Variability in FEM mesh geometry

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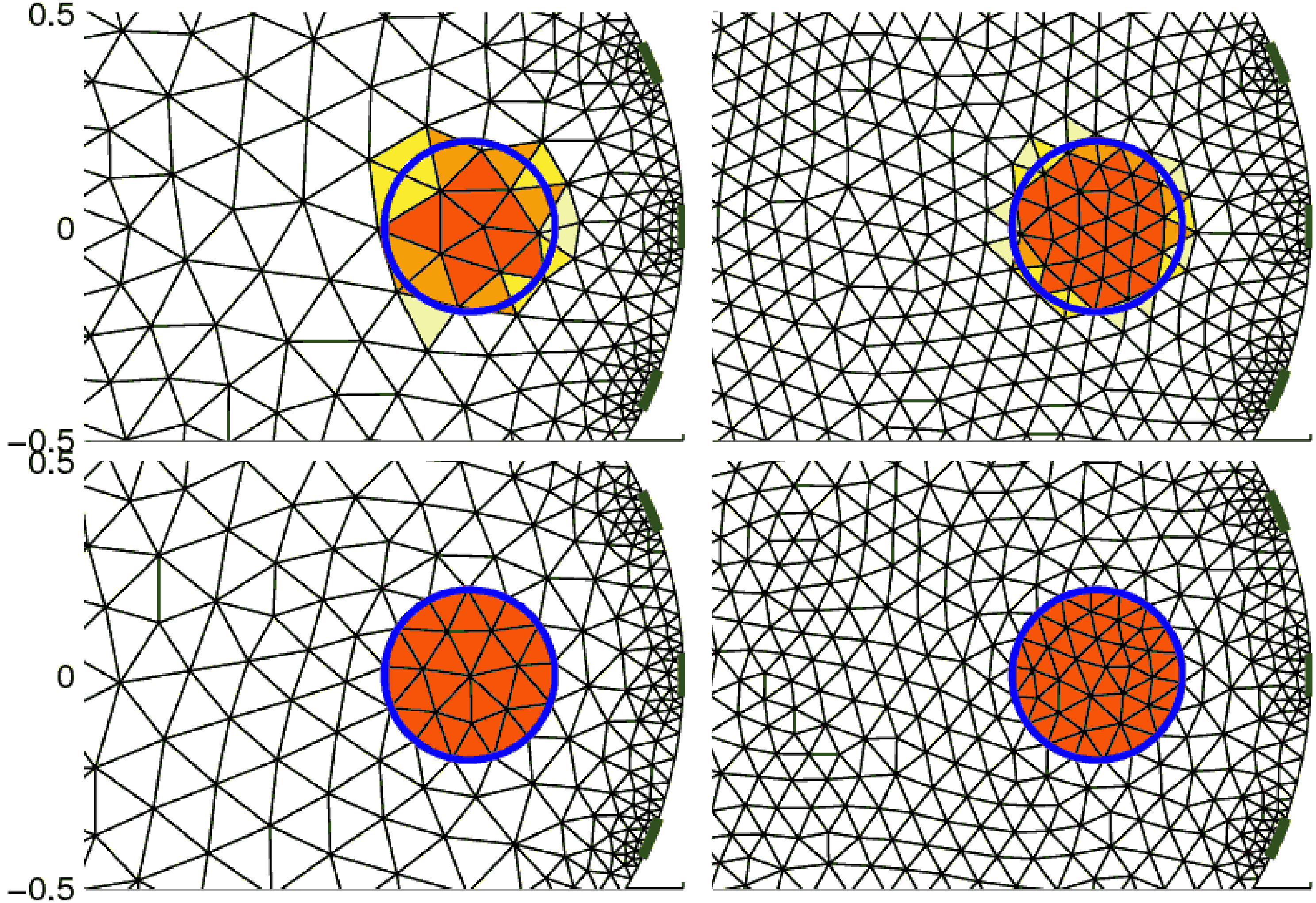
Simulating with FEMs

Interpolated Mesh

Adapted Mesh

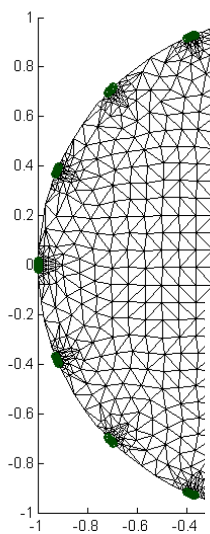


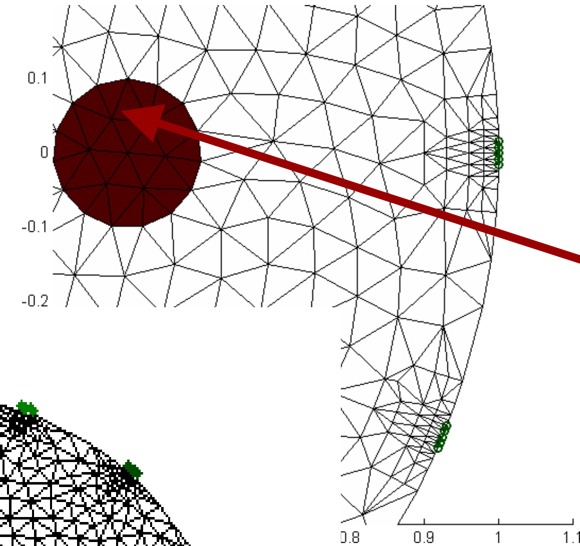
Coarse (1411 elems)

Fine (1941 elems)

What's the problem?

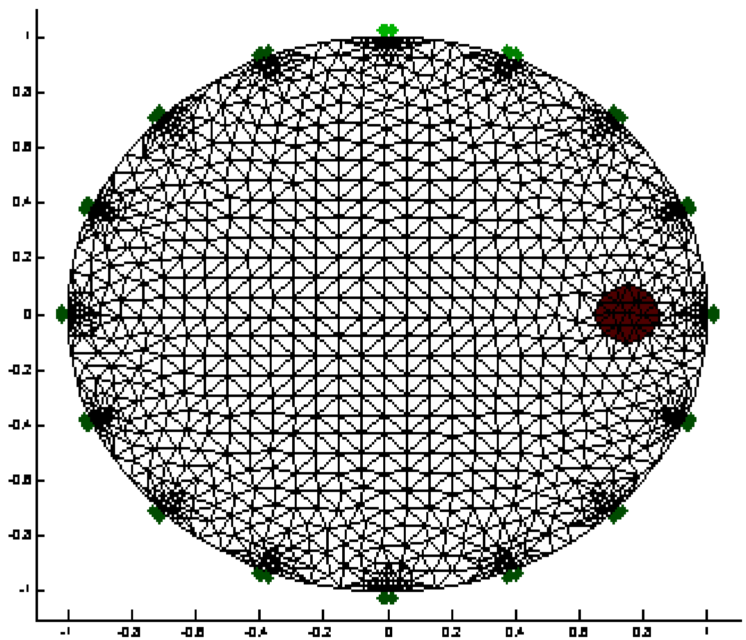
Correct simulation: remesh at each target





Geometry changes with targets

Animation

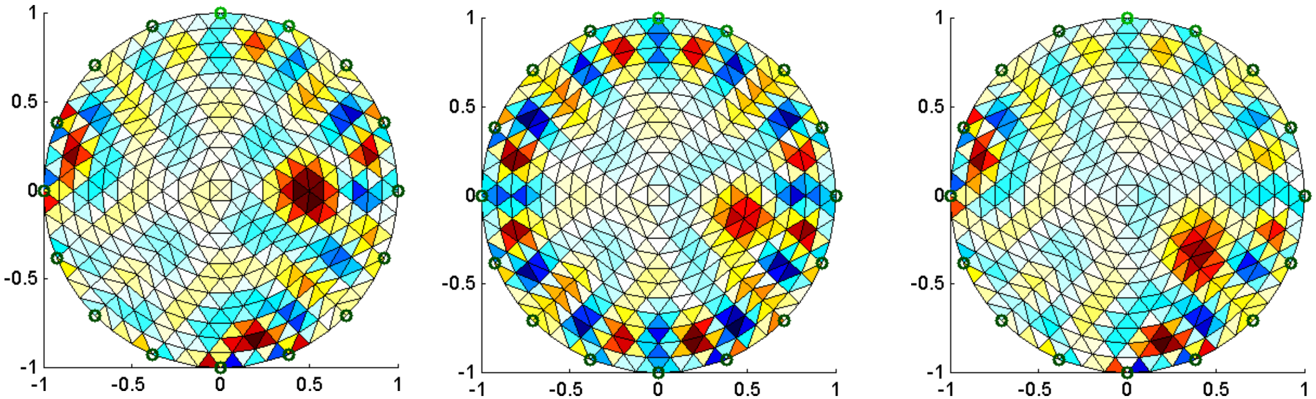


Difference imaging with changing FEM shows the model accuracy effects

What’s the problem?

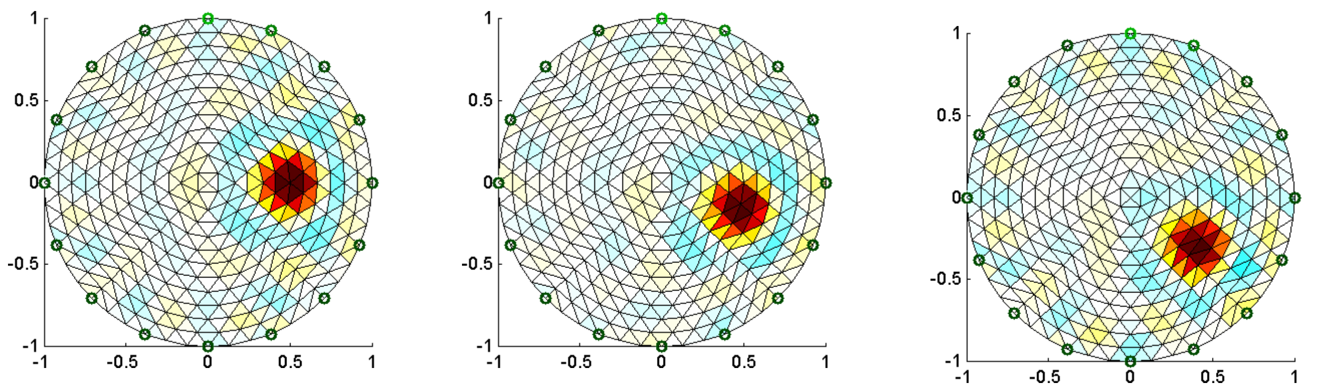
Images are awful

2000 elements

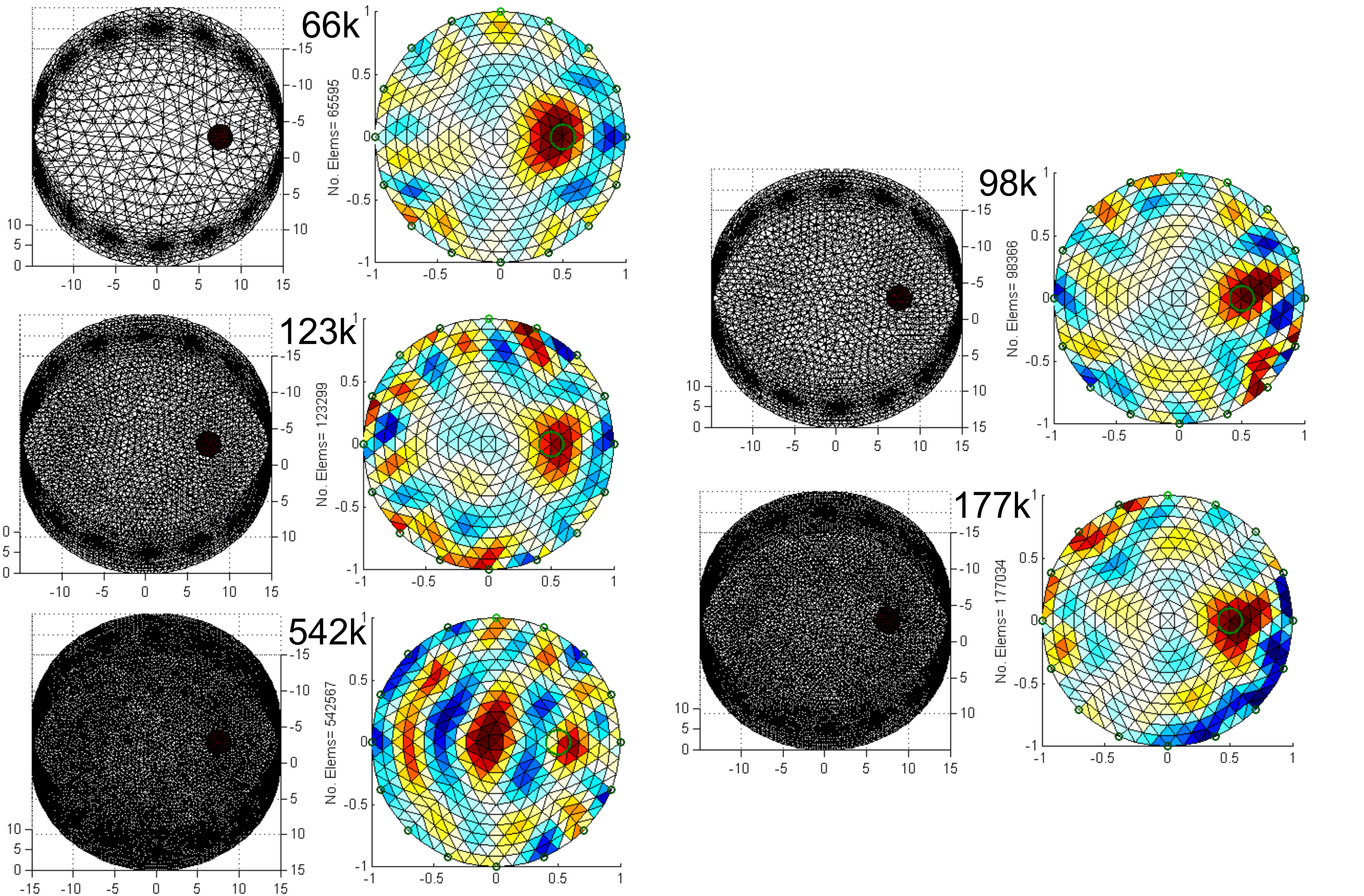


Unless you use fine FEMS

10000 elements



But it’s worse in 3D



What's causing this?

- Due to mapping of anisotropic conductivity
- A particular FEM geometry maps the (potentially anisotropic) conductivity tensors onto the system matrix
- Change in geometry projects tensor differently
- Result: slightly different *anisotropic content* into the simulated voltages.
- Reconstruction can't explain anisotropy, so projects it as noise/artefacts

How can we fix it?

- The problem is that the reconstruction can't vary the geometry to explain anisotropy.
- What if we allow the reconstruction to '*jiggle*' the vertex locations?

Formulation

$$\Delta\hat{\boldsymbol{\sigma}} = \begin{cases} \boldsymbol{\Sigma}_c \mathbf{J}_c^t \left(\mathbf{J}_c \boldsymbol{\Sigma}_c \mathbf{J}_c^t + \boldsymbol{\Sigma}_n \right)^{-1} \mathbf{y} & \text{standard} \\ \boldsymbol{\Sigma}_c \mathbf{J}_c^t \left(\mathbf{J}_c \boldsymbol{\Sigma}_c \mathbf{J}_c^t + \mathbf{J}_m \boldsymbol{\Sigma}_m \mathbf{J}_m^t + \boldsymbol{\Sigma}_n \right)^{-1} \mathbf{y} & \text{proposed} \end{cases}$$

J_c – Jacobian (Sensitivity) to Δconductivity

J_m – Jacobian (Sensitivity) to Δposition

Σ_c – Image element covariance (Prior)

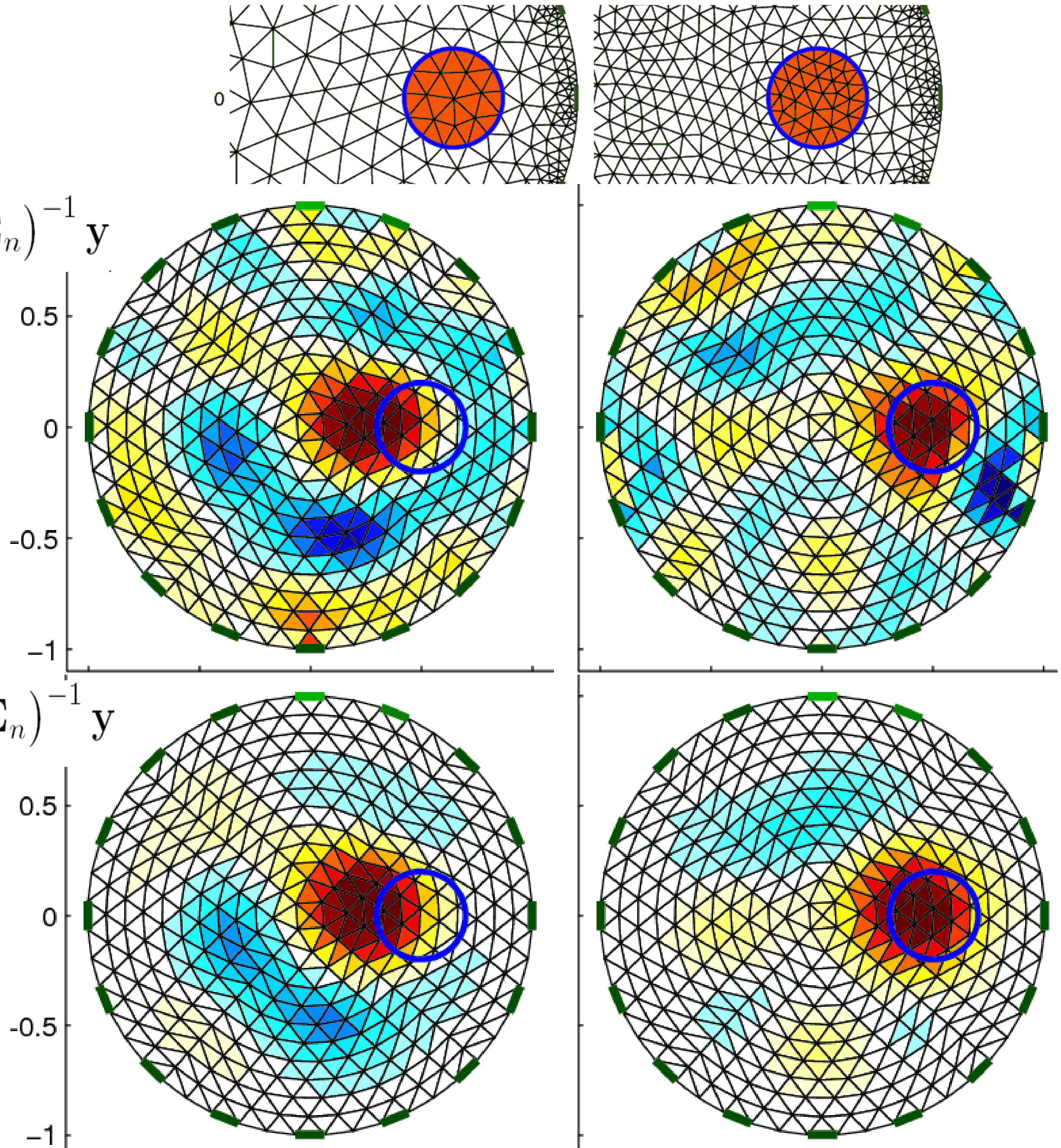
Σ_n – Channel noise variance

Results

Coarse (1411 elems) | Fine (1941 elems)

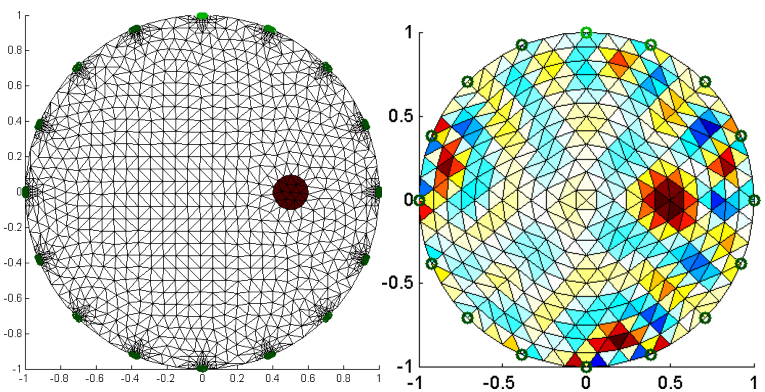
$$\boldsymbol{\Sigma}_c \mathbf{J}_c^t \left(\mathbf{J}_c \boldsymbol{\Sigma}_c \mathbf{J}_c^t + \boldsymbol{\Sigma}_n \right)^{-1} \mathbf{y}$$

$$\boldsymbol{\Sigma}_c \mathbf{J}_c^t \left(\mathbf{J}_c \boldsymbol{\Sigma}_c \mathbf{J}_c^t + \mathbf{J}_m \boldsymbol{\Sigma}_m \mathbf{J}_m^t + \boldsymbol{\Sigma}_n \right)^{-1} \mathbf{y}$$

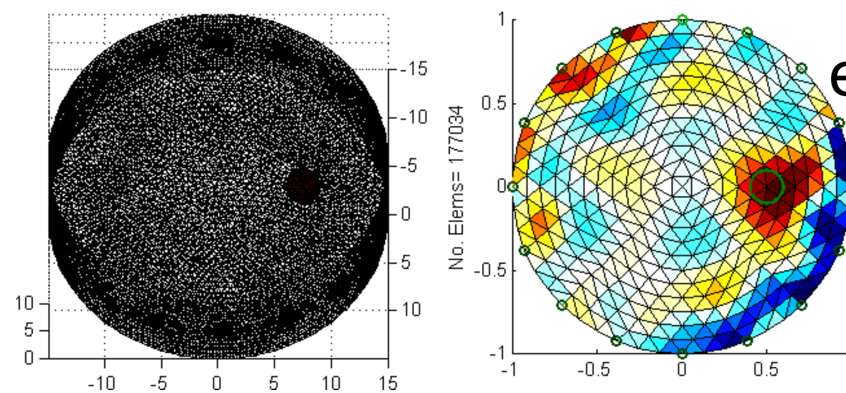


Discussion

- We think we can explain an effect that has bugged us for several years
- This may be significant. Especially for absolute imaging, and for 3D solvers



2D
2k elems



3D
177k elems

Announcing EIDORS v3.4

	Version	Lines of Code
1999	1.0 (2D)	1314
2002	2.0 (3D)	3715
2005	3.0	10685
2006	3.1	14850
2007	3.2	18127
2008	3.3	23437
2010	3.4	36554