Information content of EIT measurements

Andy Adler[1], Bill Lionheart[2]

[1]Carleton University, Ottawa, Canada
[2]University of Manchester, U.K

Question:

- How much information is in a frame of EIT measurements?

Abbreviation:

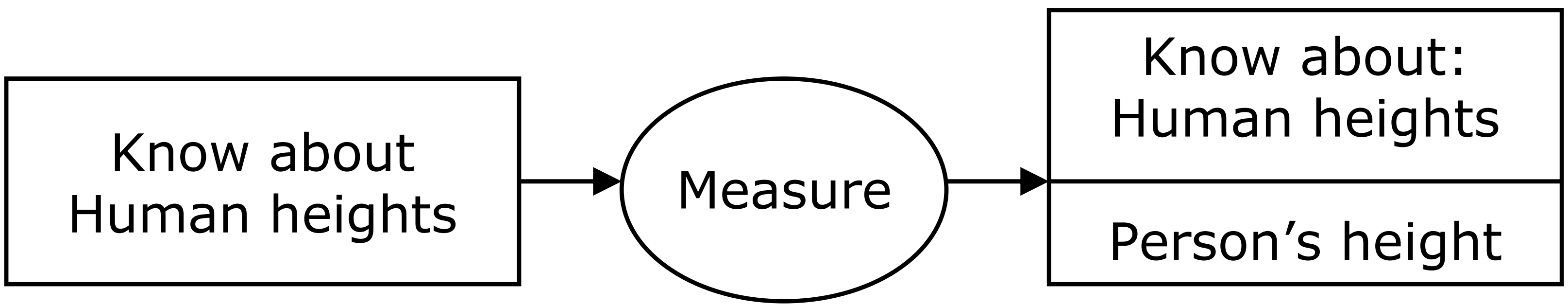
IM = Information in measurements

Motivations: measure IM?

- Regularization adds ‘prior information’ to the image
  - Can we understand and measure this information?

Definition:
Information in Measurements (IM):

- the decrease in uncertainty about the contents of a medium, due to a set of measurements.



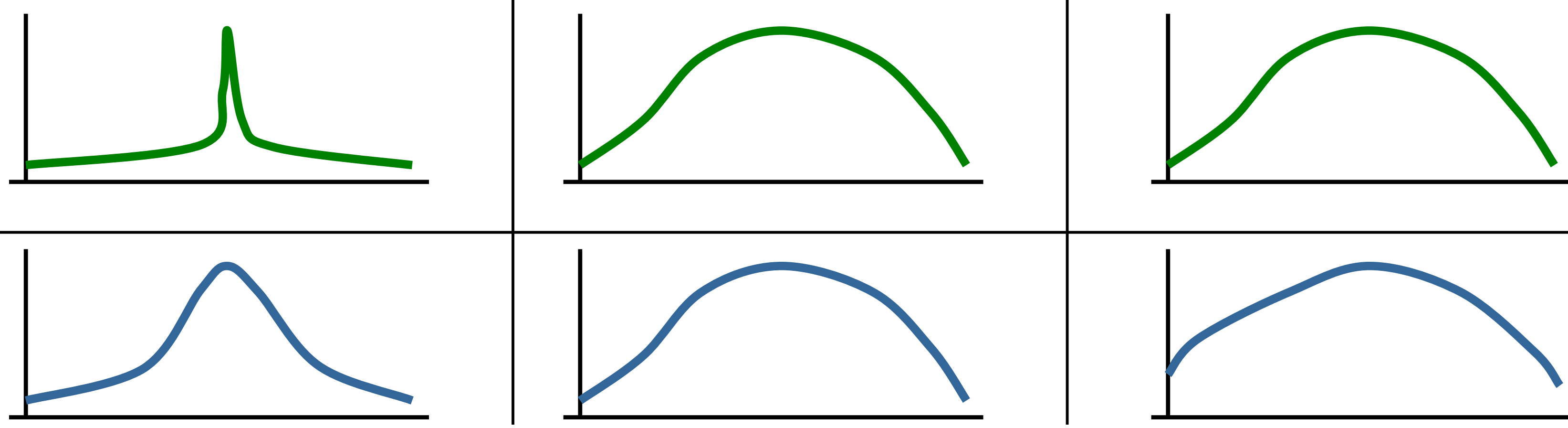
Prior:
Uncertainty is 1:6 billion

Posterior:
Uncertainty is less (meas. precision)

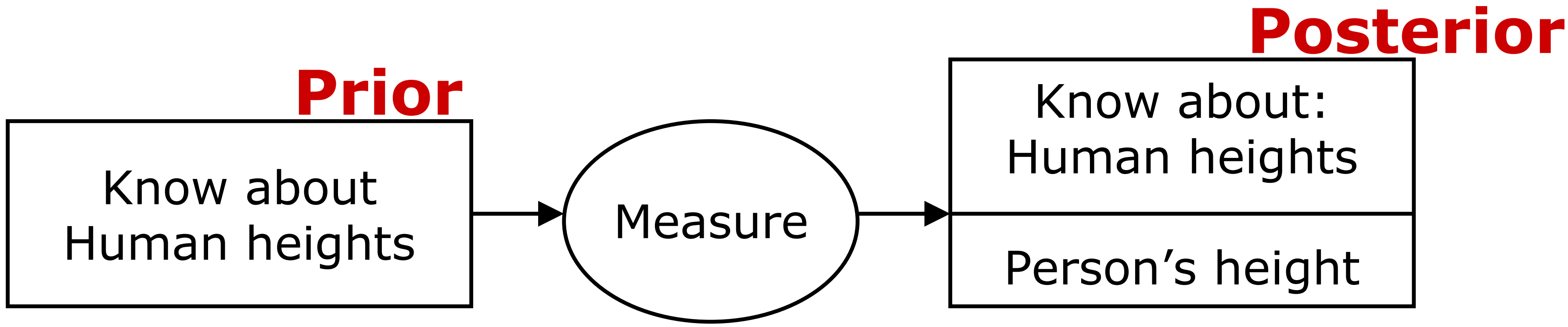
Example: measure *Height*

- Measure #1 (at doctor's office, ie. accurate)
- Measure #2 (via telescope, ie. inaccuate)

Individual Variability (device errors)	Population Variability	Overall Distribution



Example: measure *Height*



- How much MI (how distinguishable)?

	Average (5½′ tall)	Tall (6½′ tall)
Measure #1	Low	Quite a lot
Measure #2	Almost zero	Low

Kullback-Leibler divergence (KLD)

- information gain:

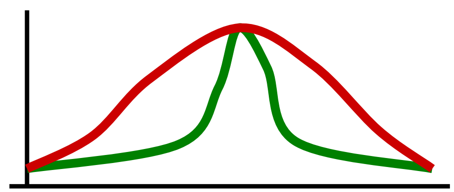
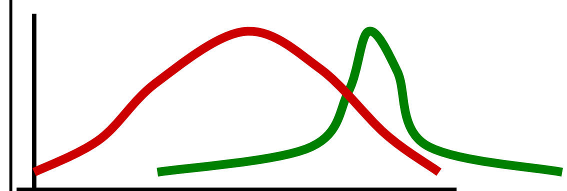
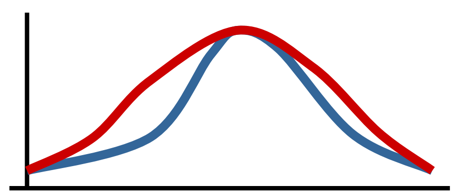
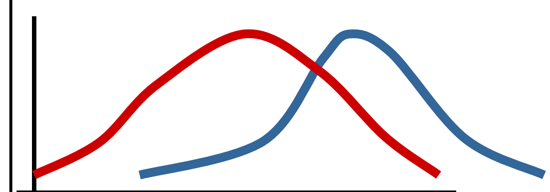
$$q(y) \Longrightarrow p(y)$$

Prior Distribution Posterior Distribution

- or the “extra bits” of information needed to represent *p(y)* wrt *q(y)*

$$D(p\|q) = \int_{\mathbf{y}} p(\mathbf{y}) log_2 \frac{p(\mathbf{y})}{q(\mathbf{y})} d\mathbf{y}$$

IM for height

	Average (5½′ tall)	Tall (6½′ tall)
Measure #1 (accurate)	 D = 0.23 bits	 D = 2.7 bits
Measure #2 (inaccurate)	 D = 0.05 bits	 D = 1.1 bits

Example: Impedance Plethysmography

- To estimate prior, measure data on several patients
  - Assume: $\mu_q = 0, \sigma_q = 800mV$
- On a specific patient
  - $\text{Measurement} = 1.0V$, $\text{Noise: } \sigma_n = 10mV$
- KLD is

$$log_2 \frac{\sigma_q}{\sigma_p} + \left(\frac{\mu_q - \mu_p}{\sigma_q}\right)^2 + \left(\frac{\sigma_p}{\sigma_q}\right)^2 - 1 = 7.9 \text{ bits}$$

Considerations

- Multi-channel measurements in EIT
  - Measurements are correlated
- Real distributions are hard to model
  - Approxiate as Gaussian
- Some signals are more likely than others, but IM should measure avg.
  - $\mathrm{IM} = \underset{q}{E}\left[D(p\|q)\right]$

Formula page …

- Thus, $\mathrm{IM} = \frac{1}{2} log_2 |\mathbf{\Sigma}_q \mathbf{\Sigma}_p^{-1}| + tr\left(\mathbf{\Sigma}_p \mathbf{\Sigma}_q^{-1}\right)$

  When signal>noise

  When noise>signal, *ignore*

  .

- EIT:

$$\mathbf{y} = \mathbf{J}\mathbf{x} + \mathbf{n}$$
$$\mathbf{\Sigma}_q = \mathbf{J}\mathbf{\Sigma}_x\mathbf{J}^T + \mathbf{\Sigma}_n$$
$$\mathbf{\Sigma}_p = \mathbf{\Sigma}_n$$

- For EIT:

$$\mathrm{IM} = \frac{1}{2} log_2 \left|\mathbf{J}\mathbf{\Sigma}_x\mathbf{J}^T\mathbf{\Sigma}_n^{-1} + \mathbf{I}\right|$$

More formulae …

- Practically, an image reconstruction algorithm has
  - Noise model: *independent noise*
  - Reconstruction prior: **R**

$$\text{Given } \mathbf{R}^{-1} \text{ normalized so } tr(\mathbf{J}\mathbf{R}^{-1}\mathbf{J}^T) = n_N$$

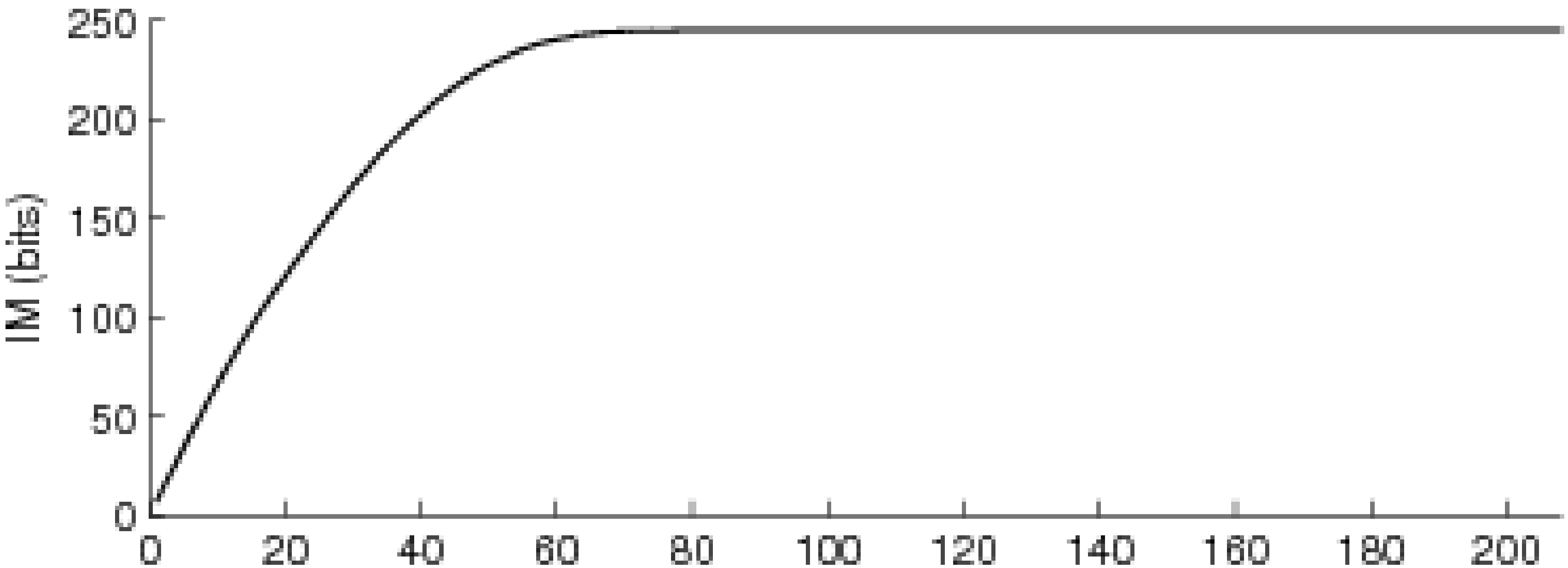
$$\text{IM} = n_N log_2\left(\frac{\sigma_x}{\sigma_n}\right) + \frac{1}{2} log_2 \left|\mathbf{J}\mathbf{R}^{-1}\mathbf{J}^T + \lambda^2 \mathbf{I}\right|$$

Num meas×SNR

Correlation between measurements + Reg. Prior

IM for sample EIT system

- Signal: golf ball in 30cm diameter tank



Number of independent measurements (via PCA)

IM for sample EIT system

- IM from system = 245.1 bits

- This value is less than

$$n_N log_2 \mathrm{SNR} = 406.3 \text{ bits}$$

(value for independent measurements)

- As λ increases, IM increases

Motivations: measure IM?

- Distinguishability may be defined in terms of the IM content from small contrasts
- Optimal current patterns may be defined in terms of maximizing IM.

Motivations: measure IM?

- Fusion of EIT with other modalities.
  - Measurements which are not independent will only add a small increment to the IM from EIT.
- Inherent limits to the compressibility of measured data.
  - Measured data cannot be stored in less space than IM.