ELG7173 – Computational Techniques in Medical Imaging:

Electrical Impedance Tomography: Image Algorithms and Applications

Andy Adler
Assistant Professor, SITE, U. Ottawa
Outline

• Electrical Impedance Tomography
• Physics
• Image Reconstruction
• Applications
• Future Work
Electrical Impedance Tomography

• Relatively new medical imaging technique (early 1990’s)
• Body Surface Electrodes apply current patterns and measure the resulting voltages
• Distribution of conductivity is calculated
EIT: Block Diagram

Medium $\Omega$

Current Source

Data acquisition Controller

Amplifiers

Imaging System
EIT: Applications

- EIT can image physiological processes involving movement of conductive fluids and gasses
- Lungs
- Heart / perfusion
- GI tract
- Brain
- Breast
EIT: Advantages

EIT is a relatively low resolution imaging modality, *but*

- Non-invasive
- Non-cumbersome
- Suitable for monitoring
- Underlying technology is low cost
Application: Breathing

Chest images of tidal breathing in normal
Application: Heart Beat

EIT signal in ROI around heart and ECG
Non-invasive

Thresholds for cutaneous perception of electric current vs. frequency and EIT system
Hardware: Electrodes

- Current stimulation is better than voltage, because it accounts for electrode contact impedance
- Traditionally EIT uses adjacent current drive.
- Some systems separate drive and measurement electrodes, using adaptive current patterns
EIT: Physics

• Within medium $\Omega$ there is $\mathbf{E}$ and $\mathbf{J}$.

\[ J_c = \sigma \mathbf{E} \]

\[ J_d = \varepsilon \varepsilon_0 \frac{d\mathbf{E}}{dt} \]

\[ \mathbf{J} = (\sigma - j \omega \varepsilon \varepsilon_0) \mathbf{E} \]
EIT: Physics

In the absence of magnetic fields

$$E = -\nabla V$$

No charge build up in conductive medium

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} = 0$$

We have

$$\nabla \cdot \left( \sigma - j\omega \varepsilon \varepsilon_0 \right) \nabla V = 0 \quad \text{in} \quad \Omega$$
EIT: Physics

Current is applied at electrodes

\[ \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = I_e \]

Body need to be grounded, somewhere

\[ V = 0 \quad \text{at some point} \]
EIT: Numerical Models

In order to calculate measurements from conductivities, we can use:

• Analytic Techniques
  – Analytic models exist for elliptic 3D media; however, numerical approximations of sums required

• Numerical Models
  – Finite Element Techniques, main method
Finite Element Models

Simple Model
with 64 elements
Used for inverse solution
Finite Element Models

“Simple” 3D Model with 768 elements
Used for inverse solution
Image Reconstruction: Static Imaging

*Static imaging* reconstructs the absolute conductivity from measurements.

Algorithms:
- Iterative (Newton-Raphson)
- Layer Stripping
Block Diagram of Iterative Algorithm

Current Injection and EIT data measurement → Real Data

Does simulation approximate real data

Finished

yes

no

Update conductivity distribution → Simulation Data → EIT data simulation → Finite Element Model

Finite Element Model

Patient
Static Imaging Difficulties

- Extremely sensitive to uncertainties in electrode position
- Ill-conditioned problem
- Numerical instability
Dynamic Imaging

• Calculate change in conductivity distribution from change in measurements
• Inverse problem *linearized*
• Much reduced sensitivity to electrode and hardware errors.
• Very suitable for physiological imaging: lung, heart, GI
Dynamic Imaging: Example

Figure 5.1: A measurement configuration with geometrical and electrode placement error
Dynamic Imaging: Example

A-B  A-D  C-D

Figure 5.2: Images from media with geometrical errors.
A: Measurements A-B
B: Measurements A-D
C: Measurements C-D
Dynamic imaging techniques

- **Backprojection**: Voltages are “projected” along the equipotential lines.

![Diagram showing current injection and equipotential region]
Dynamic imaging: Backprojection

• Technique used in early studies (mid-1980’s to early 1990’s)
• Based on analogy with C.T.
• Not appropriate because
  – Measurements don’t really come from equipotential region
  – Not symmetric
Inverse Techniques

• We can pose dynamic imaging as linear inverse, using a *sensitivity matrix*

\[
Z_j = \frac{Z(\sigma_h) - Z(\sigma_h + \delta_j)}{\delta_j}
\]

\[Z = H \Delta \sigma\]
Parametrize Conductivity

• We want to parameterize conductivity
  – So that all reconstructed valued are physically valid
  – To reflect physical importance of low and high values

• Most common parameterization is
  \[ r = \log(\text{conductivity}) \]
Parameterization

Figure 4.2: Normalised mean signal vs. change in log conductivity contrast ratio.
Inverse Techniques

• Classic least-squares inverse

\[ z = Hx \]
\[ \hat{x} = \left( H^t H \right)^{-1} H^t z \]
Matrix Techniques

However, problem is:

- ill-conditioned: measurements depend much more on data near electrodes than in centre
- ill-formed: more unknowns than measurements
Regularized Imaging

*Handwaving argument for regularization:* used for ill-posed and ill-formed problems to find a solution with:

- Low error: small \((z - Hx)\)
- Stable: small change in \(x\) for small \(\Delta z\)
- Good looking:
  - Somewhat hard to define, but includes smoothness, clean edges, etc.
MAP estimates

• MAP approach says choose $x$ such that $f(x|z)$ is maximized
  – In other words, choose the image that is most likely, considering the measured data

• Bayes Rule

$$f(x|z) = \frac{f(z|x)f(x)}{f(z)}$$
MAP estimates

\[ f(z|x) \] the distribution of measurements given an image
  - Based on forward model and noise properties

\[ f(z) \] distribution of measurements
  - Not a parameter of MAP estimate

\[ f(x) \] distribution of image
  - Based on \textit{a priori} knowledge of physically possible and likely images distributions
Regularized Imaging

Given Linear Model:

$$z = Hx + n$$

Maximum A Posteriori (MAP) estimate is:

$$\hat{x} = \left( H^t R_n^{-1} H + R_x^{-1} \right)^{-1} \left( H^t R_n^{-1} z + R_x^{-1} x_\infty \right)$$
Regularized Imaging

- Parameters $R_x$, $R_n$, $x_\infty$, represent a priori statistical knowledge of problem

\[
x_\infty = E[x]
\]

\[
R_x = E[(x - x_\infty)^t(x - x_\infty)] = E[x^tx] - x_\infty^tx_\infty
\]

\[
R_n = E[n^tn] = \begin{bmatrix}
\sigma_1^2 & 0 & \cdots \\
0 & \sigma_2^2 & \\
& \vdots & \ddots
\end{bmatrix}
\]
Choice of parameter $R_x$

- Parameter is a “penalty function”
- Many regularization approaches use a diagonal matrix
  - Tikhonov regularization uses the scaled identity matrix
  - This will penalize large amplitude pixels in image
- We choose a dense matrix
  - Penalize image frequency content above maximum possible with measurements
Choice of parameter $R_x$

- In order to avoid problems inverting $R_x$, we directly calculate the inverse
  - Since $R_x$ represents spatial low pass filter, $R_x^{-1}$ represents a high pass

- Choose a Gaussian high pass of form

$$F(u, v) = 1 - e^{-\omega_0 (u^2 + v^2)}$$
Regularization: Hyperparameters

Regularizations techniques must finally introduce a “hyperparameter” ($\mu$)

$$\hat{x} = \left( H^t W H + \mu Q \right)^{-1} \left( H^t W z + \mu Q x_\infty \right)$$

where

$$W = \frac{1}{\sigma_n^2} R_n^{-1}$$, ie. the relative noise amplitudes

$$Q = \frac{1}{\sigma_x^2} R_x^{-1}$$, ie. the relative image correlations
Regularization: Hyperparameters

\[ \mu \text{ is thus the ratio of image and noise amplitudes,} \]

\[ \mu = \frac{\sigma_x^2}{\sigma_y^2} \]

it can be interpreted as a filter noise figure
Regularized Inverse

Parameters:

• $W$: models measurement noise
• $Q$: penalizes image features which are greater than data supports
• $x_\infty$: represents the background conductivity distribution (heart, lungs, etc)
• $\mu$: “hyper-parameter” amount of regularization
Advantages of Regularization

- Stabilizes ill-conditioned inverse
- Introduction of *a priori* information
- Control of *resolution-noise* performance trade-off
- MAP inverse justifies the formulation in terms of Bayesian statistics
Noise – Resolution Tradeoff

F:  Meas: No Noise  Reconst: NF = 2.0

G:  Meas: -3dB SNR  Reconst: NF = 2.0
Noise – Resolution Tradeoff

F: Meas: No Noise  Reconst: NF = 0.4
G: Meas: -3dB SNR  Reconst: NF = 0.4
Electrode Movement

- Electrodes on chest move during breathing
- Figure shows rough movement pattern of rib cage
Electrode Movement

- FEM of electrical and mechanical properties of thorax to simulate
- Signal from expansion shown to contribute 10-20% of conductivity signal
Applications: Lung Function and Disease

Monitoring of lung function is important in ICU. Issues are:

• Distribution of ventilation
• Ventilation – perfusion match
• Lung Edema
• Flow limitation (obstructive lung disease)
Lung function tests

- Patients with obstructive lung disease (emphysema, bronchitis) tend to dynamically hyperinflate: lower inspiration than expiration.
- Simulated with PEEP (positive end expiratory pressure)
Lung Function: Protocol

- Initially, inflations to 3 kPa
- dog expires to the applied PEEP
- ventilation stopped, and dog expires against PEEP for 15 s
Lung Function

A: image of section thorax due to $\Delta VL$ 800 ml

B: image due to a $\Delta VL$ 400 where left main stem bronchus was plugged.
- Volume estimates by EIT, Pao and Pes, after step volume increases of 100ml, 500ml, 900ml.

- Note that EIT signal does not display overshoot
Pulmonary Edema

• Pulmonary Edema (lungs filled with fluid) plagues ICU patients for heart disease, accident victims
• Typical monitoring techniques are invasive (Swan-Ganz catheter)
• We looked at EIT ability to monitor level of Edema.
Pulmonary Edema: Results

- Change in lung liquid volume by EIT vs liquid volume instilled
Imaging

- Algorithms to use data from internal electrodes
- How to manage electrode artefacts
- Arbitrary electrode placements
- Image interpretation and error bounds given artefacts and noise
Applications of interest

- Longer term monitoring in ICU
- Small animal lung function
- Contrast agents
- Inhomogeneous ventilation on smaller scale