Image Reconstruction in EIT Using Advanced Regularization Frameworks

Tao Dai

Supervisor: Dr Andy Adler

Systems and Computer Engineering
Carleton University, Ottawa, Canada
EIT System
Forward Model (linearized)

\[ y = Jx + n \]

This is an underdetermined system
Inverse Model (linearized)

The image $X$ is the unknowns to be calculated:

$$\hat{x} = J^{-1}y \quad \text{NOT realizable!}$$

$$\hat{x} = (J^T J)^{-1} J^T y \quad \text{Naïve Solution, NOT stable!}$$

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**Hadamard Criteria**

1. solution existence.
2. solution uniqueness.
3. solution stability.

Solution stabilization -- Regularization
\[
\hat{x} = \operatorname*{arg\,min}_{x} \|y - Jx\|_{\Sigma_n^{-1}}^2 + \|x - x_0\|_{\Sigma_x^{-1}}^2
\]
Problems

- Image quality of EIT is poor.
  - Low spatial resolution
  - High noise level
  - Large artefacts

- EIT is sensitive to system and measurement errors
  - Model error
  - Electrode movement
  - Electrode malfunction
Objectives

- General objective:
  Develop algorithms to improve EIT image reconstruction performance

  - Object 1: improve image quality
  
  - Object 2: improve robustness against system and measurement errors
M1: Temporal Regularization

Current image is correlated to past and future images

\[ \gamma^n \text{ is the interframe correlation between two images with delay } n \]

- \( \gamma^n \) is the interframe correlation between two images with delay \( n \)


M1 Application: Temporal Regularization on electrode Motion Analysis

- Temporal reconstruction of conductivity changes and electrode movements simultaneously

\[ y = Jx \]

M2: 4-D regularization

- Temporal and 3-D spatial regularization

M3: Iterative Solution for L1 Norm Minimization

\[ \hat{x} = \text{argmin}_{x} \|y - Jx\|_2^2 + \|x - x_0\|_2^2 \]

Linearizable as fast one-step reconstruction

\[ \hat{x} = \text{argmin}_{x} \|y - Jx\|_1 + \|x - x_0\|_1 \]

Robust against measurement errors

Edge preservation, less artefacts

• “Electrical impedance tomography reconstruction using l1 norm on data and image terms”
Miscellaneous contributions

- A scheme of in vivo blood characterization using bioimpedance spectroscopy

To develop a novel in vivo measurement technique to calculate bioelectrical properties of blood

- Bioimpedance spectroscopy
- Cole-Cole model
- Nonlinear curve fitting

Miscellaneous contributions

- Variable step size affine projection algorithm with a weighted and regularized projection matrix
  - This is part of iterative system identification research
  - Temporal weights in the projection matrix
  - Tracking the latest behaviour of error signal.

- Any Questions?
- Thank you very much!!!
Publications: Peer-Reviewed Journals


Publications: conferences


Direct temporal forward model

\[
\begin{pmatrix}
0 \\
\vdots \\
0
\end{pmatrix}
\times
\begin{pmatrix}
\begin{bmatrix}
\text{Measurement sequence}
\end{bmatrix}
\end{pmatrix}
= 
\begin{pmatrix}
\begin{bmatrix}
\text{Augmented Jacobian}
\end{bmatrix}
\end{pmatrix}
\times
\begin{pmatrix}
\begin{bmatrix}
\text{Image sequence}
\end{bmatrix}
\end{pmatrix}
\]
Direct temporal inverse model
Normally, non-diagonal elements are zeros based on assumption that images are independent.
Temporal Boundary Element Motion Analysis

GN Method with EM calculated (without temporal regularization)

GN Method with EM calculated (with temporal regularization)
Contributions
**Advantages:**

1. Edge preservation
2. Data error robustness

(a)L-2 norm solution   (b)L-1 norm solution
M3: Iterative Solution for L1 Norm Minimization

\[ x_k = x_{k-1} + w_k (y - Jx_{k-1}) \]
Parametric images for virtual biopsy:

Multi-frequency EIT

\[ m = \begin{bmatrix} R_0 & R_{\infty} & f_c & \alpha \end{bmatrix} \]

Multi-frequency EIT
EIT: A phantom experiment
Use BIS to differentiate prostate tissues
Upper: different spectra; Lower: different parametric characteristics

**CaP:** Cancerous prostate; **BPH:** Benign Prostatic Hyperplasia
**Gl:** Glandular tissue; **Str:** Stroma