ELG7173 – Computational Techniques in Medical Imaging:

Electrical Impedance Tomography: Image Algorithms and Applications

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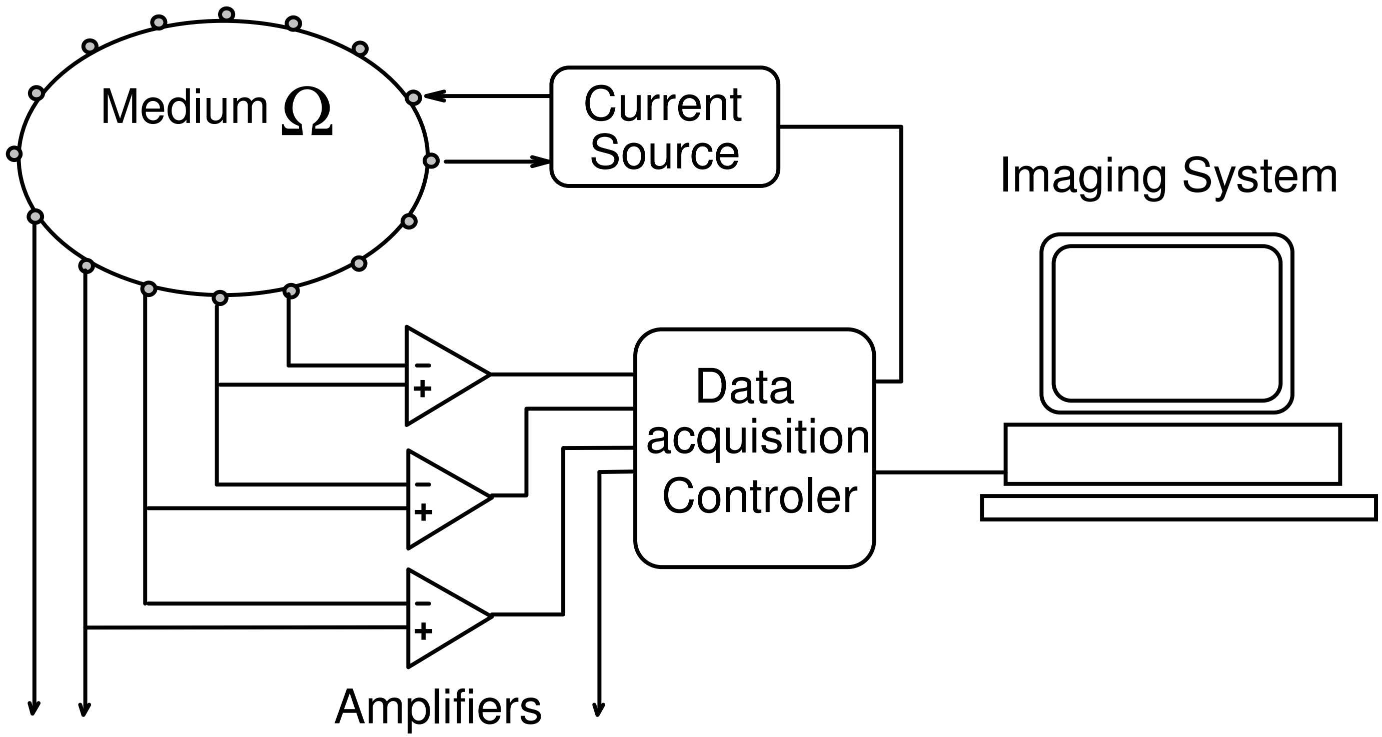
Outline

- Electrical Impedance Tomography
- Physics
- Image Reconstruction
- Applications
- Future Work

Electrical Impedance Tomography

- Relatively new medical imaging technique (early 1990's)
- Body Surface Electrodes apply current patterns and measure the resulting voltages
- Distribution of conductivity is calculated

EIT: Block Diagram



EIT: Applications

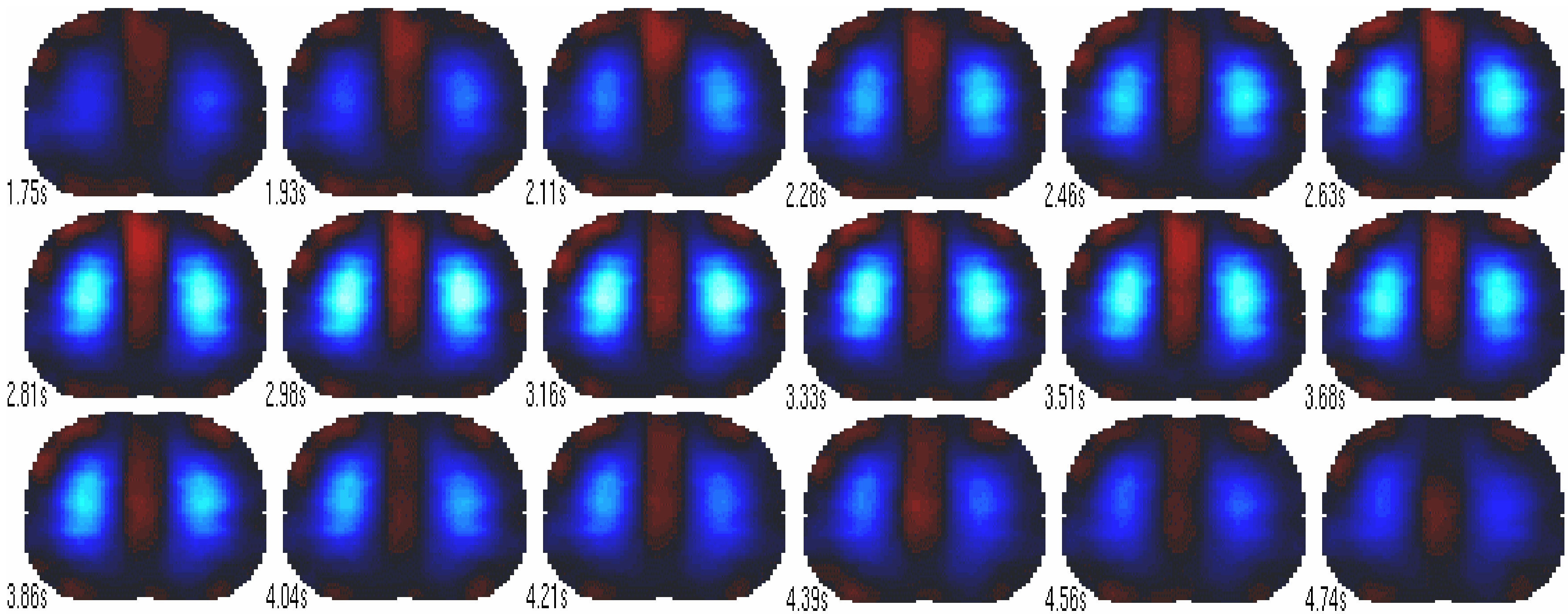
- EIT can image physiological processes involving movement of conductive fluids and gasses
- Lungs
- Heart / perfusion
- GI tract
- Brain
- Breast

EIT: Advantages

EIT is a relatively low resolution imaging modality, *but*

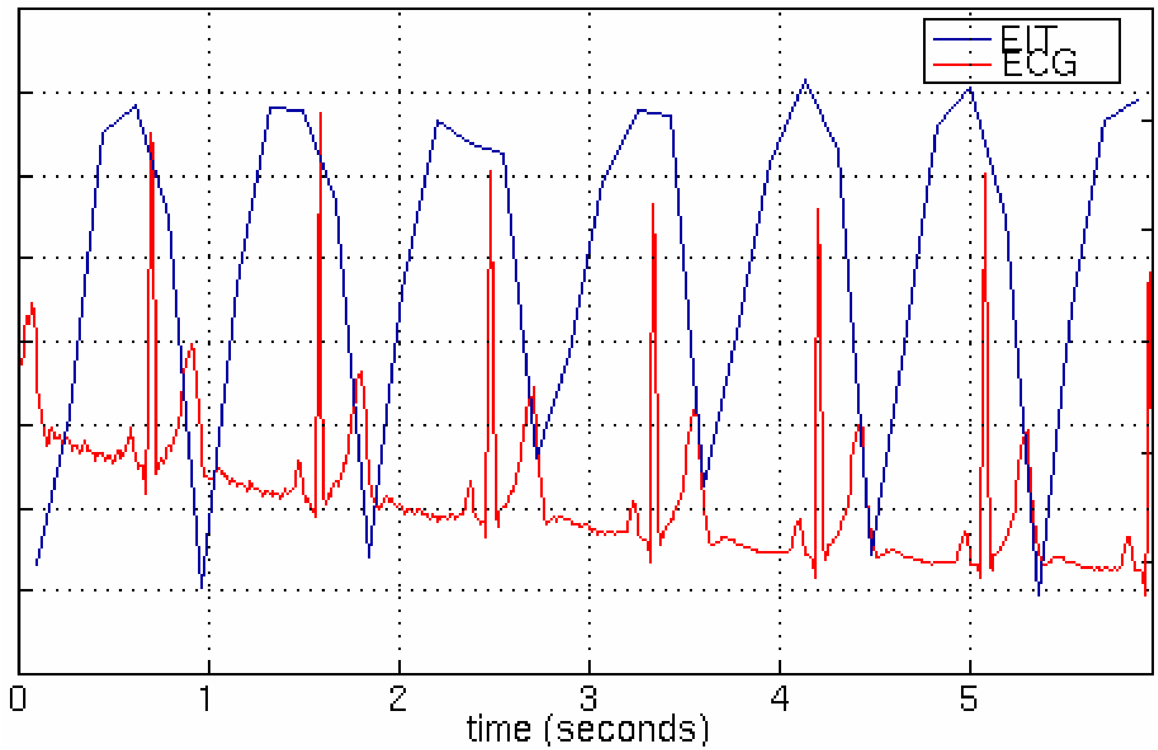
- Non-invasive
- Non-cumbersome
- Suitable for monitoring
- Underlying technology is low cost

Application: Breathing



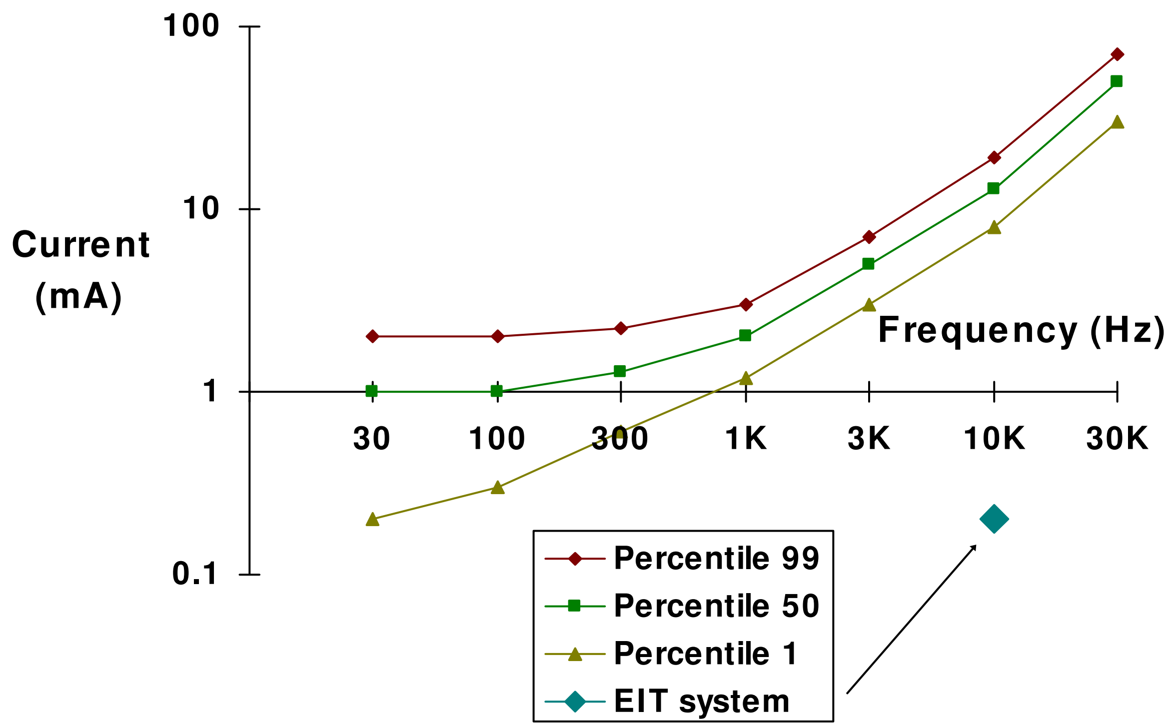
Chest images of tidal breathing in normal

Application: Heart Beat



EIT signal in ROI around heart and ECG

Non-invasive



Thresholds for cutaneous perception of electric current vs. frequency and EIT system

Hardware: Electrodes

- Current stimulation is better than voltage, because it accounts for electrode contact impedance
- Traditionally EIT uses adjacent current drive.
- Some systems separate drive and measurement electrodes, using adaptive current patterns

EIT: Physics

- Within medium Ω there is **E** and **J**.
- $$\mathrm{J}_c = \sigma \mathrm{E}$$
  $$\mathrm{J}_d = \varepsilon\varepsilon_o \frac{d\mathrm{E}}{dt}$$

$$\mathbf{J} = (\sigma - j\omega\varepsilon\varepsilon_o)\mathbf{E}$$

EIT: Physics

In the absence of magnetic fields

$$\mathrm{E} = -\nabla V$$

No charge build up in conductive medium

$$\nabla \bullet \mathrm{J} = -\frac{\partial \rho}{\partial t} = 0$$

We have

$$\nabla \bullet (\sigma - j\omega\varepsilon\varepsilon_o)\nabla V = 0 \qquad \text{in } \Omega$$

EIT: Physics

Current is applied at electrodes

$$\nabla \bullet \mathbf{J} = -\frac{\partial \rho}{\partial t} = I_e$$

Body need to be grounded, somewhere

$$V = 0 \qquad \text{at some point}$$

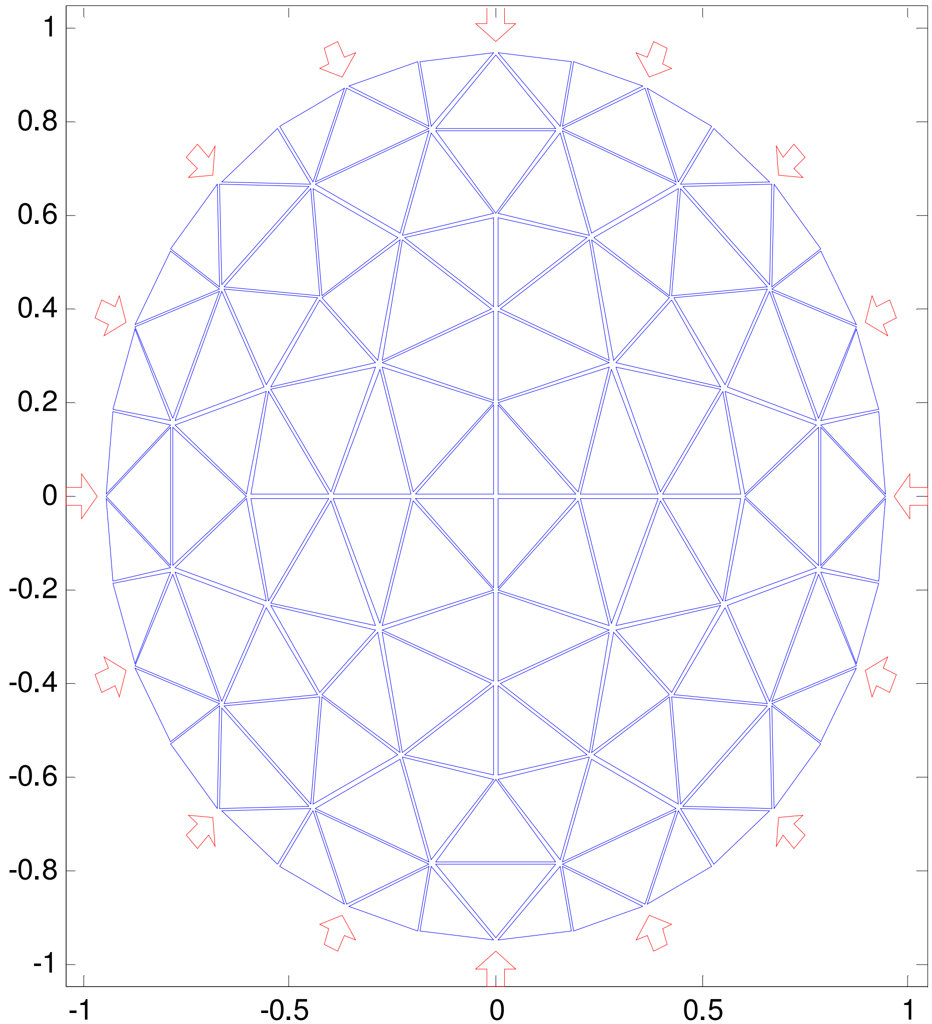
EIT: Numerical Models

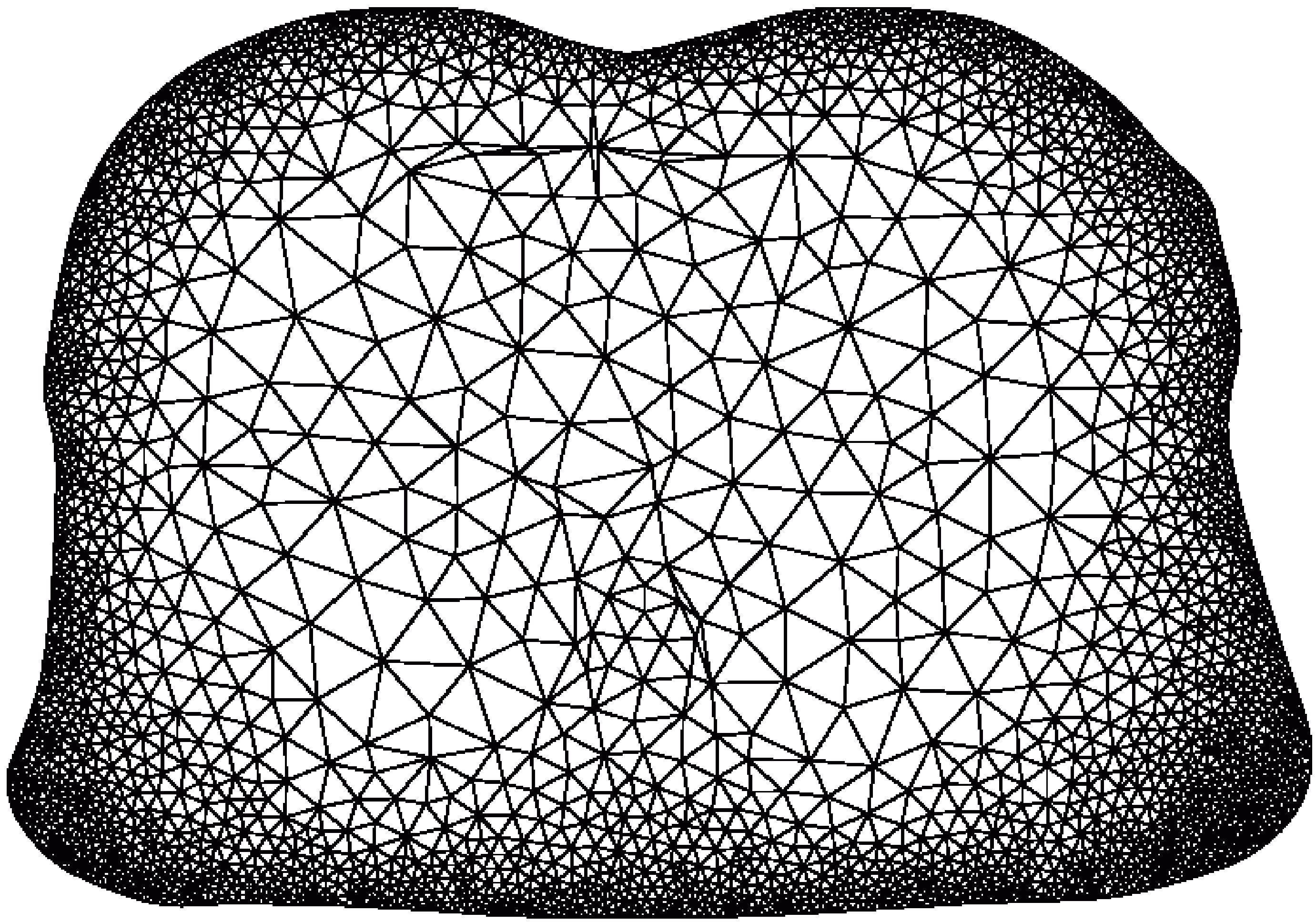
In order to calculate measurements from conductivities, we can use:

- Analytic Techniques
  - Analytic models exist for elliptic 3D media; however, numerical approximations of sums required
- Numerical Models
  - Finite Element Techniques, main method

Finite Element Models

- Simple Model with 64 elements
- Used for inverse solution





Model of Borsic, *Physiol Meas, 22:77-83, 2002*

Finite Element Models

- "Simple" 3D Model with 768 elements
- Used for inverse solution

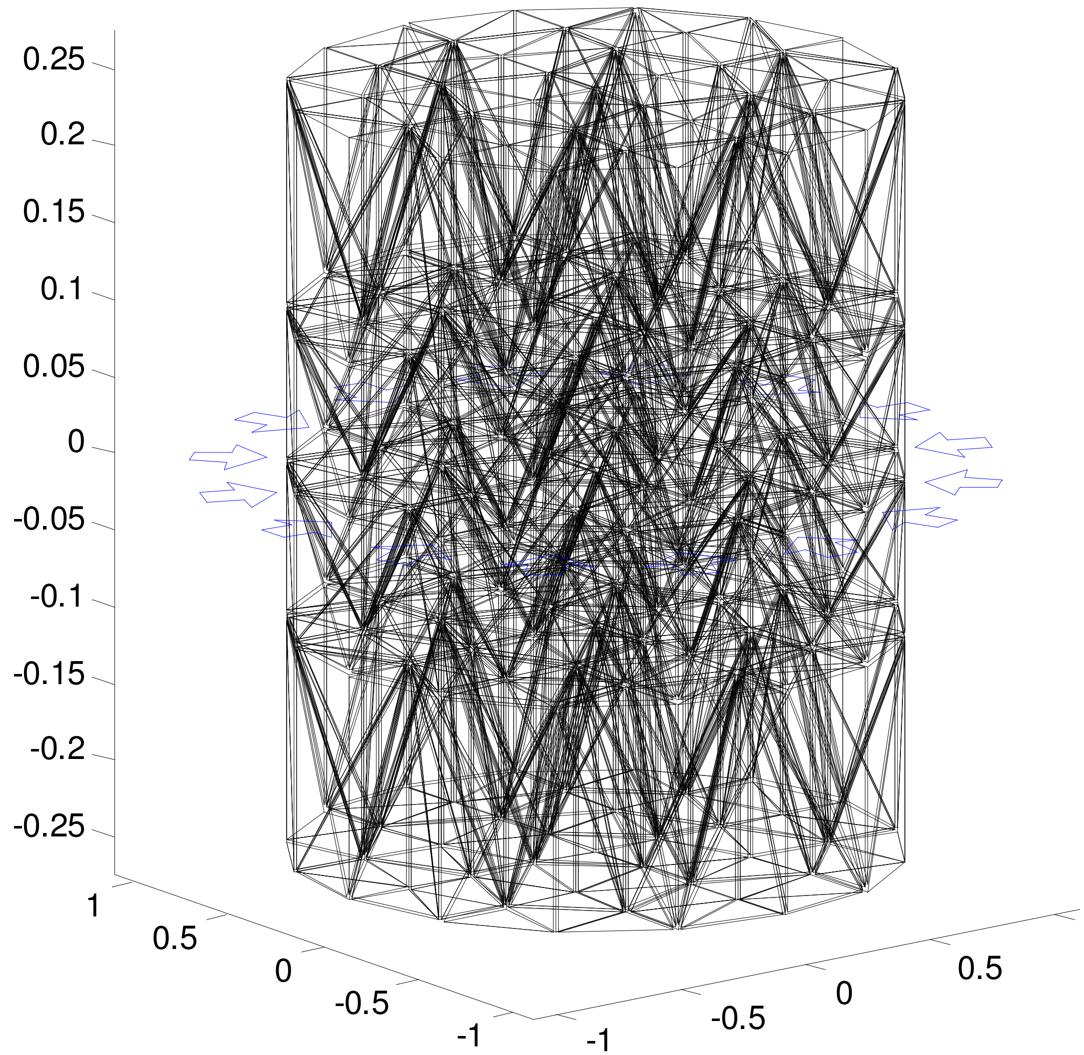


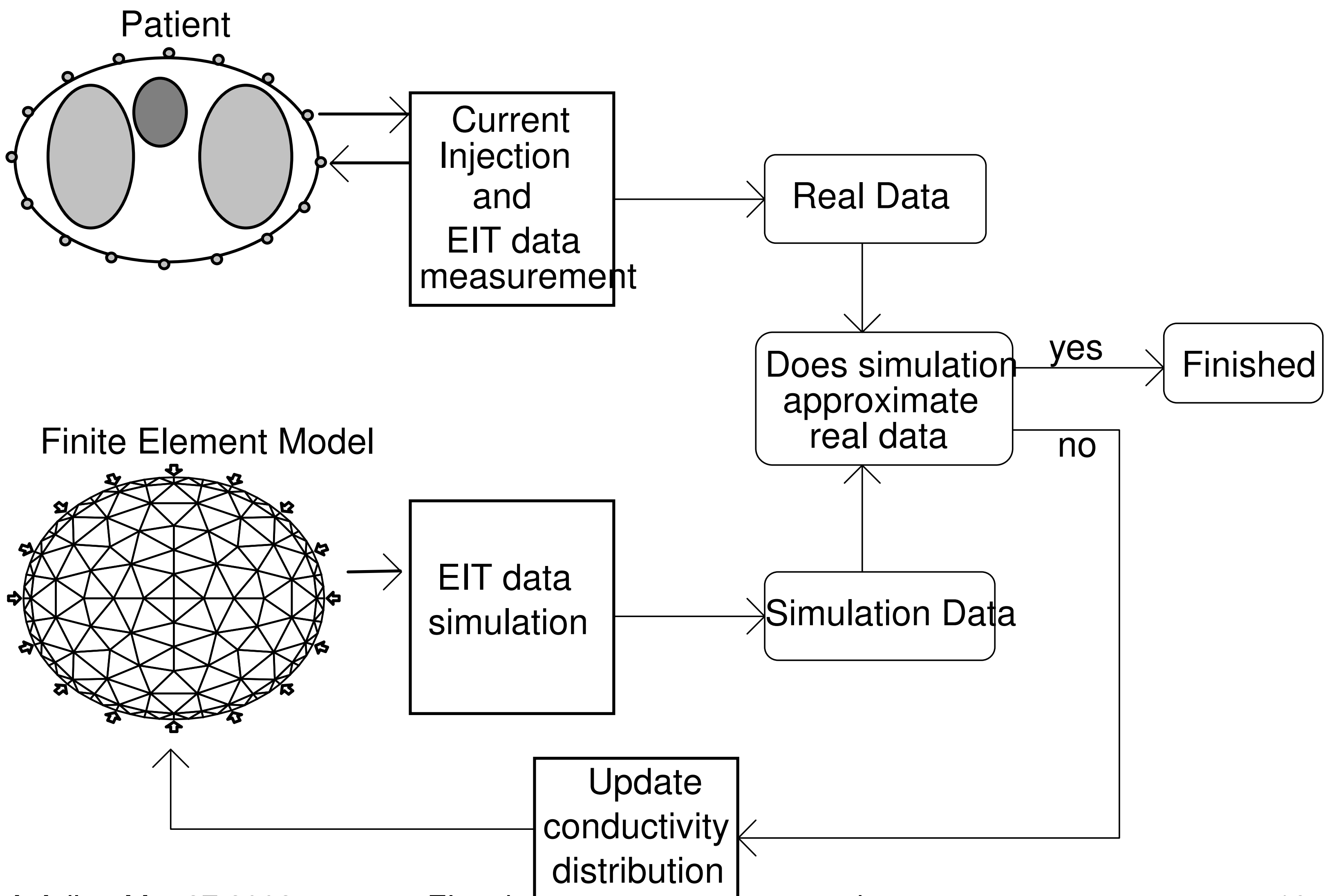
Image Reconstruction: Static Imaging

Static imaging reconstructs the absolute conductivity from measurements.

Algorithms:

- Iterative (Newton-Raphson)
- Layer Stripping

Block Diagram of Iterative Algorithm



Static Imaging Difficulties

- Extremely sensitive to uncertainties in electrode position
- Ill-conditioned problem
- Numerical instability

Dynamic Imaging

- Calculate change in conductivity distribution from change in measurements
- Inverse problem *linearized*
- Much reduced sensitivity to electrode and hardware errors.
- Very suitable for physiological imaging: lung, heart, GI

Dynamic Imaging: Example

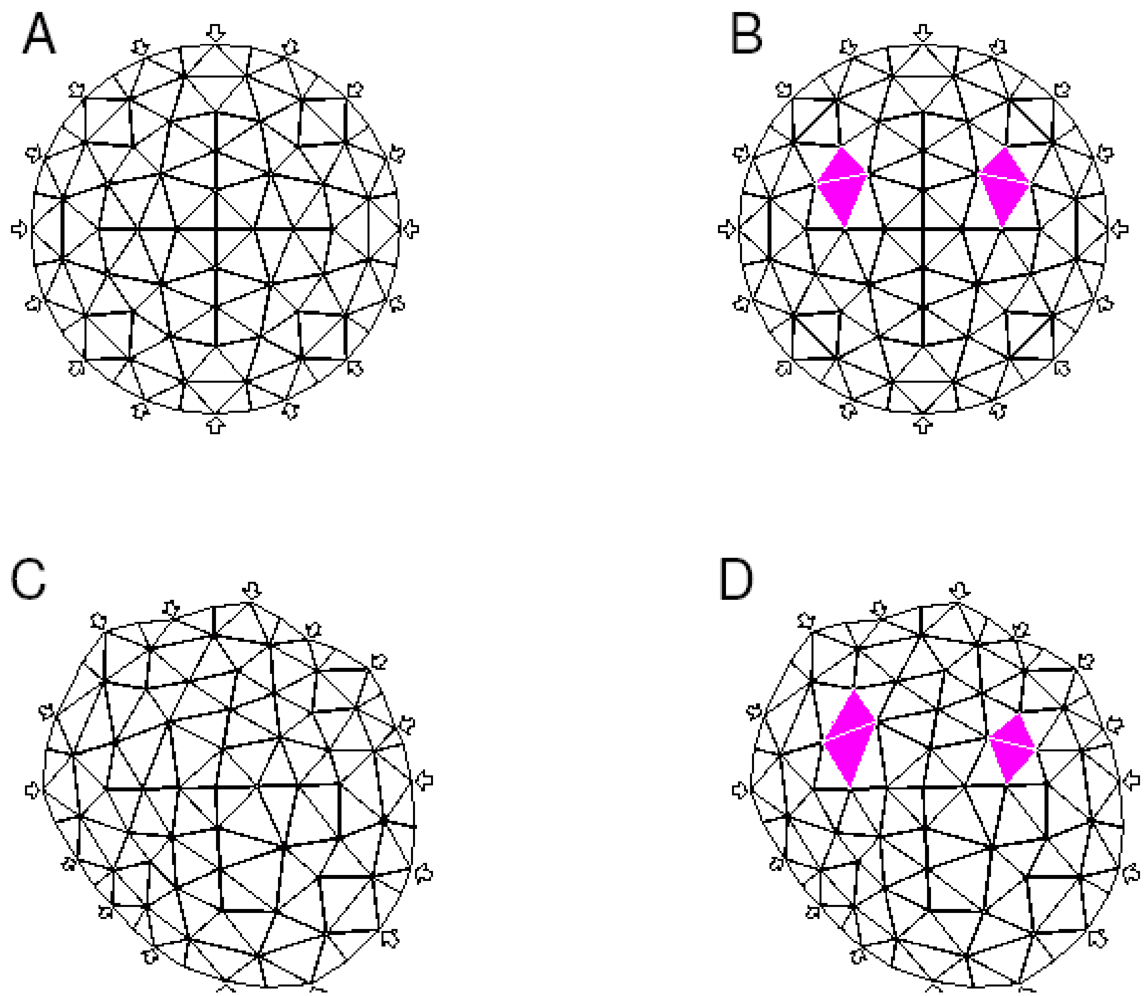


Figure 5.1: A measurement configuration with geometrical and electrode placement error

Dynamic Imaging: Example

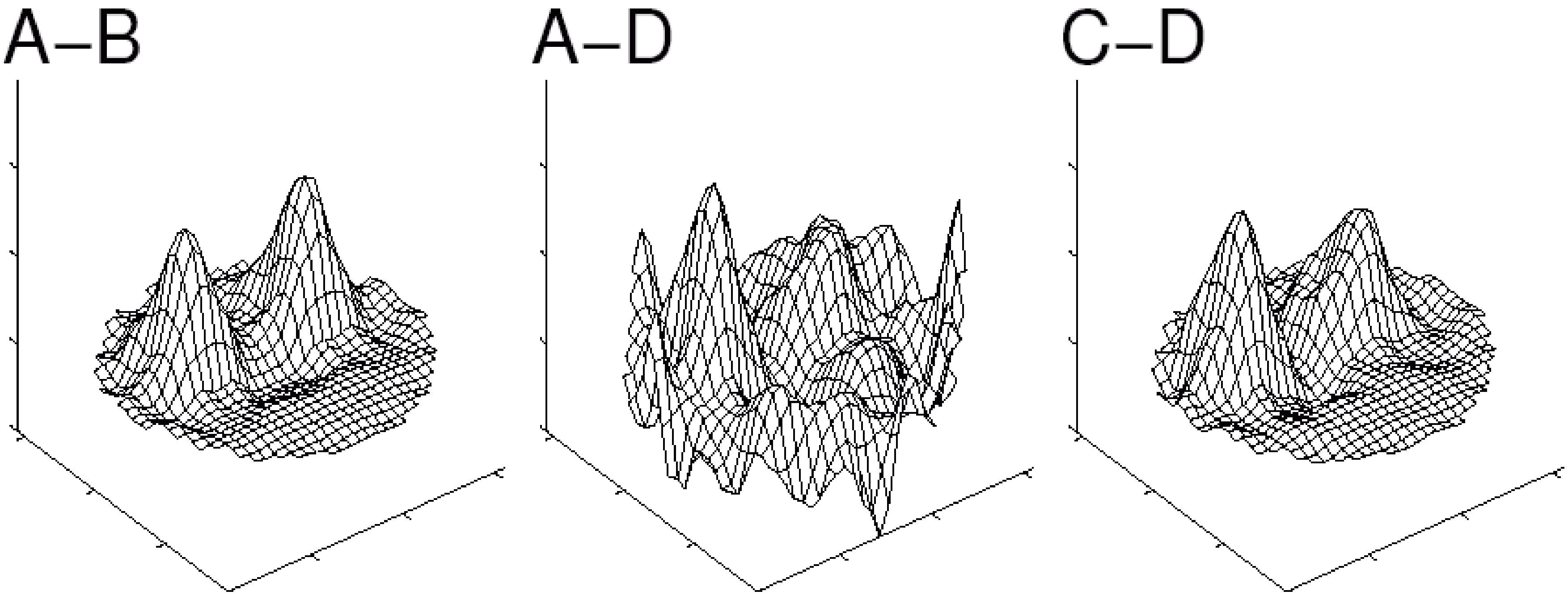
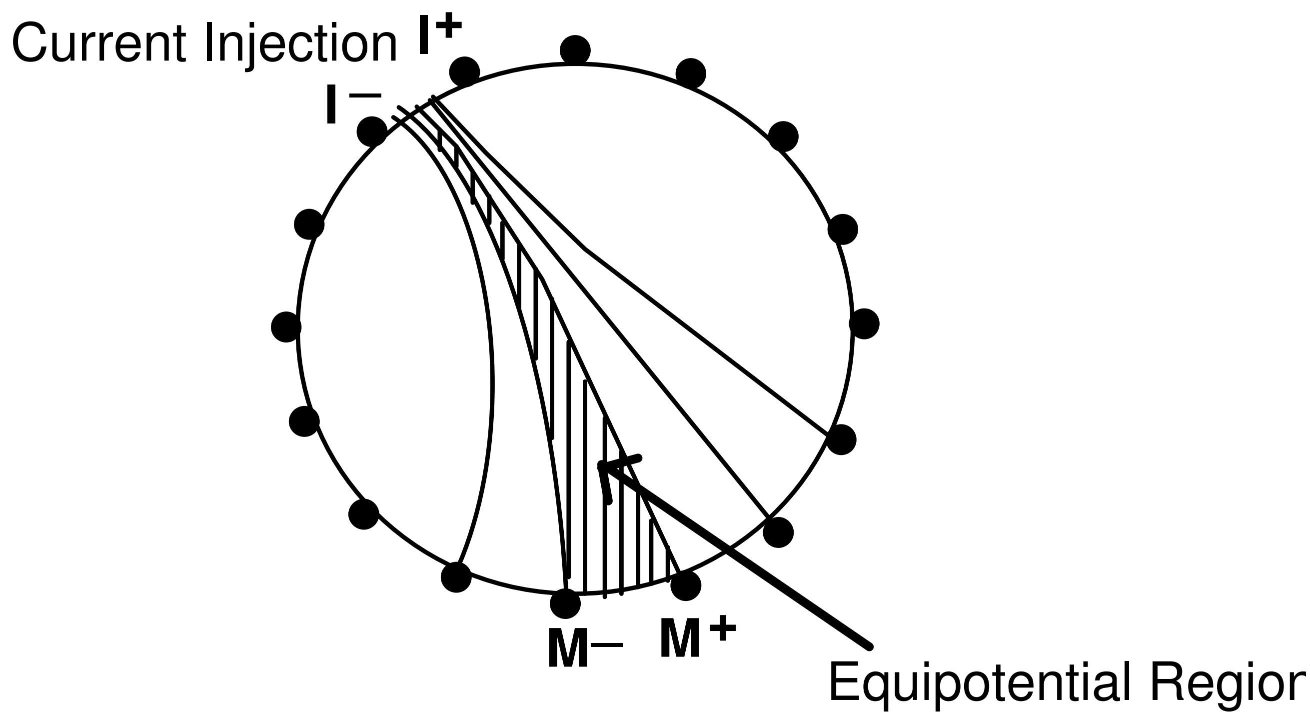


Figure 5.2: Images from media with geometrical errors.
A: Measurements A-B
B: Measurements A-D
C: Measurements C-D

Dynamic imaging techniques

- *Backprojection:* Voltages are "projected" along the equipotential lines.



Dynamic imaging: Backprojection

- Technique used in early studies (mid-1980's to early 1990's)
- Based on analogy with C.T.
- Not appropriate because
  - Measurements don't really come from equipotential region
  - Not symmetric

Inverse Techniques

- We can pose dynamic imaging as linear inverse, using a *sensitivity matrix*

$$\mathbf{z}_j = \frac{\mathbf{z}(\sigma_h) - \mathbf{z}(\sigma_h + \delta_j)}{\delta_j}$$

$$\mathbf{z} = \mathbf{H}\Delta\sigma$$

Parametrize Conductivity

- We want to parameterize conductivity
  - So that all reconstructed valued are physically valid
  - To reflect physical importance of low and high values
- Most common parameterization is r = log(conductivity)

Parameterization

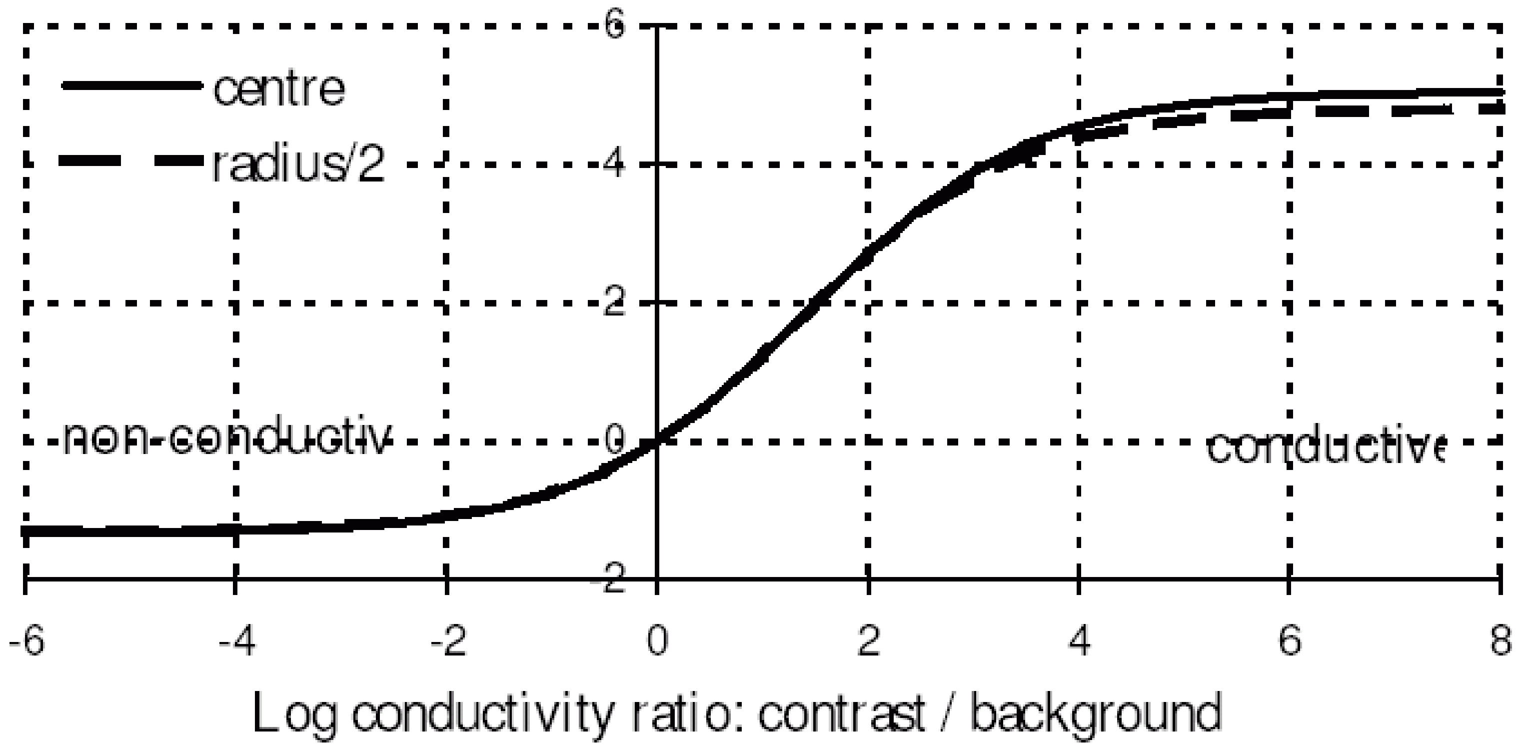


Figure 4.2: Normalised mean signal vs. change in log conductivity contrast ratio.

Inverse Techniques

- Classic least-squares inverse

$$\mathbf{z} = \mathbf{H}\mathbf{x}$$

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t\mathbf{H}\right)^{-1}\mathbf{H}^t\mathbf{z}$$

Matrix Techniques

However, problem is:

- ill-conditioned: measurements depend much more on data near electrodes than in centre
- ill-formed: more unknowns than measurements

Regularized Imaging

Handwaving argument for regularization:
used for ill-posed and ill-formed problems to find a solution with:

- Low error: small ($\mathbf{z} - \mathbf{H}\mathbf{x}$)
- Stable: small change in $\mathbf{x}$ for small $\mathbf{\Delta z}$
- Good looking:
  - Somewhat hard to define, but includes smoothness, clean edges, etc.

MAP estimates

- MAP approach says choose **x** such that $f(\mathbf{x}|\mathbf{z})$ is maximized
  - In other words, choose the image that is most likely, considering the measured data
- Bayes Rule

$$f(\mathbf{x}|\mathbf{z}) = \frac{f(\mathbf{z}|\mathbf{x})\, f(\mathbf{x})}{f(\mathbf{z})}$$

MAP estimates

$f(\mathbf{z}|\mathbf{x})$ the distribution of measurements given an image

- Based on forward model and noise properties

$f(\mathbf{z})$ distribution of measurements

- Not a parameter of MAP estimate

$f(\mathbf{x})$ distribution of image

- Based on *a priori* knowledge of physically possible and likely images distributions

Regularized Imaging

Given Linear Model:

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Maximum A Posteriori (MAP) estimate is:

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t \mathbf{R}_n^{-1} \mathbf{H} + \mathbf{R}_x^{-1}\right)^{-1} \left(\mathbf{H}^t \mathbf{R}_n^{-1} z + \mathbf{R}_x^{-1} \mathbf{x}_\infty\right)$$

Regularized Imaging

- Parameters $\mathbf{R_x}$, $\mathbf{R_n}$, x_∞ , represent *a priori* statistical knowledge of problem

$$\mathbf{x}_\infty = E[\mathbf{x}]$$

$$\mathbf{R_x} = E\left[(\mathbf{x} - \mathbf{x}_\infty)^t (\mathbf{x} - \mathbf{x}_\infty)\right] = E[\mathbf{x}^t \mathbf{x}] - \mathbf{x}_\infty{}^t \mathbf{x}_\infty$$

$$\mathbf{R_n} = E[\mathbf{n}^t \mathbf{n}] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots \\ 0 & \sigma_2^2 & \\ \vdots & & \ddots \end{bmatrix}$$

Choice of parameter $\mathbf{R_x}$

- Parameter is a “penalty function”
- Many regularization approaches use a diagonal matrix
  - Tikhonov regularization uses the scaled identity matrix
  - This will penalize large amplitude pixels in image
- We choose a dense matrix
  - Penalize image frequency content above maximum possible with measurements

Choice of parameter $\mathbf{R_x}$

- In order to avoid problems inverting $\mathbf{R_x}$, we directly calculate the inverse
  - Since $\mathbf{R_x}$ represents spatial low pass filter, $\mathbf{R_x^{-1}}$ represents a high pass
- Choose a Gaussian high pass of form

$$F(u,v) = 1 - e^{-\omega_0\left(u^2 + v^2\right)}$$

Regularization: Hyperparameters

Regularizations techniques must finally introduce a “hyperparameter” (μ)

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t \mathbf{W} \mathbf{H} + \mu \mathbf{Q}\right)^{-1} \left(\mathbf{H}^t \mathbf{W} \mathbf{z} + \mu \mathbf{Q} \mathbf{x}_\infty\right)$$

where

$$\mathbf{W} = \frac{1}{\sigma_n^2} \mathbf{R}_\mathbf{n}^{-1}$$, ie. the relative noise amplitudes

$$\mathbf{Q} = \frac{1}{\sigma_x^2} \mathbf{R}_\mathbf{x}^{-1}$$, ie. the relative image correlations

Regularization: Hyperparameters

μ is thus the ratio of image and noise amplitudes,

$$\mu = \frac{\sigma_x^2}{\sigma_y^2}$$

it can be interpreted as a filter noise figure

Regularized Inverse

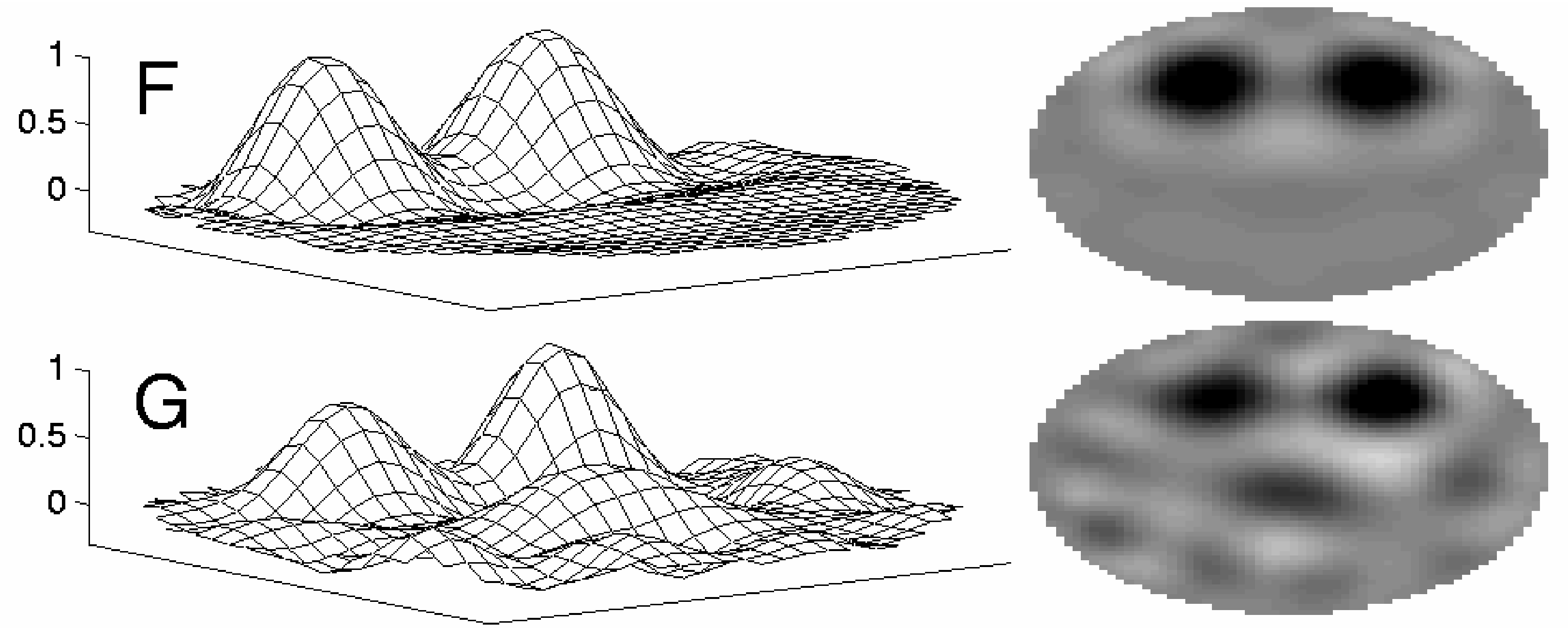
Parameters:

- **W**: models measurement noise
- **Q**: penalizes image features which are greater than data supports
- $\mathbf{x}_\infty$: represents the background conductivity distribution (heart,lungs,etc)
- **μ**: “hyper-parameter” amount of regularization

Advantages of Regularization

- Stabilizes ill-conditioned inverse
- Introduction of *a priori* information
- Control of *resolution-noise* performance trade-off
- MAP inverse justifies the formulation in terms of Bayesian statistics

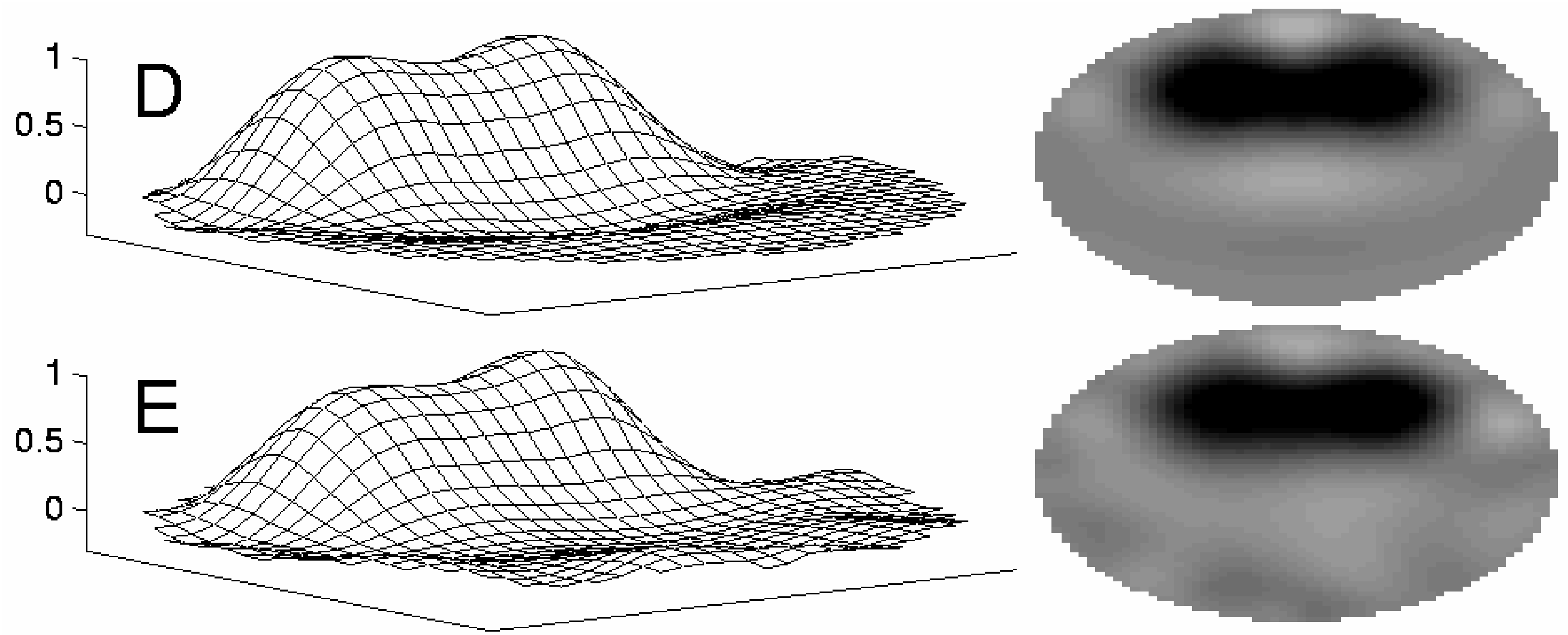
Noise – Resolution Tradeoff



F: *Meas:* No Noise *Reconst:* NF= 2.0

G: *Meas:* -3dB SNR *Reconst:* NF= 2.0

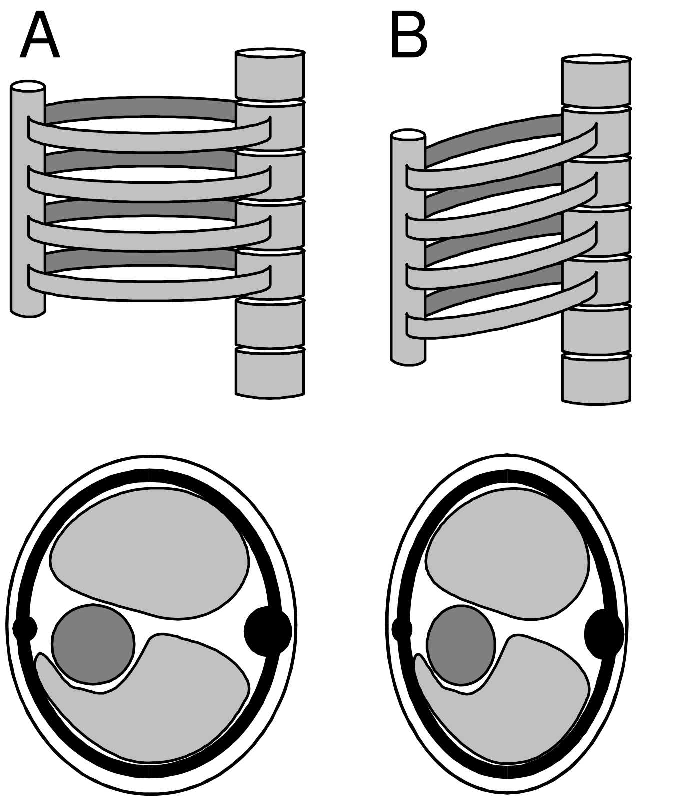
Noise – Resolution Tradeoff



F: *Meas:* No Noise *Reconst:* NF= 0.4

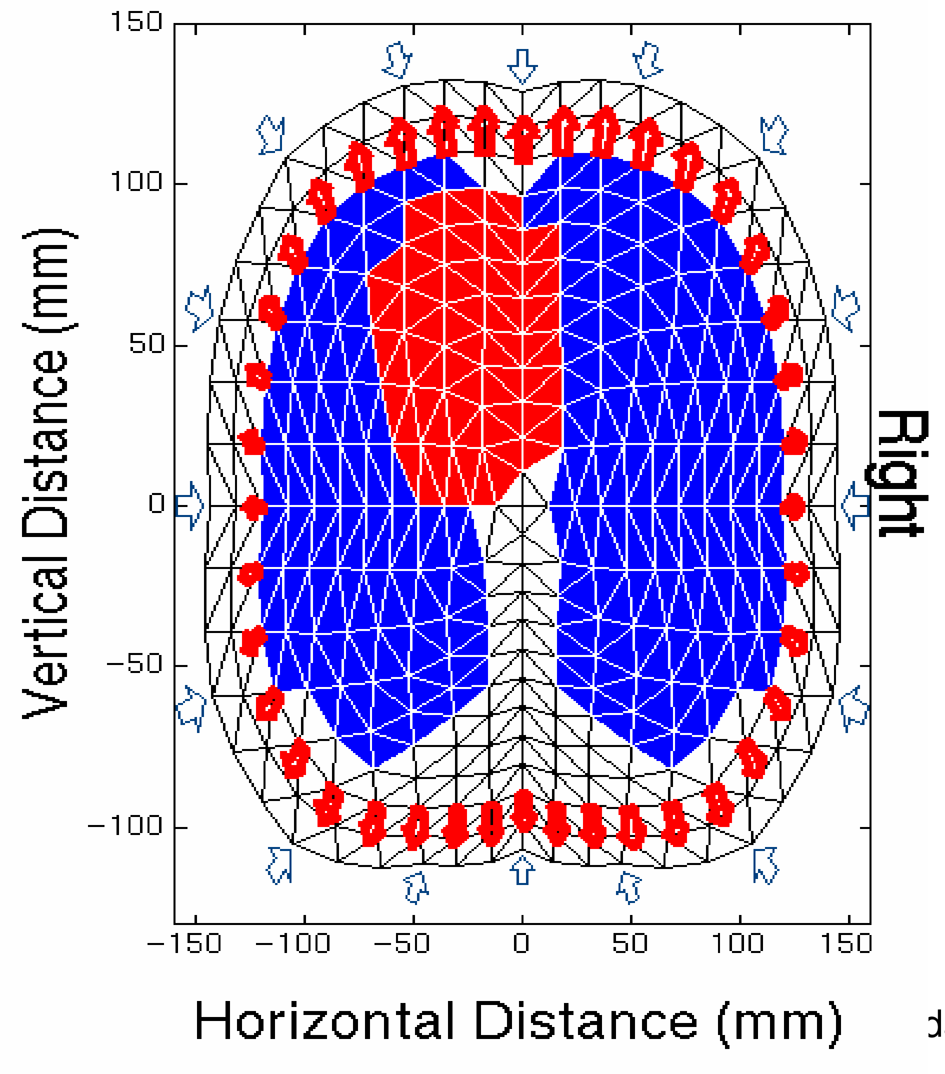
G: *Meas:* -3dB SNR *Reconst:* NF= 0.4

Electrode Movement



- Electrodes on chest move during breathing
- Figure shows rough movement pattern of rib cage

Electrode Movement



- FEM of electrical and mechanical properties of thorax to simulate
- Signal from expansion shown to contribute 10-20% of conductivity signal

Applications: Lung Function and Disease

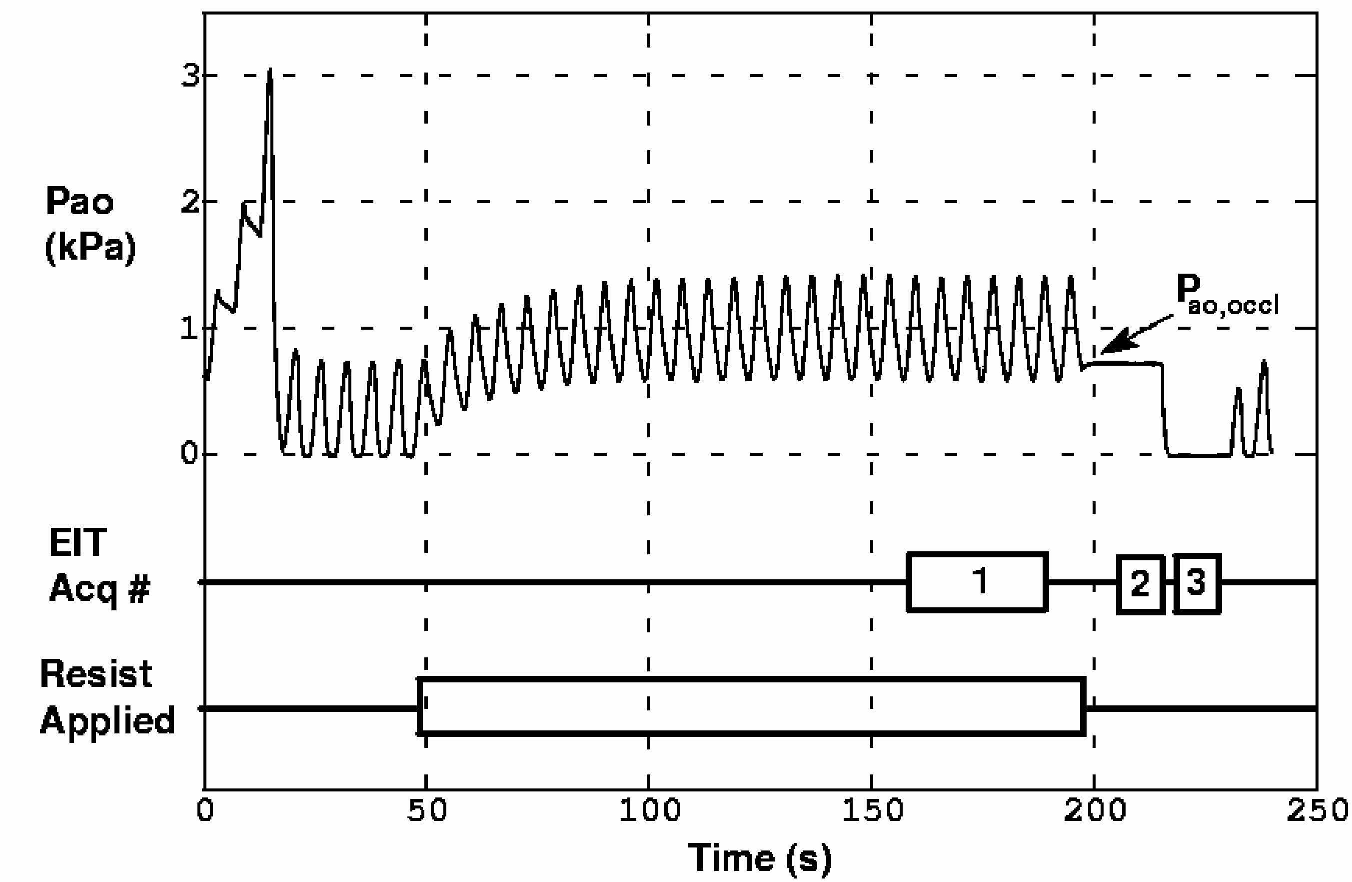
Monitoring of lung function is important in ICU. Issues are:

- Distribution of ventilation
- Ventilation – perfusion match
- Lung Edema
- Flow limitation (obstructive lung disease)

Lung function tests

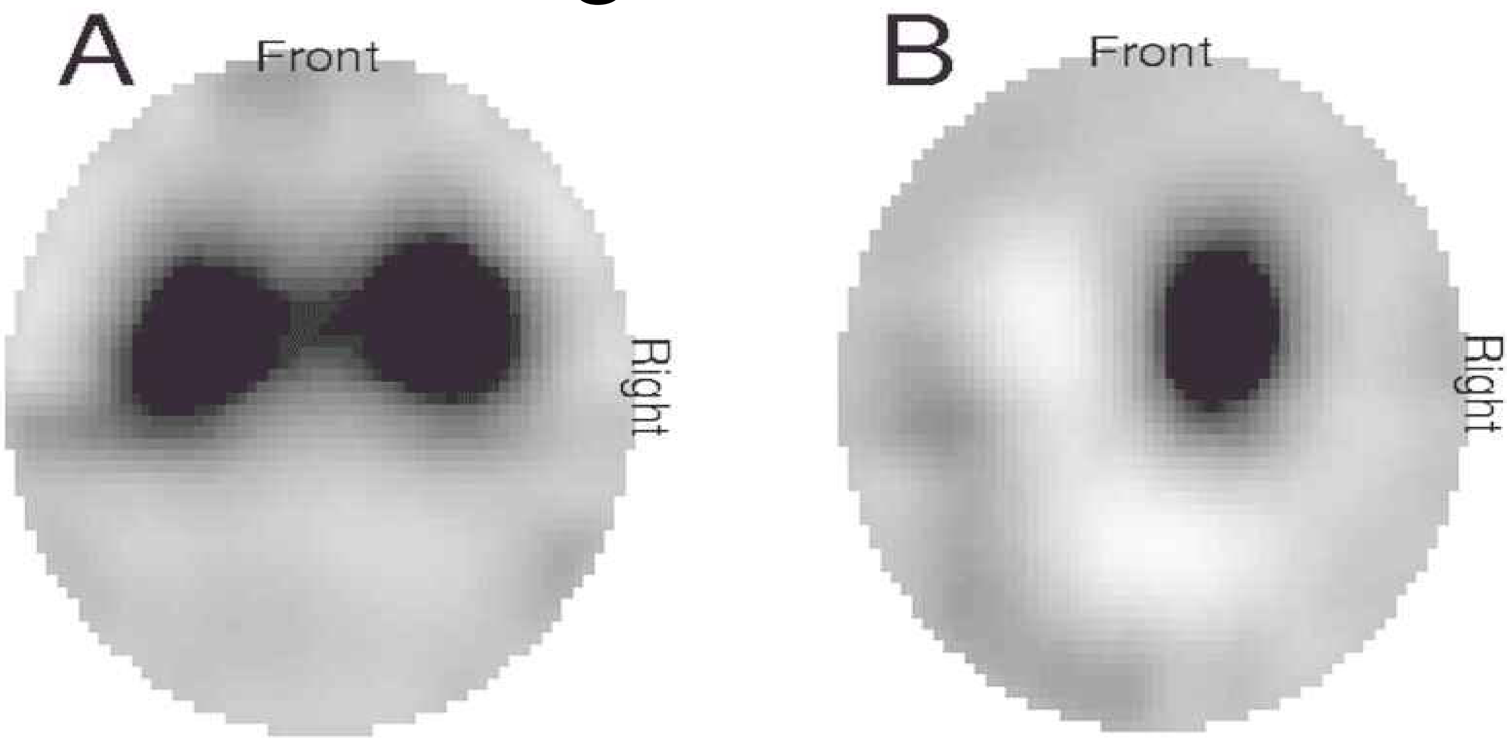
- Patients with obstructive lung disease (emphysema, bronchitis) tend to dynamically hyperinflate: lower inspiration than expiration.
- Simulated with PEEP (positive end expiratory pressure)

Lung Function: Protocol



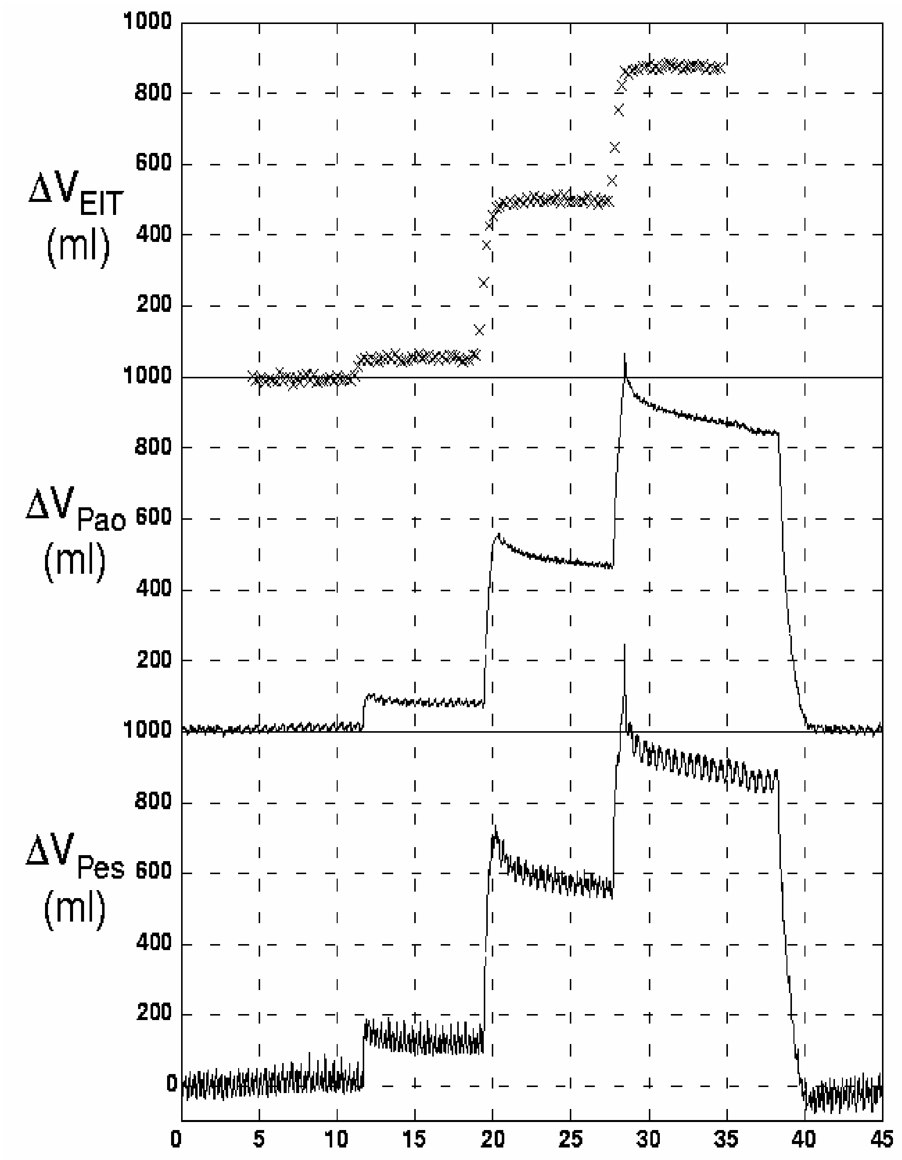
- Initially, inflations to 3 kPa
- dog expires to the applied PEEP
- ventilation stopped, and dog expires against PEEP for 15 s

Lung Function



A: image of section thorax due to ΔVL 800 ml

B: image due to a ΔVL 400 where left main stem bronchus was plugged.

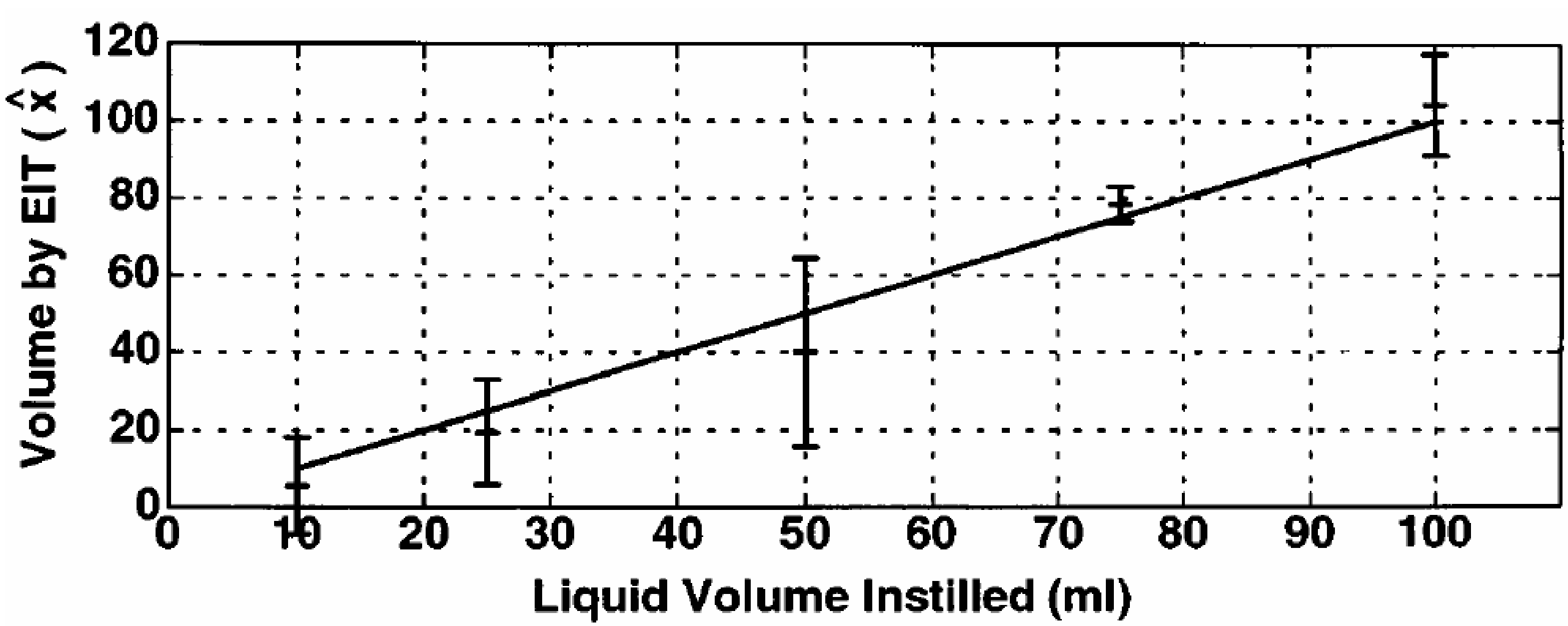


- Volume estimates by EIT, Pao and Pes, after step volume increases of 100ml, 500ml, 900ml.
- Note that EIT signal does not display overshoot

Pulmonary Edema

- Pulmonary Edema (lungs filled with fluid) plagues ICU patients for heart disease, accident victims
- Typical monitoring techniques are invasive (Swan-Ganz catheter)
- We looked at EIT ability to monitor level of Edema.

Pulmonary Edema: Results



- Change in lung liquid volume by EIT vs liquid volume instilled

Imaging

- Algorithms to use data from internal electrodes
- How to manage electrode artefacts
- Arbitrary electrode placements
- Image interpretation and error bounds given artefacts and noise

Applications of interest

- Longer term monitoring in ICU
- Small animal lung function
- Contrast agents
- Inhomogeneous ventilation on smaller scale