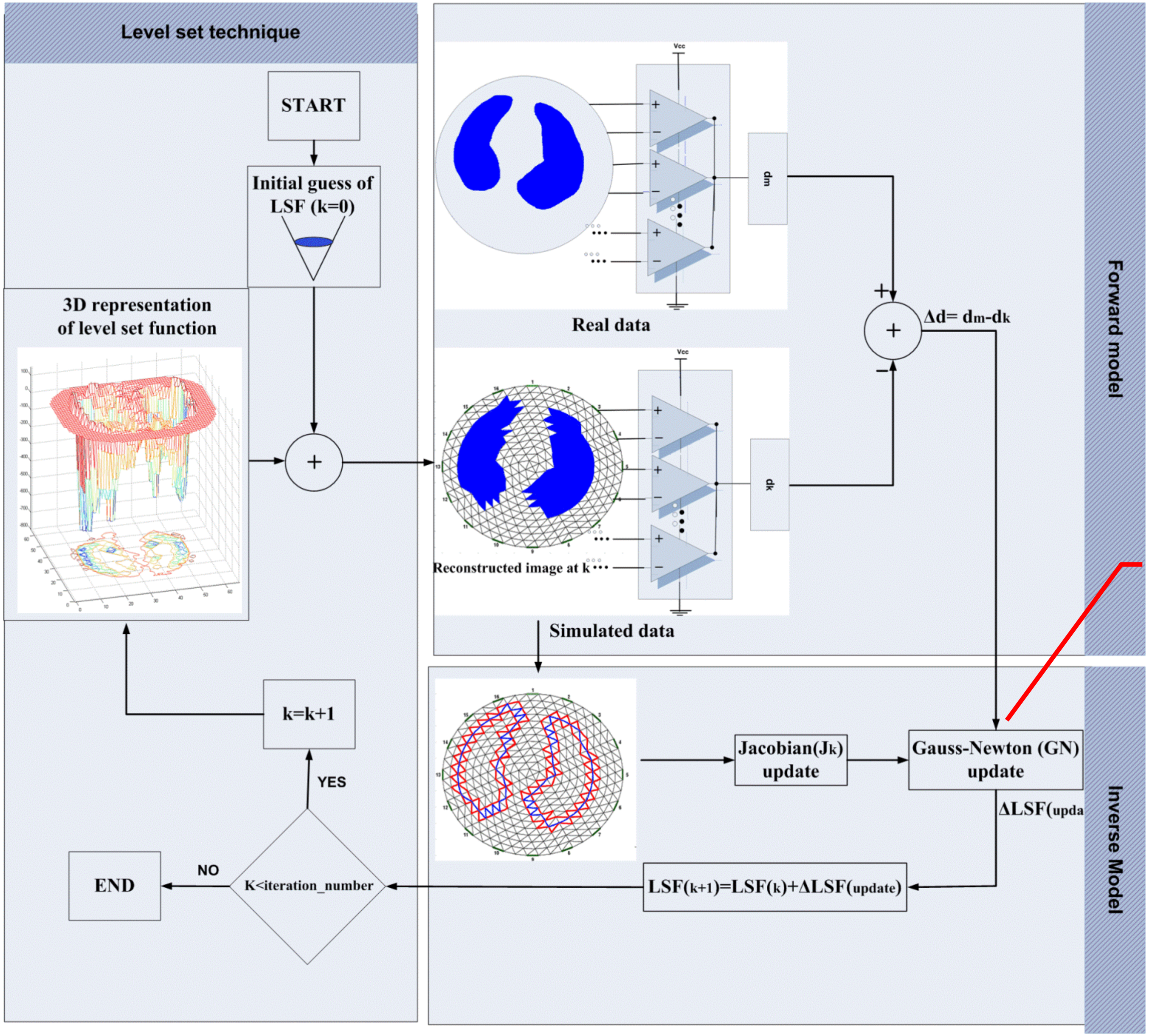
- Solution:

Flexible representations of the shapes is needed!

→ LEVEL SET technique can do this for us! ☺

What we proposed in 2011

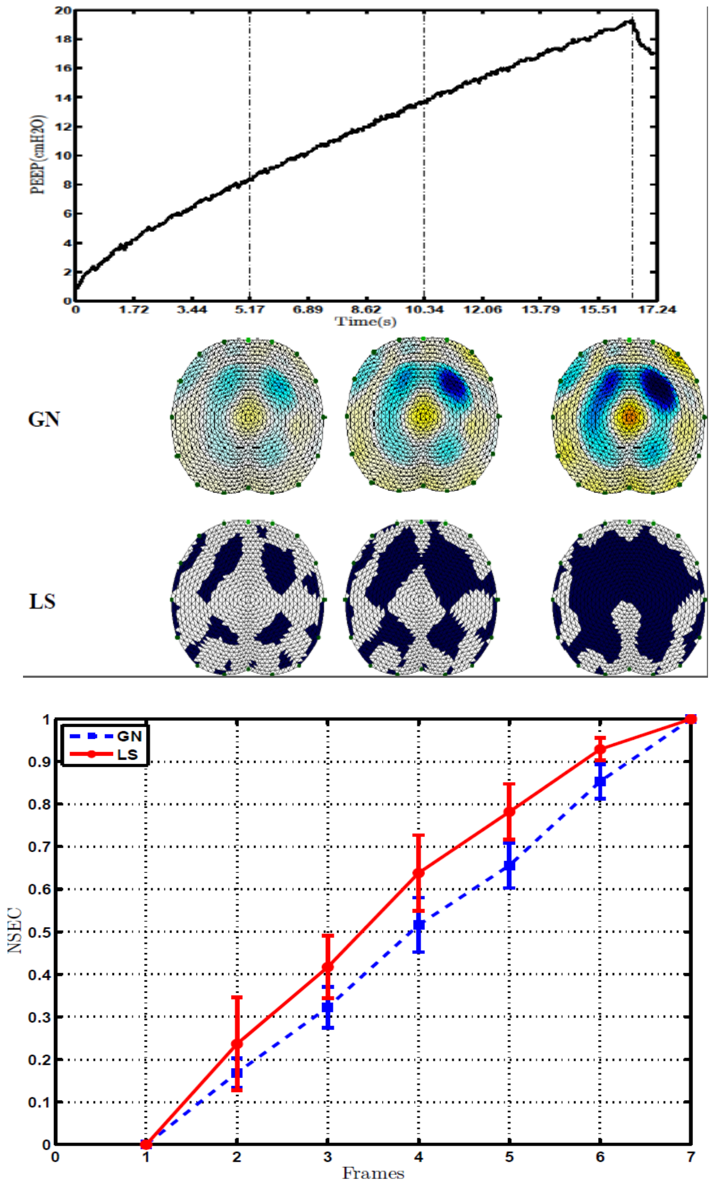
Level Set-based Reconstruction Algorithm using difference solver



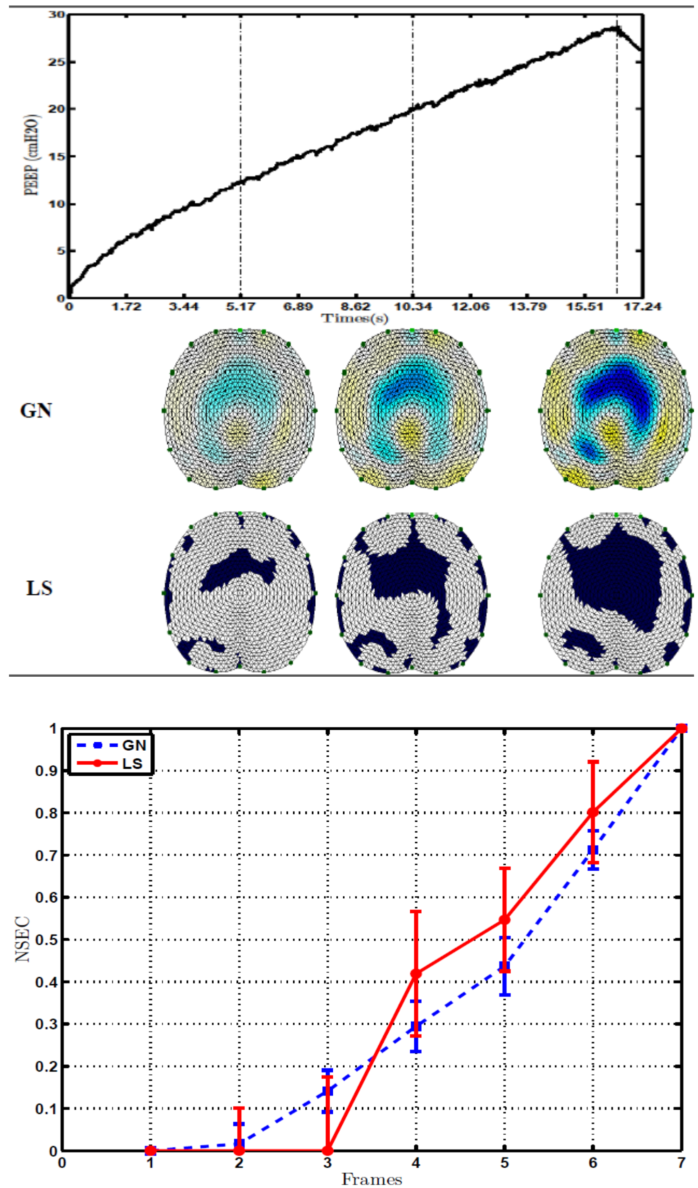
L2 NORM based update equation

Shape based reconstruction algorithm (L2 norm)_ First clinical results (2011)

Lung Healthy Patients (8 patients)



Acute Lung Injury Patients (18 patients)



Contribution #2: Level Set Technique for High Contrast Image Reconstruction (LSPDIPM, 2013)

- Problem:

L2 norms are sensitive to spatial noise and data outliers!

- Solution:

Redefine the cost functional based on L1 norms, instead of L2 norms:

$$\Delta\Phi(\Psi) = \left| d - h(\Phi(\Psi)) \right|_1^1 + \left| \Phi(\Psi) - \Phi(\hat{\Psi}) \right|_1^1$$

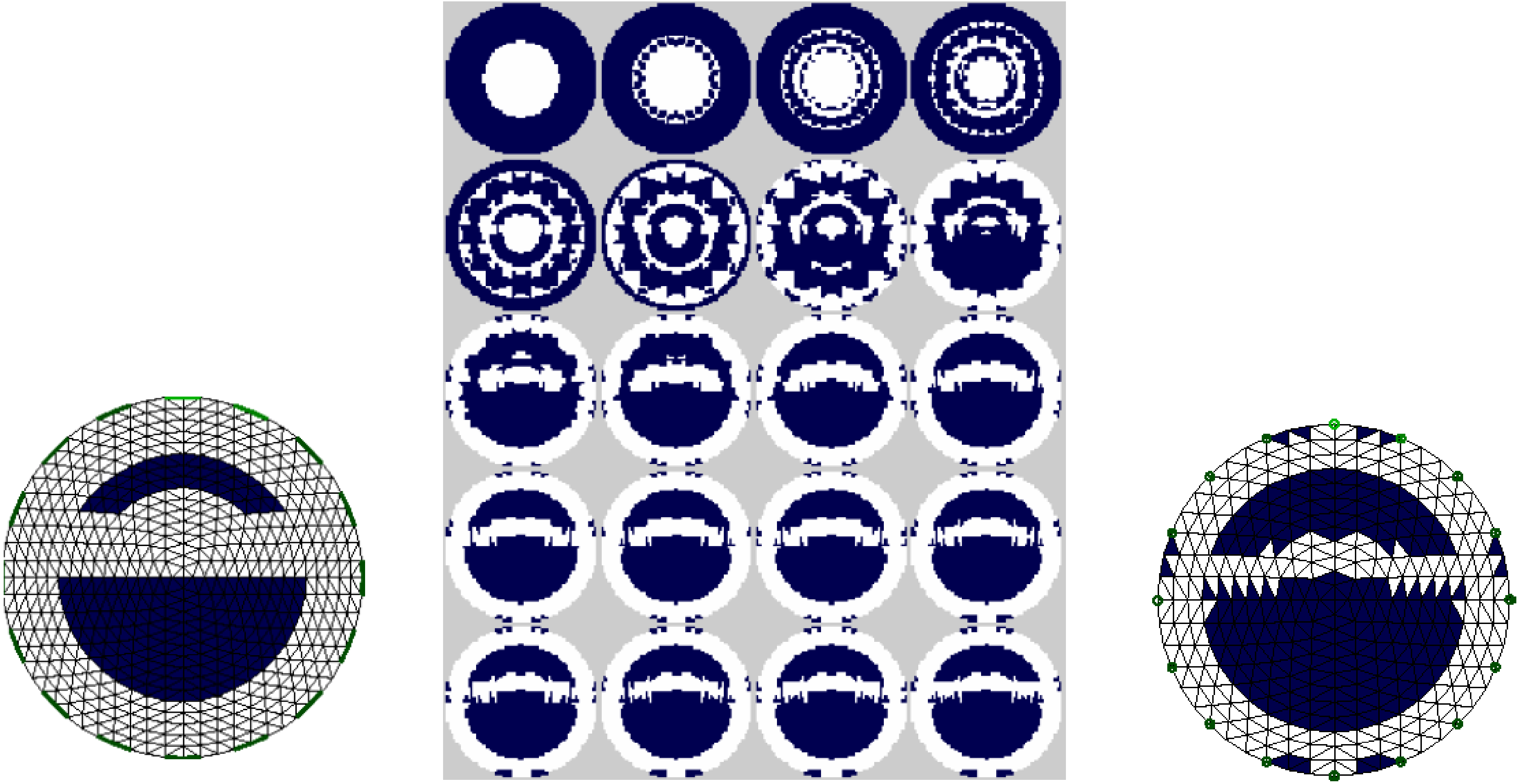
Where Φ is a mapping function such as a step function, and Ψ is the level set function,

- Difficulty:

Minimization of the L1 norm based cost functional is not computationally easy!

Primal-Dual Interior Point Method (PDIPM)

Results (LSPDIPM)



(a) (b) (c)

Contribution #3: A Generalized Inverse Problem with Weighted L1 and L2 Norms on Data and Regularization Terms (GPDIPM, 2013)

Error function:

$$\arg\min_x \{\Delta x = \zeta \underbrace{\left| h(x) - d_{real} \right|_1^1}_{\text{Weighted L1 norm based Data Term}} + (1-\zeta) \underbrace{\left\| h(x) - d_{real} \right\|_2^2}_{\text{Weighted L2 norm based Data Term}} + \eta \underbrace{\left| \lambda(x - x_{prior}) \right|_1^1}_{\text{Weighted L1 norm based Image Term}} + (1-\eta) \underbrace{\left\| \lambda(x - x_{prior}) \right\|_2^2}_{\text{Weighted L2 norm based Image Term}} \}$$

where ξ and η are weighting parameters within the range [0,1].

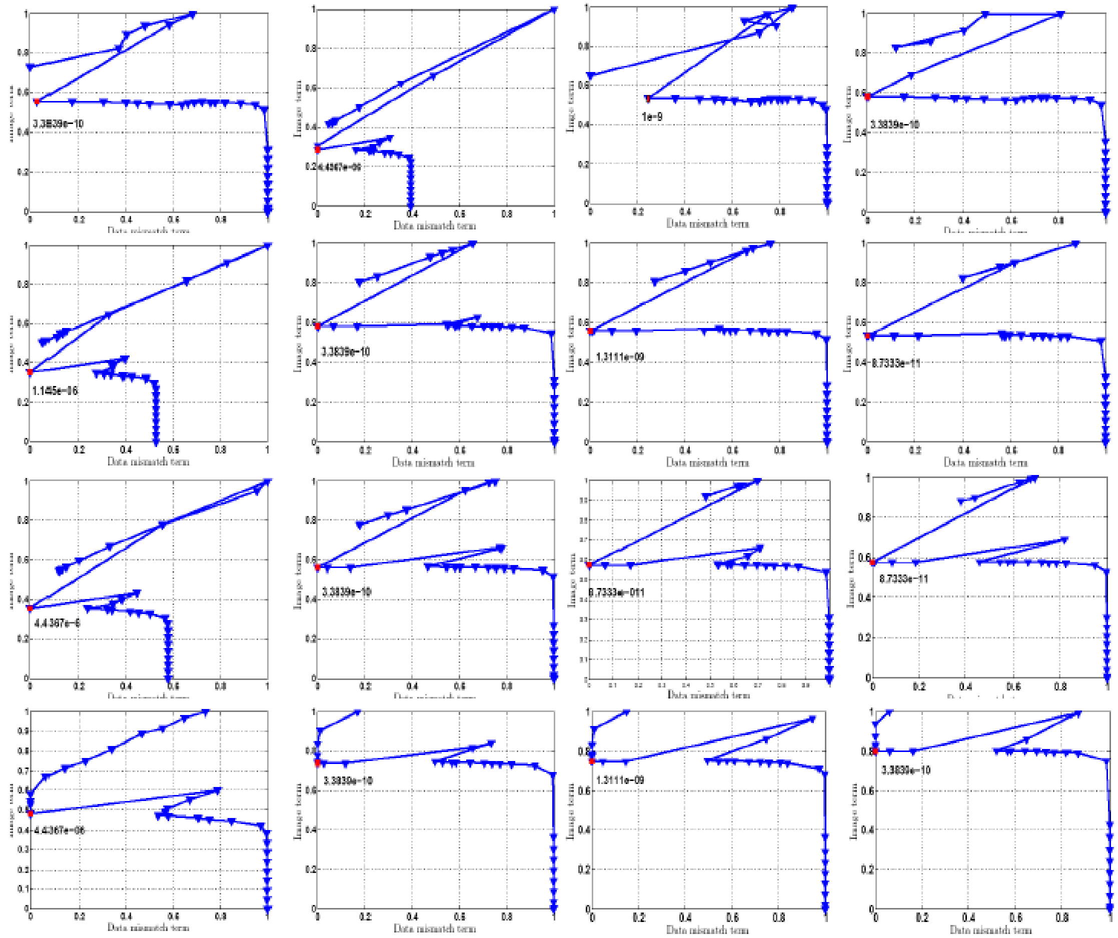
Weighting Parameters Selection

η \ ξ	0	0.3	0.6	1
0	L2+L2	(0.3*L1+0.7*L2+L2) ?	(0.6*L1+0.4*L2+L2) ?	L1+L2
0.3	(L2+0.3*L1+0.7*L2) ?	(0.3*L1+0.7*L2+0.3*L1 +0.7*L2) ?	(0.6*L1+0.4*L2+0.3*L1 +0.7*L2) ?	(L1+0.3*L1+0.7*L2) ?
0.6	(L2+0.6*L1+0.4*L2) ?	(0.3*L1+0.7*L2+0.6*L1 +0.4*L2) ?	(0.6*L1+0.4*L2+0.6*L1 +0.4*L2) ?	(L1+0.6*L1+0.4*L2) ?
1	L2+L1	(0.3*L1+0.7*L2+L1) ?	(0.6*L1+0.4*L2+L1) ?	L1+L1

Question to answer:

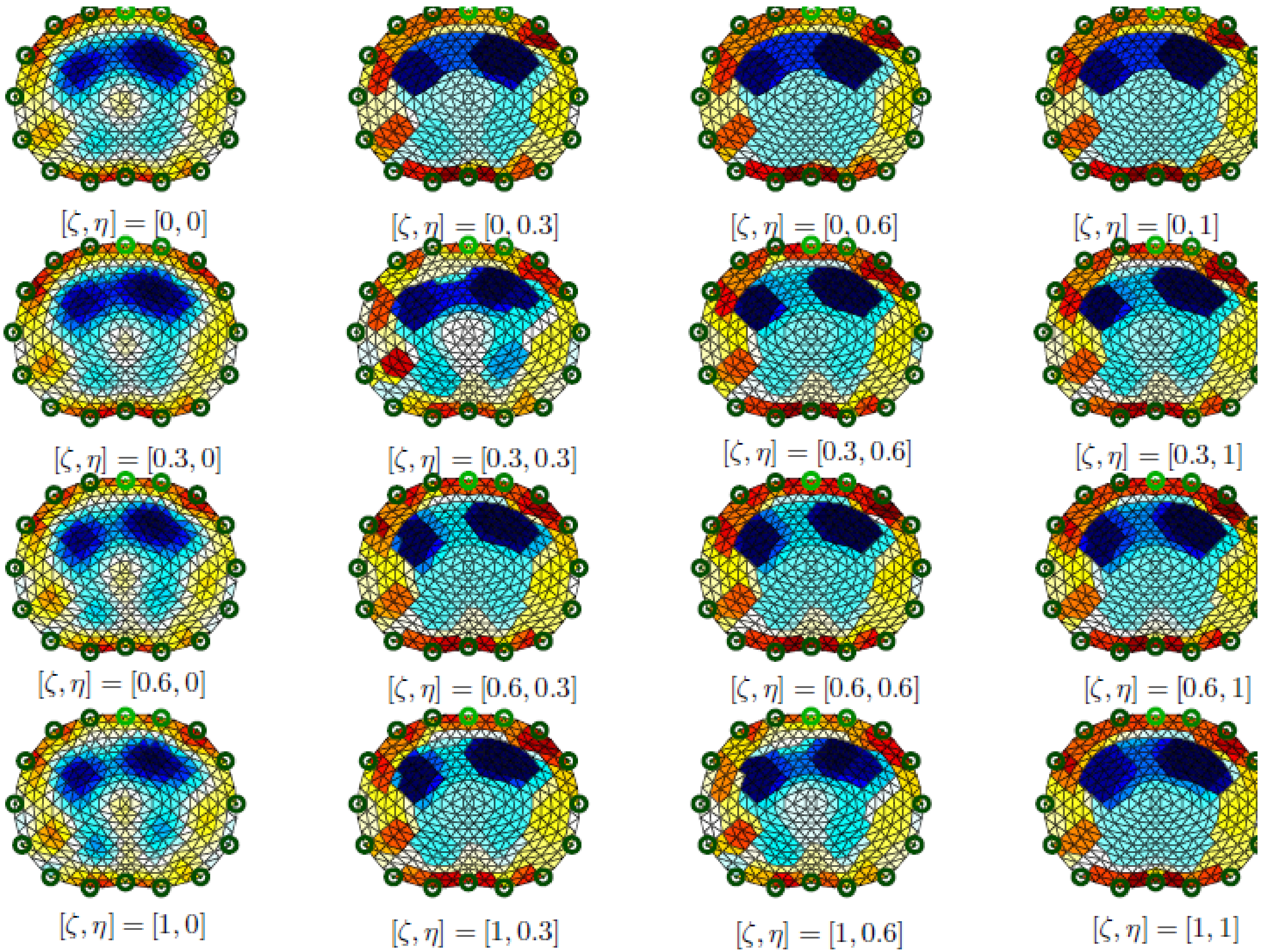
How different selection of weighting parameters affects the reconstructed image?!

Hyperparameter (λ) Selection using the L-Curve Method



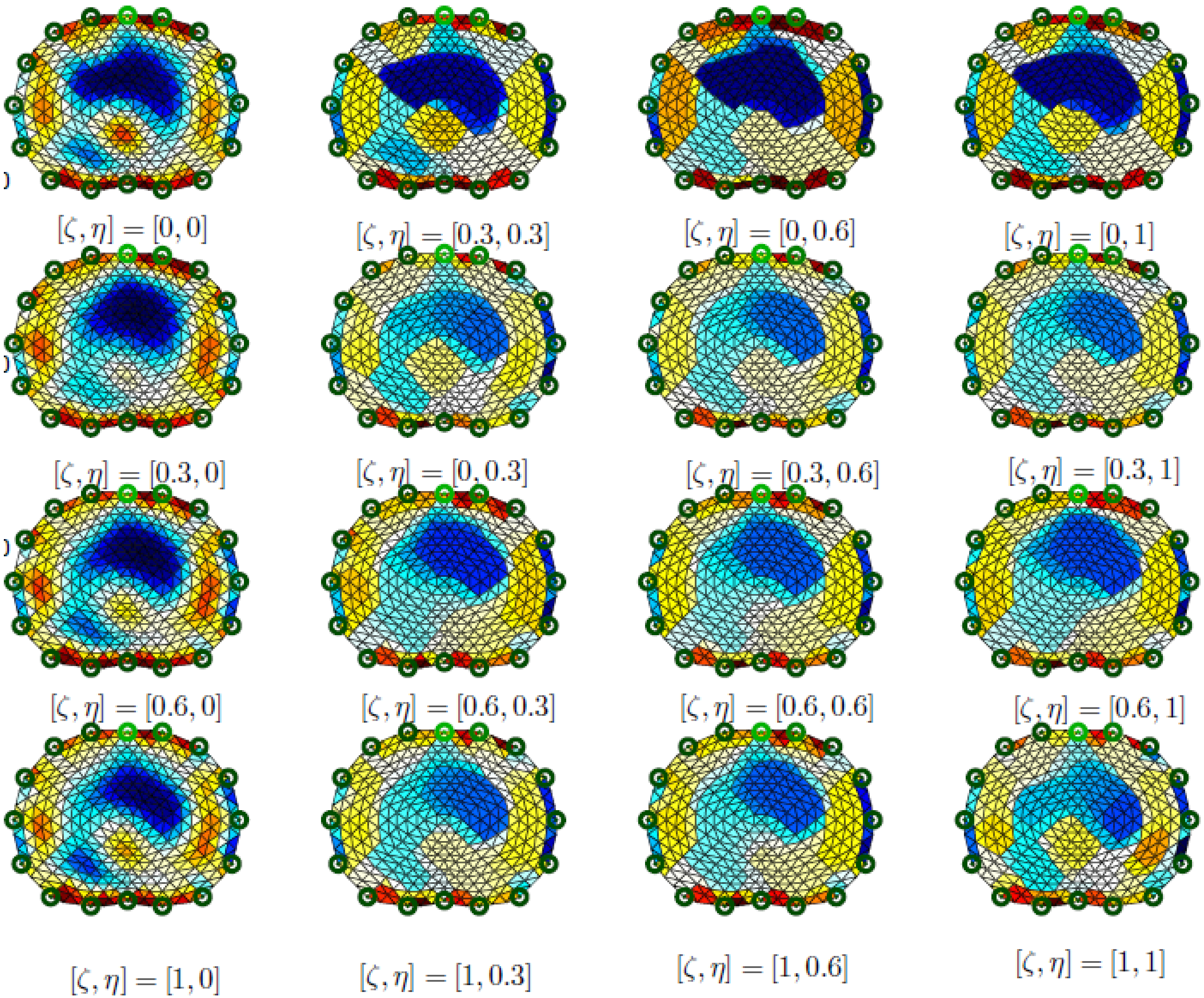
EIT Clinical data

Patient with healthy lungs



EIT Clinical data

Patient with Acute Lung Injury (ALI)

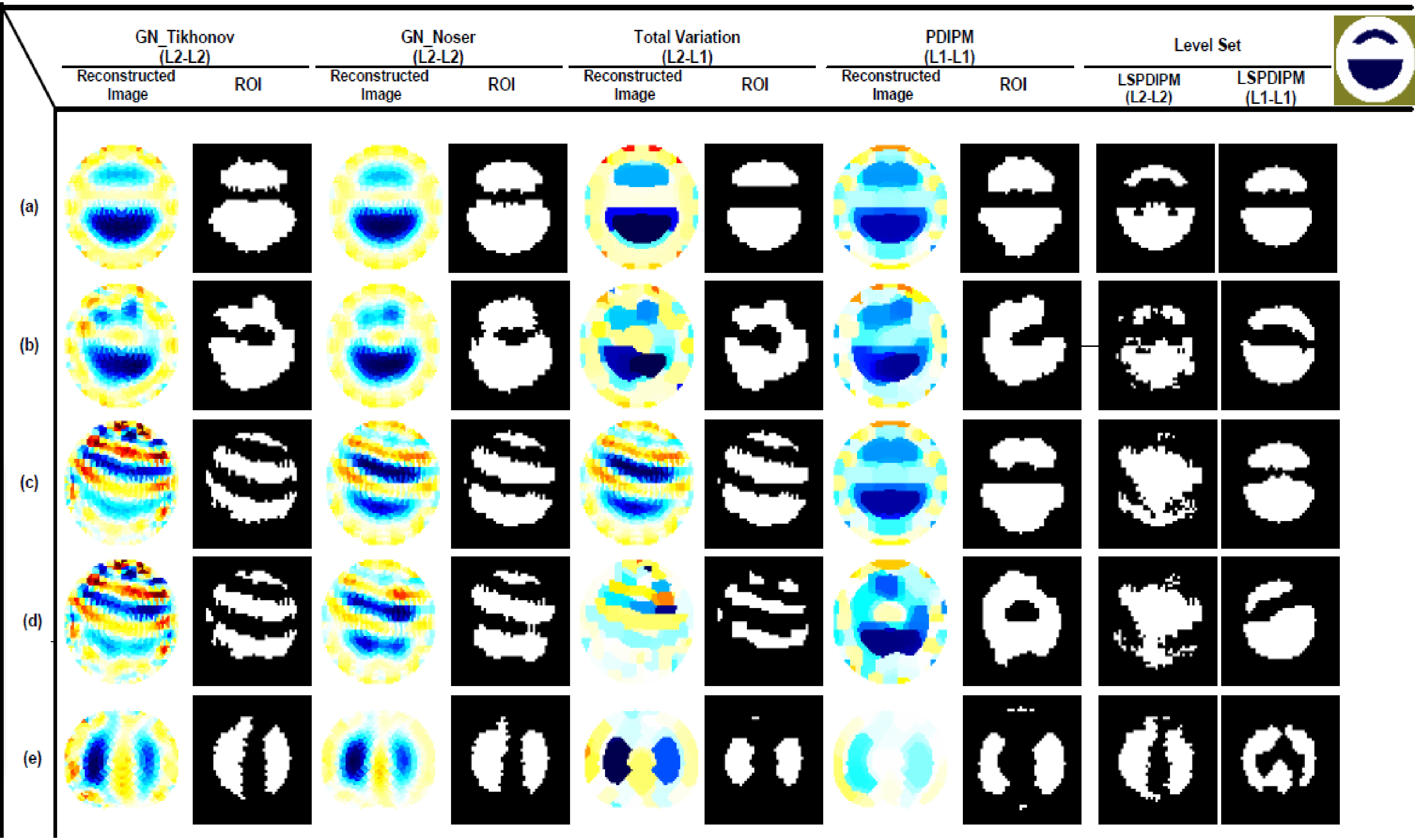


Evaluation Framework

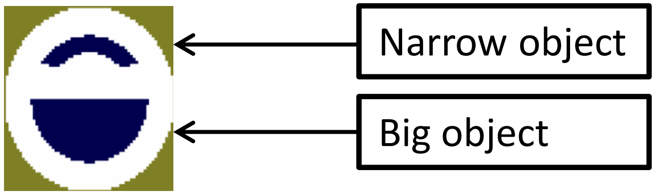
Qualitative Evaluation which includes ROI based comparisons.

Quantitative Evaluation which includes: Robustness metric (NM), and Morphological (or shape) Features.

Qualitative Evaluation



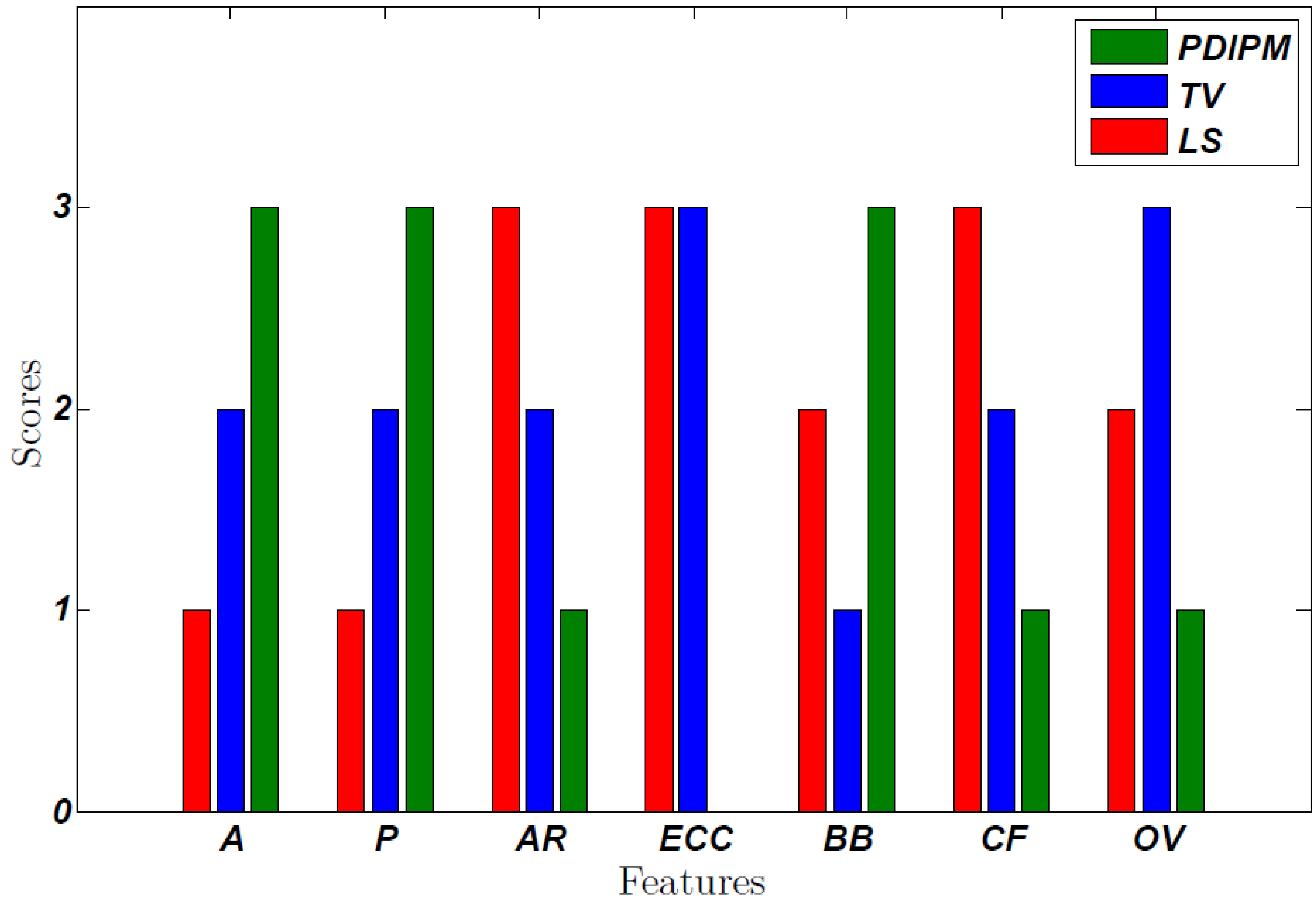
Quantitative Evaluation
(Accuracy Measurement using Morphological and Shape Features)

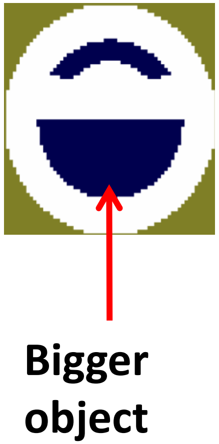


Reconstruction Algorithms	Inclusion	Morphological Features					Shape Features	
		Area	Perimeter	Axis-Ratio	Eccentricity	Bounding-Box	Compactness	Overlap
Ground Truth	Big object	734	113	1.889	0.848	[10.5,32.5,44,22]	0.2747	1
	Narrow object	200	82.4	3.571	0.960	[14.5,10.5,36,11]	0.6301	1
Tikhonov	Big object	925	142	1.757	0.822	[8.5,28.5,48,27]	0.4262	0.7781
	Narrow object	421	111	2.415	0.910	[14.5,7.5,36,17]	0.5654	0.4681
Noser	Big object	964	144	1.692	0.807	[9.5,28.5,46,28]	0.4148	0.7578
	Narrow object	432	91.6	2.291	0.9	[14.5,8.5,36,17]	0.3530	0.4630
TV	Big object	682	104	1.859	0.843	[12.5,32.5,40,21]	0.2067	0.9292
	Narrow object	370	84.3	2.66	0.927	[14.5,8.5,36,14]	0.3455	0.5405
PDIPM	Big object	780	121	1.709	0.811	[9.5,32.5,46,25]	0.3307	0.8878
	Narrow object	481	96.2	2.099	0.879	[14.5,7.5,36,18]	0.3466	0.4158
$LSPDIPM_{L2L2}$	Big object	676	137	1.854	0.843	[11.5,29.5,42,24]	0.55	0.889
	Narrow object	188	78.4	3.604	0.961	[15.5,11.5,34,10]	0.6159	0.8131
$LSPDIPM_{L1L1}$	Big object	602	98.8	1.92	0.854	[12.5,32.5,39,20]	0.2245	0.8202
	Narrow object	321	82.4	2.701	0.929	[14.5,11.5,35,14]	0.4063	0.4158

In each column, the light gray indicates the most accurate method, the medium gray shows the second accurate method, and the dark gray indicates the third accurate method!

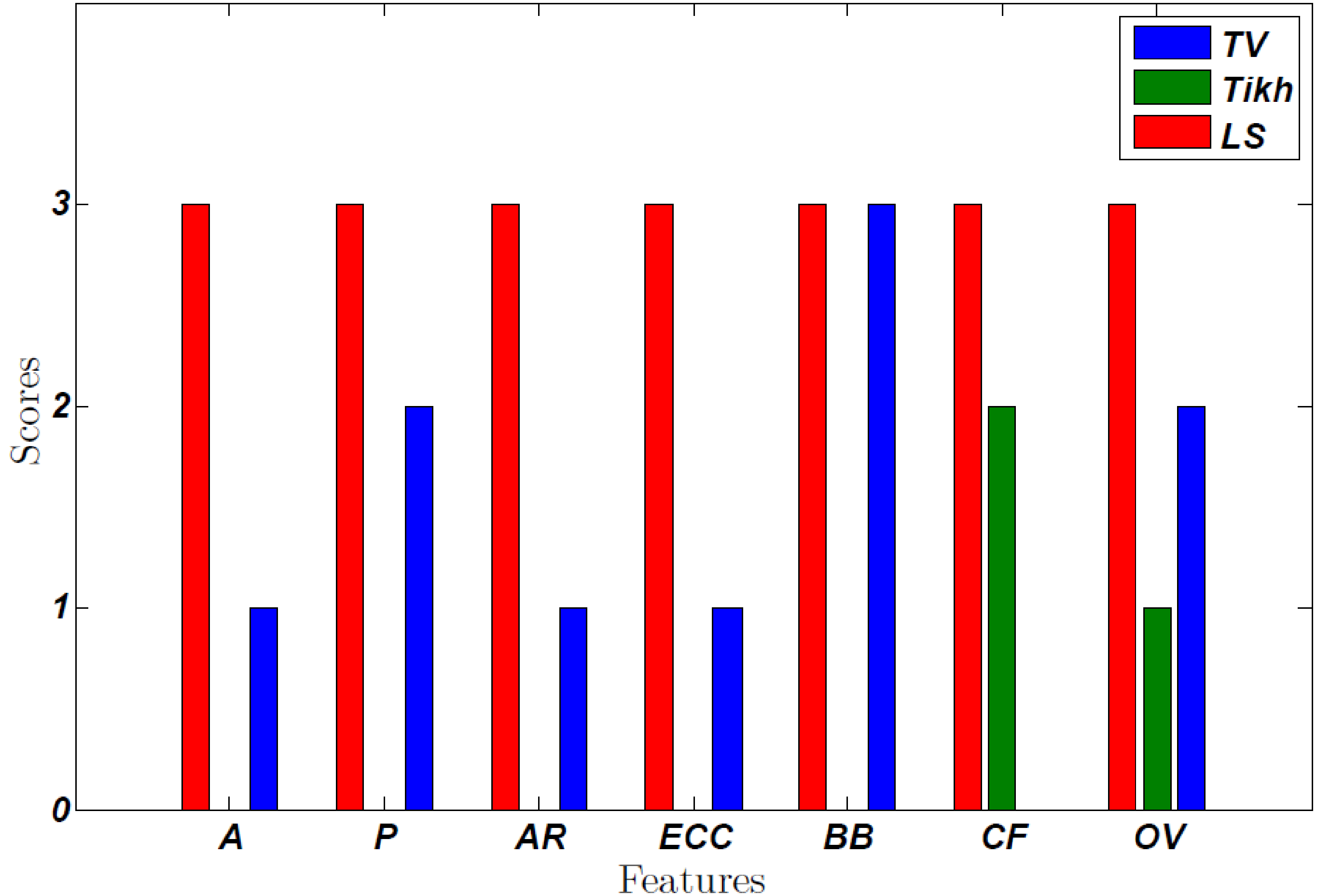
Quantitative Evaluation
(Accuracy measurement for the bigger object)

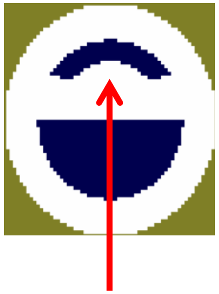




Overall average accuracy score of 2.57 (out of 3) for the proposed LSPDIPM, vs. 1.78 for the Total Variation

Quantitative Evaluation
(Accuracy measurement for the smaller object)





Smaller object

Overall average accuracy score of 3 (out of 3) for the proposed LSPDIPM, vs. 1.42 for the Total Variation!

Quantitative Evaluation (Robustness Measurement for the LSPDIPM)

Measurement Conditions	Reconstruction Algorithms	Robustness Metrics NM	NMB (dB)
Additional Noise	Tikhonov	3551	81.7
	Noser	2219	77.04
	TV	4006	82.95
	PDIPM	2718	79.07
	$\mathrm{LSPDIPM}_{L2L2}$	2408	77.86
	$\mathrm{LSPDIPM}_{L1L1}$	1398	72.42
Data Outliers	Tikhonov	1020	69.2
	Noser	882	67.8
	TV	1133	70.3
	PDIPM	112	47.1
	$\mathrm{LSPDIPM}_{L2L2}$	919	68.2
	$\mathrm{LSPDIPM}_{L1L1}$	79	43.6
Noise and Outliers	Tikhonov	973	68.8
	Noser	862	67.6
	TV	874	67.7
	PDIPM	414	60.2
	$\mathrm{LSPDIPM}_{L2L2}$	929	68.3
	$\mathrm{LSPDIPM}_{L1L1}$	307	57.2

Proposed LSPDIPM shows the highest robustness against the uncertainties with a NMB of 57.2 dB, vs. 60.2 dB for the PDIPM.

Quantitative Evaluation (Robustness Measurement for the GPDIPM)

Measurement Conditions	Reconstruction Method	Weighting Parameters	Robustness Metrics	
			NM	NMB (dB)
Additional Noise	Tikhonov	$[\zeta, \eta] = [0, 0]$	31006	103.4
	GPDIPM	$[\zeta, \eta] = [0, 1]$	12604	94.4
Data Outliers	Tikhonov	$[\zeta, \eta] = [0, 0]$	1009	69.1
	GPDIPM	$[\zeta, \eta] = [0.6, 1]$	24	31.7
Noise and Outliers	Tikhonov	$[\zeta, \eta] = [0, 0]$	816	67
	GPDIPM	$[\zeta, \eta] = [1, 0]$	631	64.4

Proposed GPDIPM with bigger values for either of its weighting parameters offers higher robustness against uncertainties.

For example, the GPDIPM with \zeta=1, \eta=0 has an NMB of 64.4 dB, vs. 67 dB for the traditional Tikhonov method.

Conclusion

- Novel edge-preserving image reconstruction methods (EPIRMs) using either level set technique or the L1 norm based inverse problems proposed.

- EPIRMs were applied on EIT clinical data and led to physiologically plausible results.

- An evaluation framework is proposed to measure the accuracy and the robustness of the EPIRMs against noise and data outliers.

- The Level set based reconstruction method using the L1 norms preserves the edges and is robust against noise and outliers (LSPDIPM).

Conclusion

- An overall average accuracy score of 2.57 (out of 3) for the proposed LSPDIPM, vs. 1.78 for the Total Variation.

- An average robustness score of 3 (out of 3), averaged over three different measurement conditions, for the proposed LSPDIPM, vs. 1.33 for the PDIPM.

- The proposed GPDIPM with bigger values for either of its weighting parameters (\zeta or \eta) tends to offer higher robustness against the uncertainties (noise and outliers).

- The L1 norm minimization is computationally expensive (10-15 iterations).

THANK YOU

Future work: Combining Level sets and Primal-Dual Interior Point Framework for Image Reconstruction in Inverse Problems.

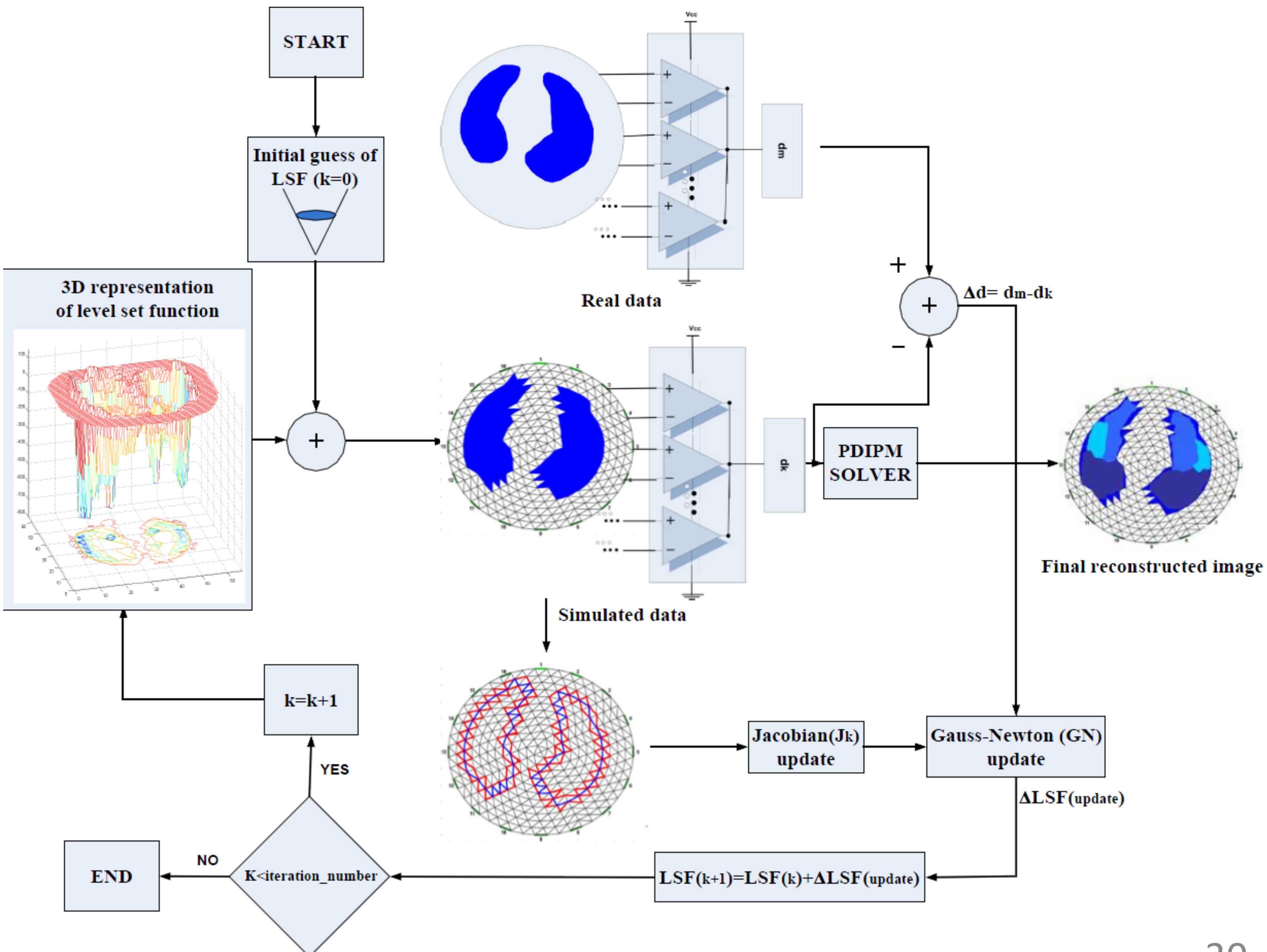
- Problem:

  The assumption of constant piece-wise pixel illumination values is to discriminate between two regions with sharp intensity transition. **However**, it may not be a realistic assumption when there are smooth conductivity gradients inside each region as well.

- Solution:

  Hybrid regularization method (HRM), which is a two steps solution, to solve ill-posed, non-linear inverse problem containing both sharp and smooth coefficients.

Hybrid Regularization Method



Deformable model regularization method.

- Problem:

Due to the boundary movement, it is not only the electrodes which change their coordination but the nodes inside the mesh as well.

- Solution:

The following model parameters are calculated as part of the inverse solution:

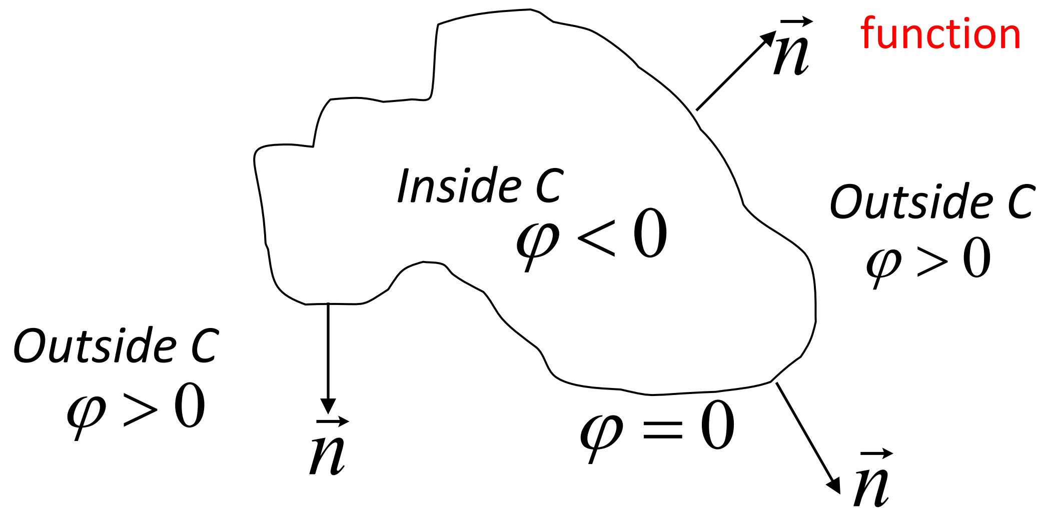
- 1) The conductivity image,
- 2) The electrode displacements,
- 3) The node displacements inside the mesh.

The formulation of such deformable model regularization method is proposed in this thesis as future work.

Level Set Representation

$$C = \{(x, y) \mid \varphi(x, y) = 0\}$$

Level set function is a Signed distance function



C= boundary of an open domain

Advantages

- Automatically detects interior contours!

- Works very well for concave objects

- Allows for automatic change of topology

Proposed GPDIPM

- Advantages:
  - The weighted L1 and L2 norms can be independently applied over the data mismatch and the regularization terms (image term) of an inverse problem.
  - Preserve edges (non-smooth optimization),
  - Robust against measurement errors (noise and outliers).

- Difficulty:
  - Computationally more expensive than the GN method.

    GN (Non-linear) ⇨ 3-5 iterations

    GPDIPM (Proposed method) ⇨ 10-15 iterations

Traditional Image reconstruction method

We need to minimize the following error function to find the best estimate of x, which is the solution of inverse problem.

Error function:

$$\arg\min_x \{\Delta x = \left\| \overbrace{h(x) - d_{real}}^{\text{Data mismatch term}} \right\|_m^m + \left\| \overbrace{\lambda(x - x_{prior})}^{\text{Image mismatch term}} \right\|_n^n \}$$

where

- m, n =1 (L1 norm) or 2 (L2 norm) ,
- f(x) is measured data,
- d_{real} is real data,
- x is pixel intensity,
- x_{prior} is expected pixel intensity.
- λ is the regularization factor.

Traditional Image reconstruction method

One-step Gauss Newton method (L2 norm)

Advantage:

- Simple to implement,

Drawbacks:

- Smoothed edges,
- Sensitive to measurement errors (noise+outliers).