Four-Dimensional Regularization for EIT Imaging

Tao Dai[1], Manuch Soleimani[2], Andy Adler[1]

[1]Systems and Computer Engineering, Carleton University, Ottawa, Canada

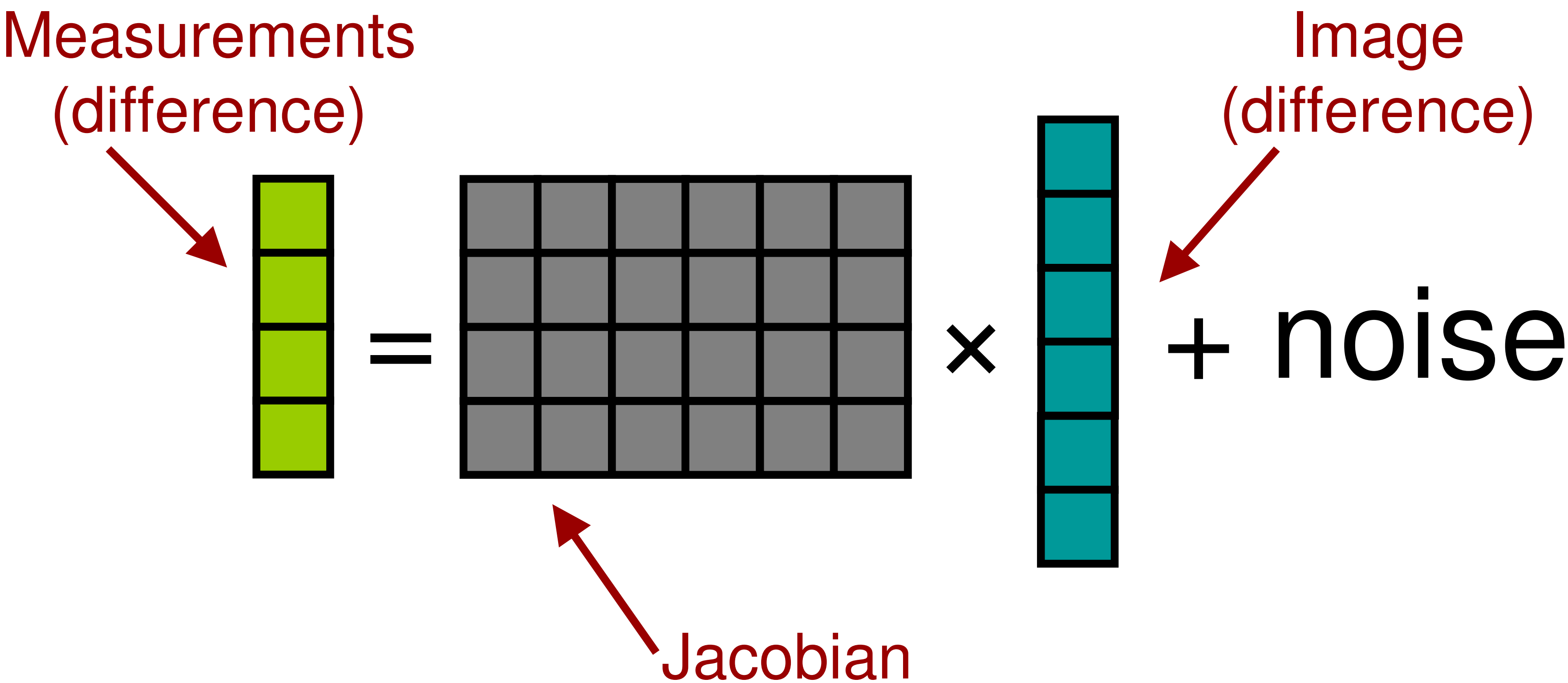
[2]Electronic & Electrical Engineering
University of Bath, U.K.

Motivation

- Traditionally, EIT treats model elements independently, we use correlations among elements as *a prior*
- One good thing about EIT is that the data collection is fast (up to 1000 fps)
- It should be possible to use the *temporal* information to improve the images

Image Reconstruction

- Forward Model (linearized)



System is underdetermined

Image Reconstruction

- Inverse Model (linearized)

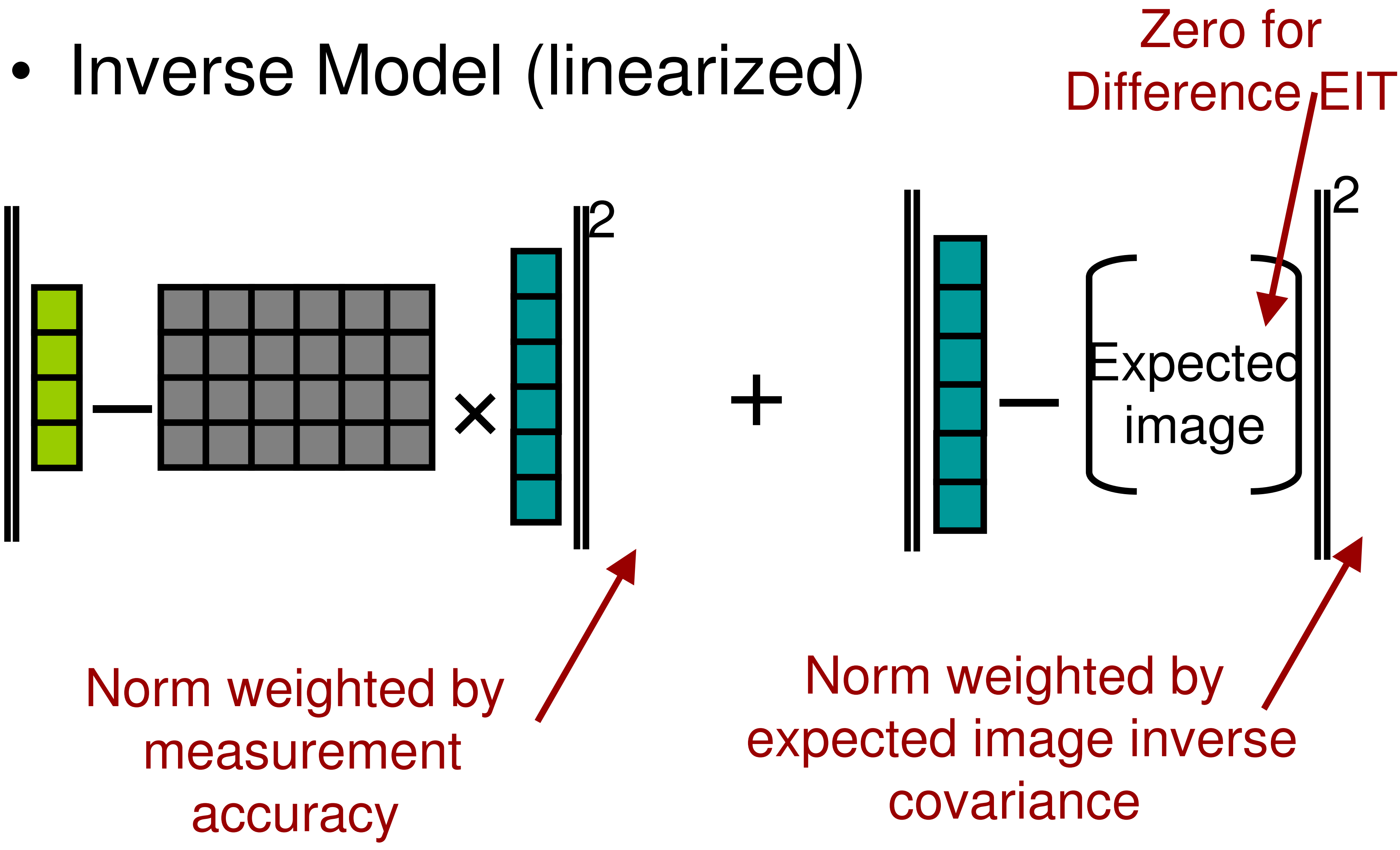


Image Reconstruction

- Penalty functions: Image Amplitude

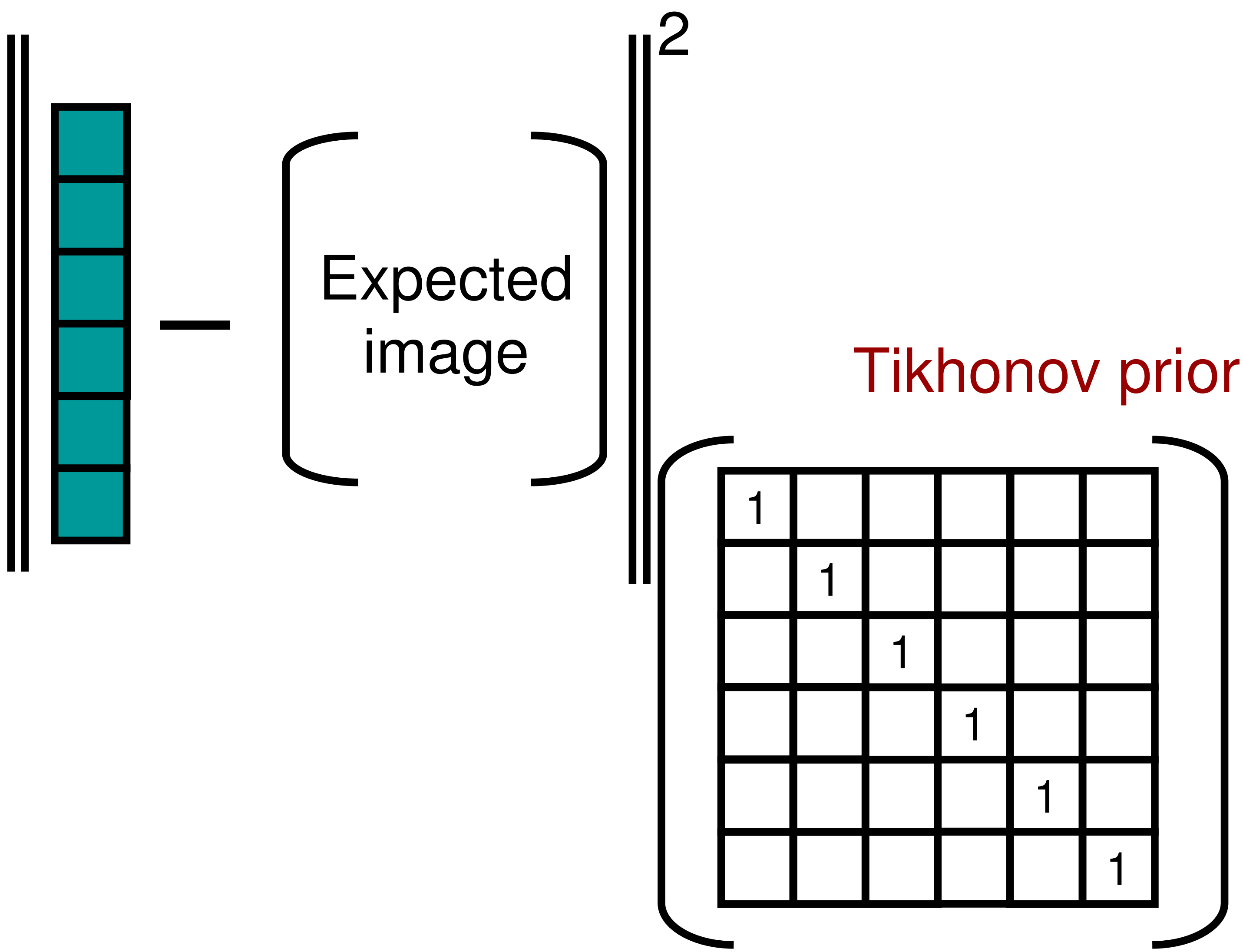
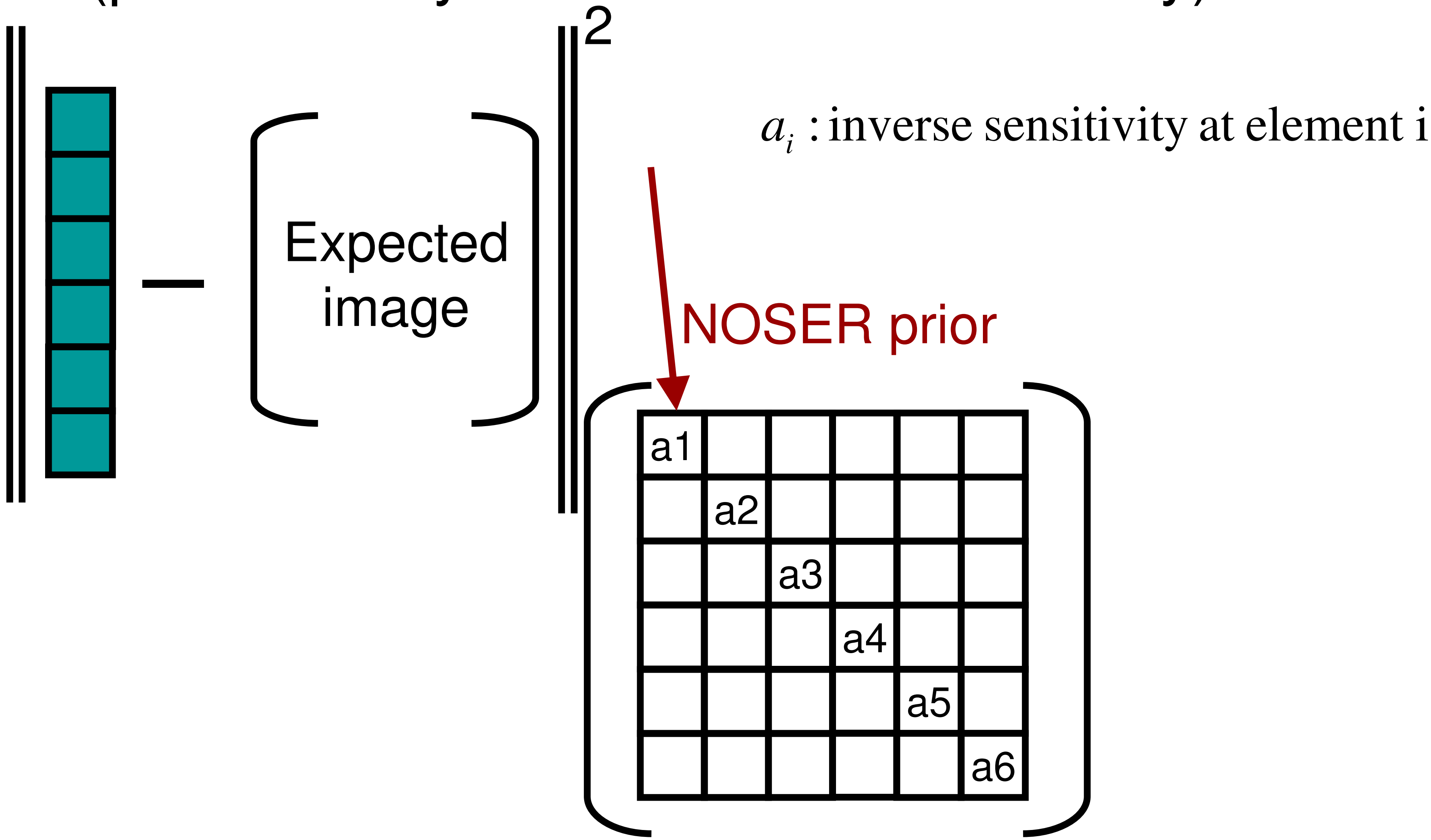
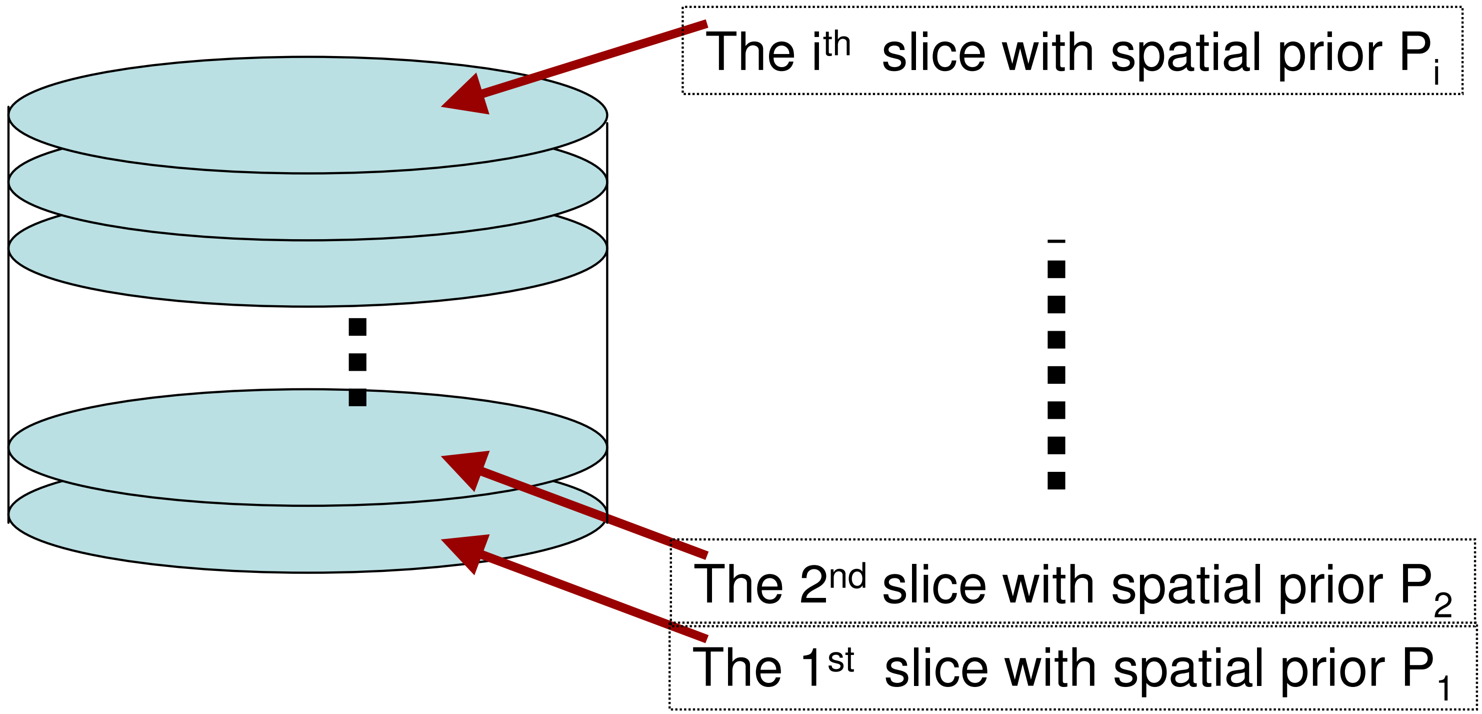


Image Reconstruction

- Penalty functions: Image Amplitude (penalized by model element sensitivity)

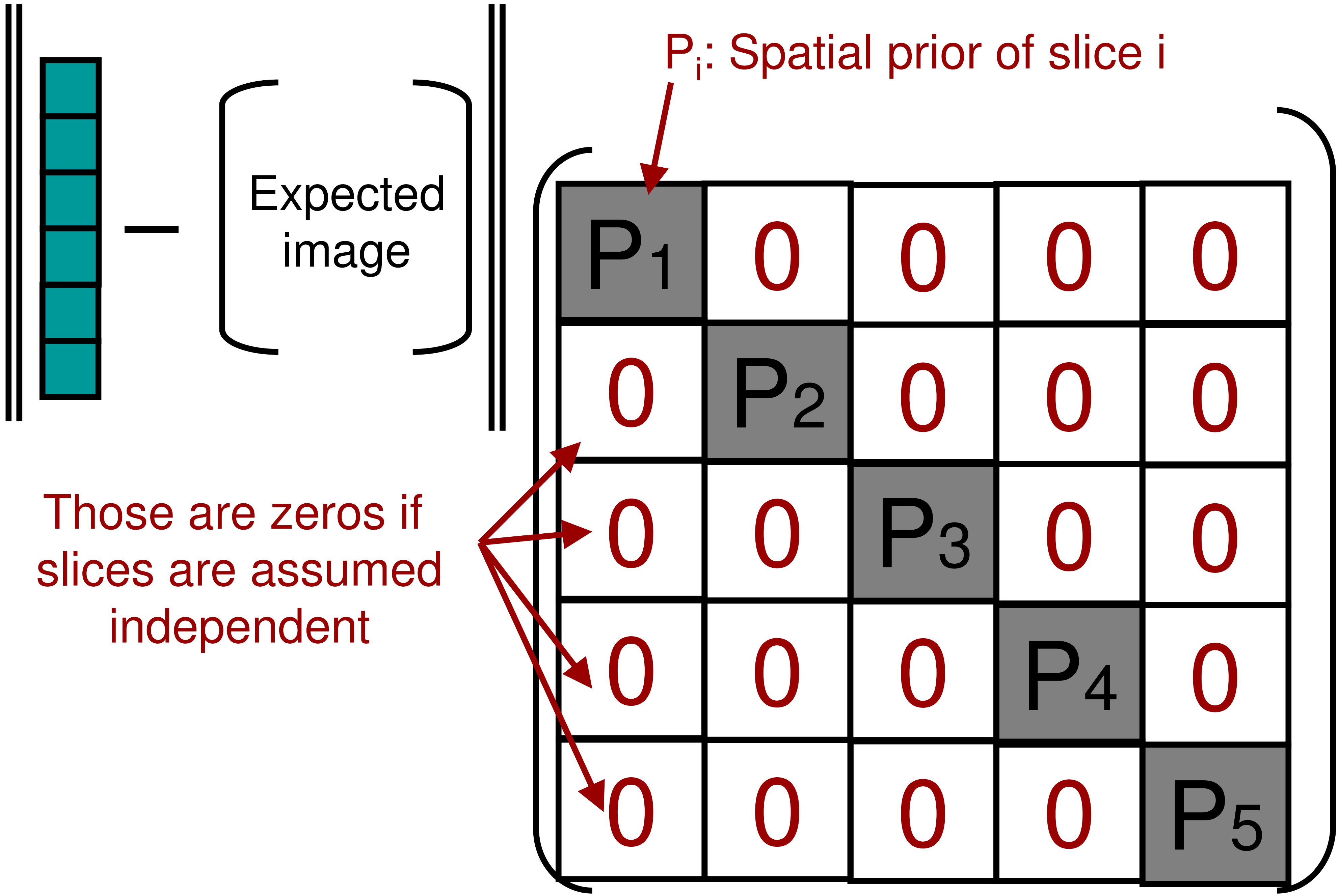


3-D Image Model

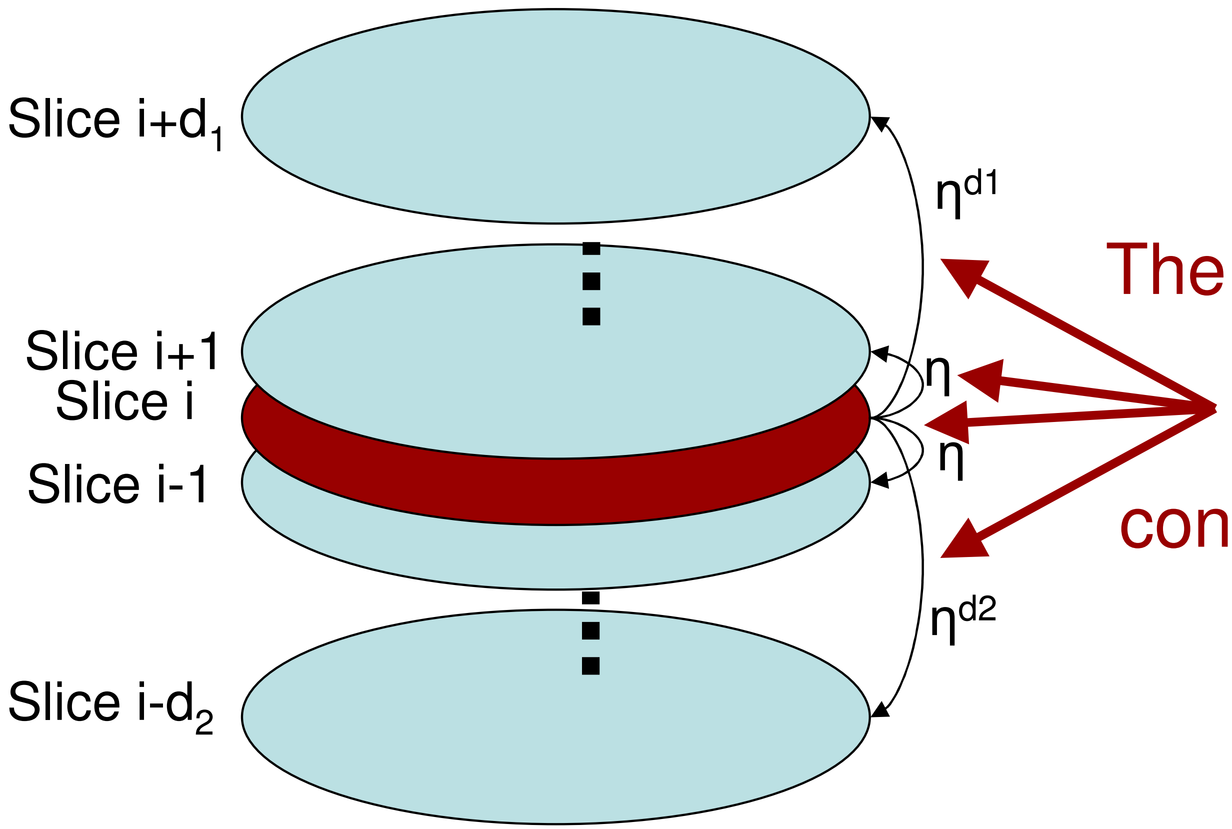


Often, image slices are assumed independent…

3-D spatial prior with independent slices



What about slices are not independent?



The conductivity change of the i^{th} slice is correlated with conductivity changes of slices nearby

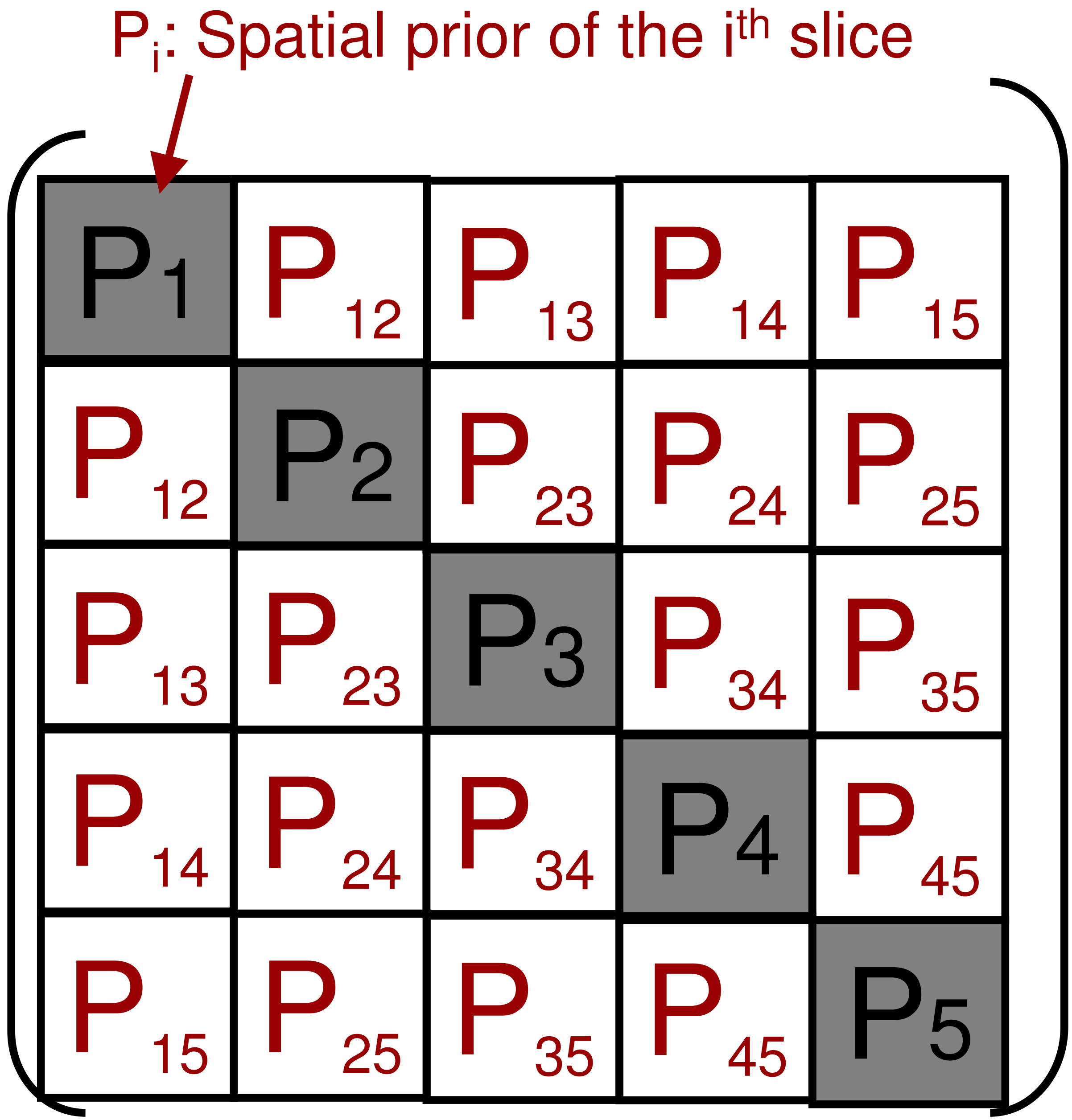
η is correlation factor of adjacent slices

3-D spatial prior with interslice dependence

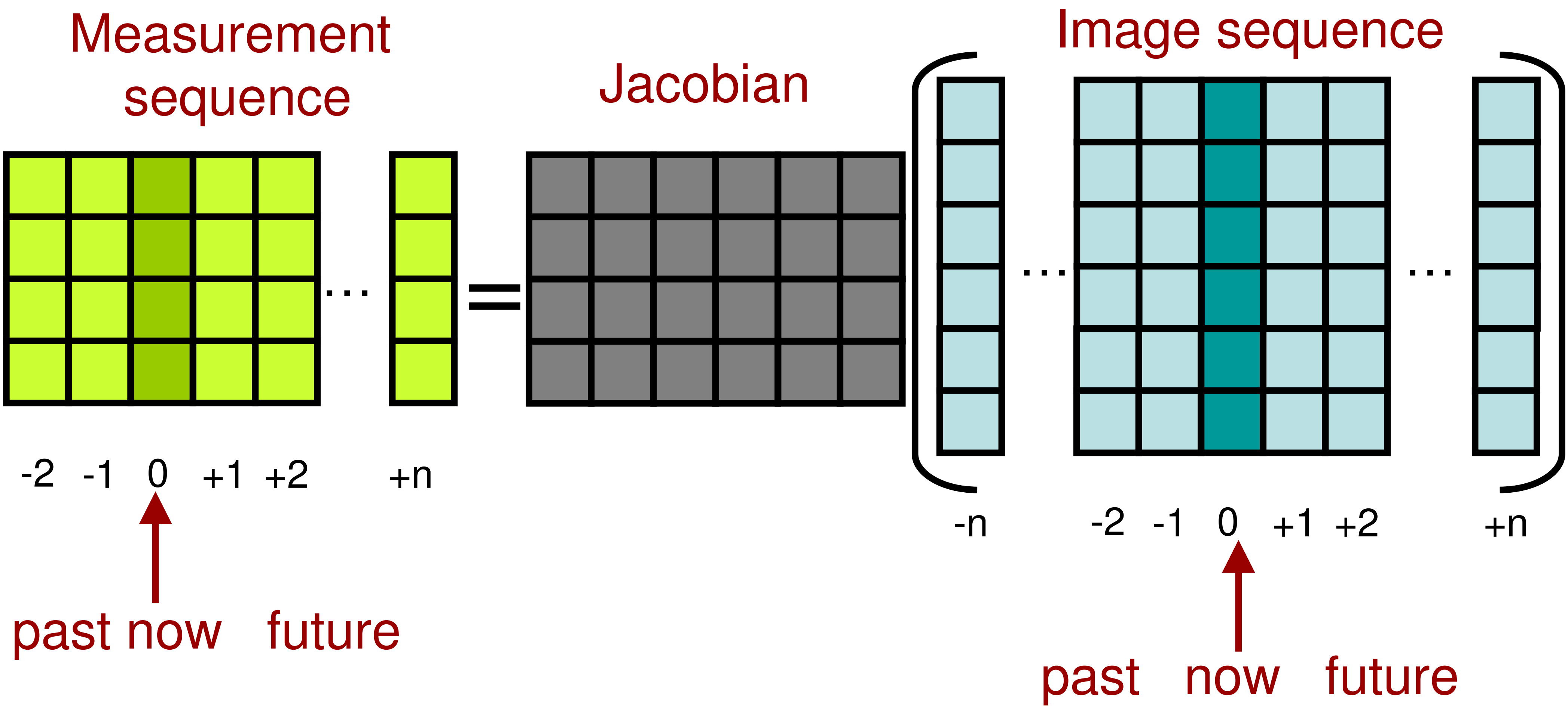
$$P_{ij} = \eta_{ij}\sqrt{(P_i .* P_j)}$$

P_{ij}: Spatial prior of slice i and j

η_{ij}: correlation coefficient of slice i and j

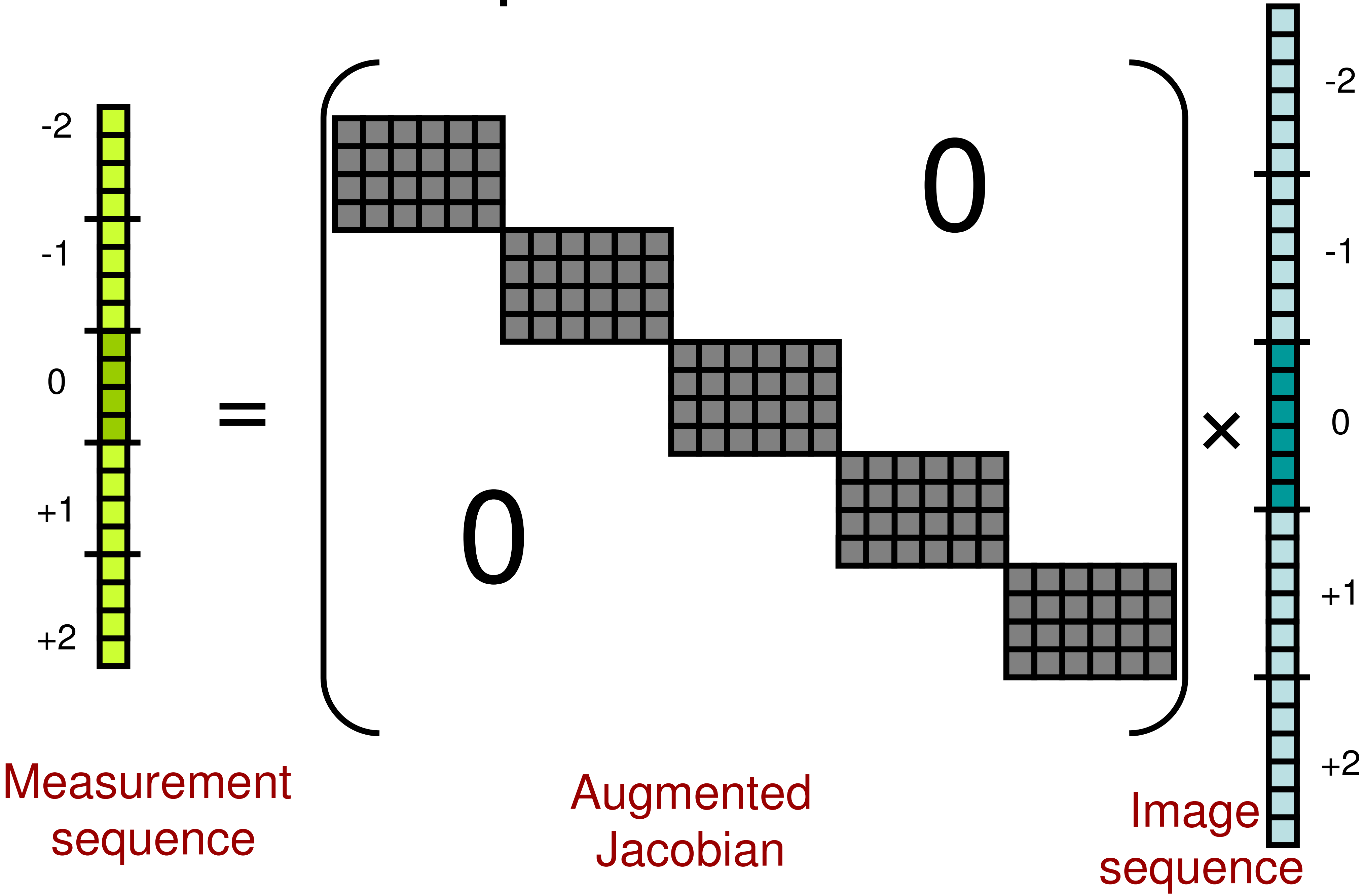


What about time?

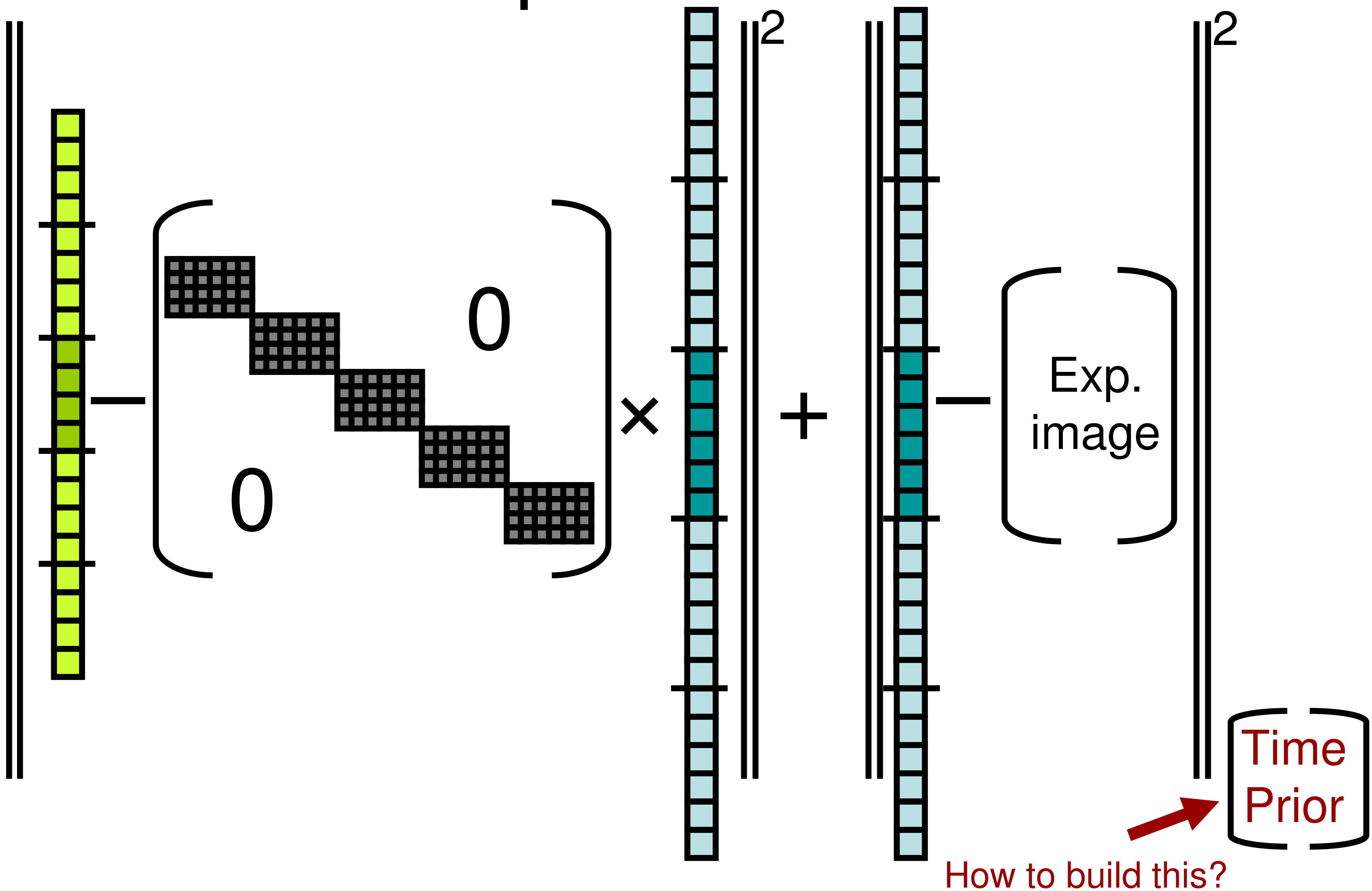


Rewrite as ...

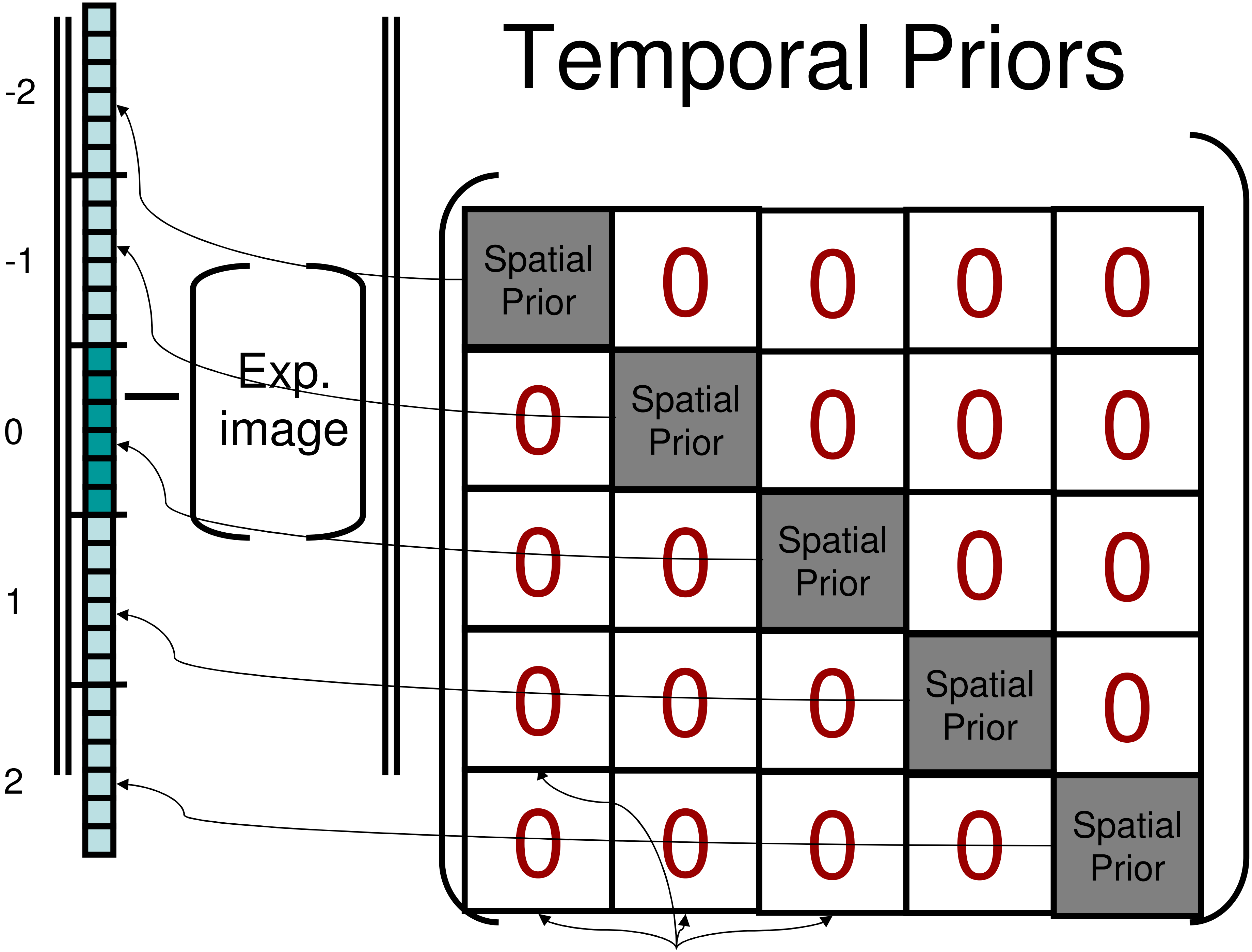
Direct temporal forward model



Direct temporal inverse model

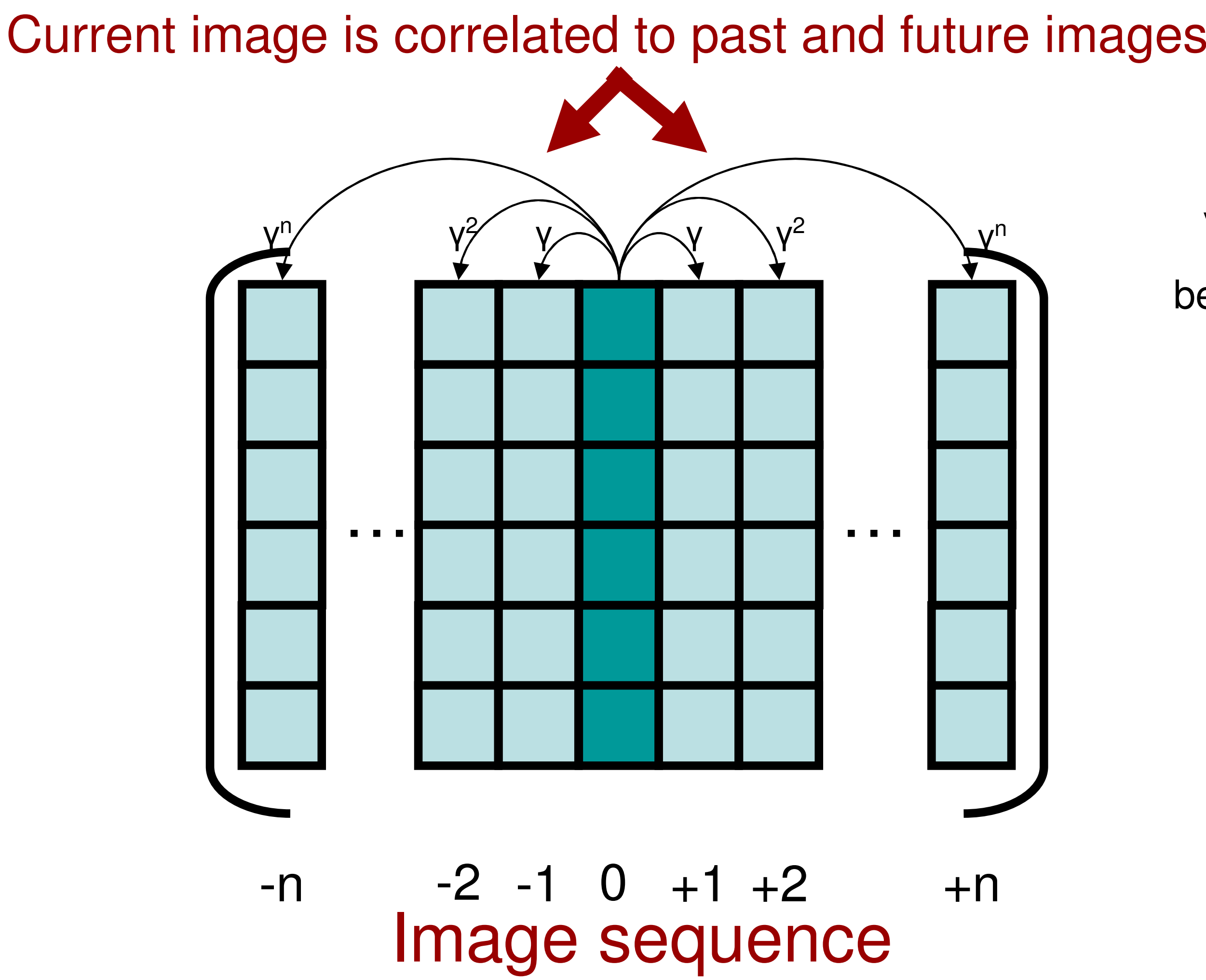


Temporal Priors



Normally, non-diagonal elements are zeros based on assumption that images are independent

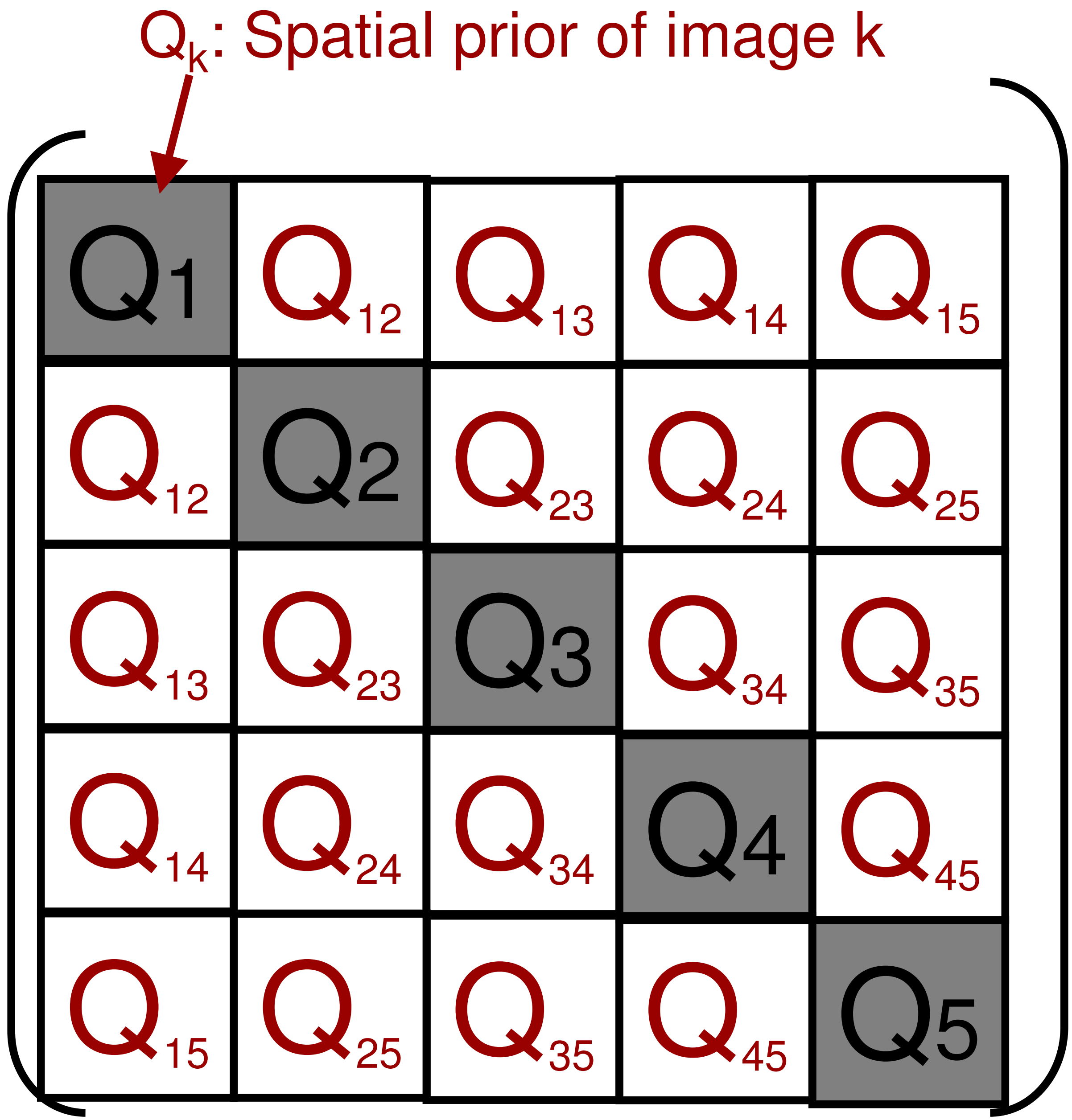
However, images in the sequence are not independent



γ^k is the interframe correlation between two images with delay k

Prior with temporal dependence

$$Q_{ij} = \gamma_{ij}\sqrt{(Q_i .* Q_j)}$$



Q_{ij}: prior
of images at time i and j

γ_{ij}: correlation coefficient
of images at time i and j

How to determine γ

$$\gamma = \arg\min_{\gamma} \left\| \Gamma_y \otimes \Sigma_y - \mathrm{I} \otimes \Sigma_n - \Gamma \otimes \left(\Sigma_y - \Sigma_n \right) \right\|_F^2$$

Γ_y : correlation matrix of data sequence

Σ_y : covariance matrix of data sequence

Σ_n : covariance matrix of noise

$[\Gamma]_{i,j} = \gamma^{|i-j|} \qquad i, j = 1,2,\ldots,2d+1$

d : length of the image sequence

One-step inverse

We formulate the one step inverse as:

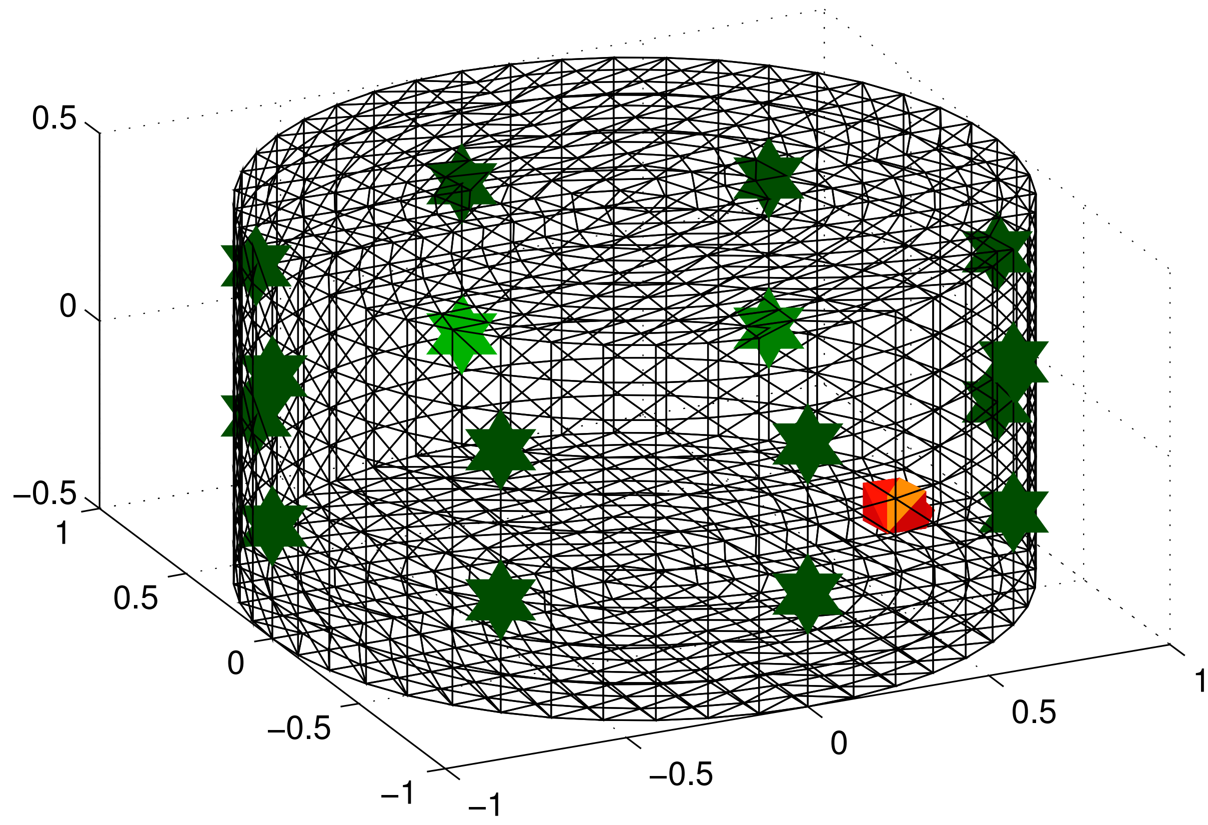
$$\left\|\mathbf{z}-\mathbf{H}\mathbf{x}\right\|_{\mathbf{W}}^{2}+\lambda^{2}\left\|\mathbf{x}\right\|_{\mathbf{R}}^{2}$$

$$\hat{\mathbf{x}}=\left(\mathbf{H}^{t}\mathbf{W}\mathbf{H}+\lambda^{2}\mathbf{R}\right)^{-1}\mathbf{H}^{t}\mathbf{W}\mathbf{z}$$

Data Form:

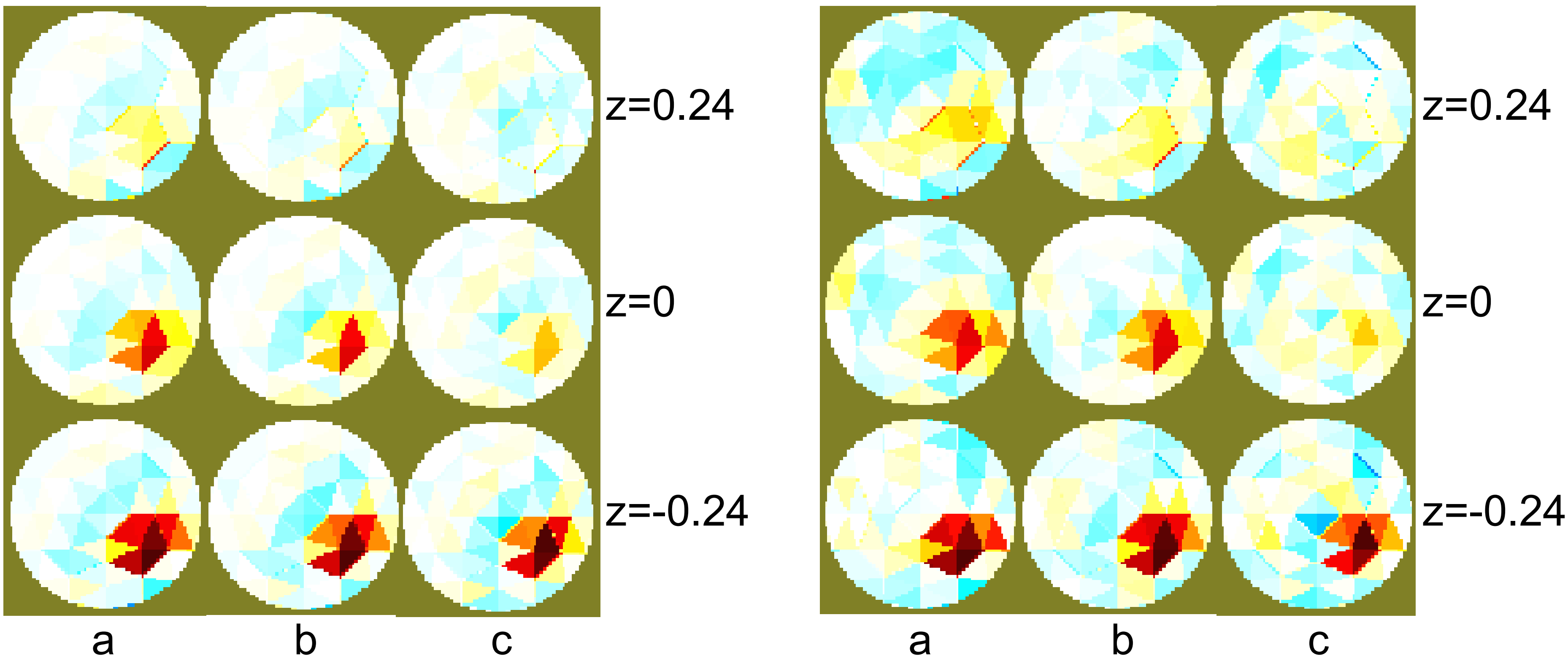
$$\hat{\mathbf{x}}=\mathbf{R}^{-1}\mathbf{H}^{t}\left(\mathbf{H}\mathbf{R}^{-1}\mathbf{H}^{t}+\lambda^{2}\mathbf{W}^{-1}\right)^{-1}\mathbf{z}$$

Simulation: Forward Model



3-D 10-slice finite element model. Electrodes are indicated by green stars, while the conductive target is shown in red.

Simulations: Comparisons



Algorithm comparison: NSR=0(left) and 2(right). NF=0.1. η=0.8
(a) conventional GN solver; (b) Temporal solver; (c) 4-D prior solver

Discussion

- Temporal priors can improve EIT image quality
- Considering interslice correlation, 3-D spatial prior can improve EIT image quality
- The one-step reconstruction can be computationally efficient
  - We're also looking at efficient iterative implementations, allowing reconstruction of entire frame sequence simultaneously