SYSC 4405: Midterm Exam #1 (Version #:1) October 22, 2012
Carleton University, Systems and Computer Engineering

Background: After graduation (in about 6 months!!), you get a job as an engineer at a company designing security systems. They want to be able to record any sound which occurs at the site where clients install their systems. However, they don't want to continuously record the input, since much of the time there is no sound. Your task is to design a system to detect when the sound amplitude passes a threshold and to trigger the recording of sound.

Consider that the input sound $x(t)$ at the microphone is as follows:

$$x(t) = 0.1\,\sin\big(2\pi(110\text{ Hz})(t-1.5\text{s})\big)u(t-1.5\text{s}) + 0.02\,\sin\big(2\pi(1110\text{ Hz})(t-1.5\text{s})\big)u(t-1.5\text{s})\ \ \text{Volts}$$

The signal is acquired by an ADC, based on a Sample/Hold (S/H) circuit and an 8-bit flash converter. The sampling rate is 1.0 kSample/sec.

1. (1 point) Your exam is version # **1**. **Write down this number.**

   ANSWER:
   This should be easy

2. (5 points) **Sketch the ADC system and label the components,** including the analog input, Sample/Hold (S/H), converter and digital output. **Sketch the signal for** $1.498 \le t \le 1.502$ **s. at the input (**$x(t)$**) and at the output of the S/H (**$x_{\text{SH}}(t)$**).** *Note:* The values of $x(t)$ do not need to be exact, but the values of $x_{\text{SH}}(t)$ should be.

   ANSWER:
   x(1.498s) = x[1.498×1.0kHz] = x[1498] = 0
   x(1.499s) = x[1.499×1.0kHz] = x[1499] = 0
   x(1.500s) = x[1.500×1.0kHz] = x[1500] = sin(?×$(t-1.5s)$) = 0
   x(1.501s) = x[1.501×1.0kHz] = x[1501] = 0.1sin($2\pi \times 110 \times .001s$) + 0.02sin($2\pi \times 1110 \times .001s$) = .1*sin(2*pi*110*.001) + .02*sin(2*pi*1110*.001) = 0.076491
   x(1.502s) = x[1.502×1.0kHz] = x[1502] = 0.1sin($2\pi \times 110 \times .002s$) + 0.02sin($2\pi \times 1110 \times .002s$) = .1*sin(2*pi*110*.002) + .02*sin(2*pi*1110*.002) = 0.11787

3. (5 points) An 8-bit differential ADC is used with $V_{\max} = 1.2$ V, and outputs a quantized signal, $x_q[n]$. **What are the values of** $x_q[n]$ **corresponding to** $1.498 \le t \le 1.502$ **s?**

   ANSWER:
   $\Delta = 2V_{max}/2^L = 2 * 1.2V/256 = 0.00938V$
   $x[1498] = 0 \Rightarrow x_q[1498] = 0$
   $x[1499] = 0 \Rightarrow x_q[1499] = 0$
   $x[1500] = 0 \Rightarrow x_q[1500] = 0$
   $x[1501] = 0.0765$. Quantization Levels = $0.0765/.00938 = 8.2$ Thresholds will be at 7.5 and 8.5, so this is at quantization level 8 above 0.
   $x_q[1501] = 8 \times \Delta = 0.0750$
   $x[1502] = 0.118$. Quantization Levels = $0.118/.00469 = 12.6$ Thresholds will be at 12.5 and 13.5, so this is at quantization level 12 above 0.
   $x_q[1502] = 25 \times \Delta = 0.1219$.

4. (5 points) **Does clipping occur for this signal.** Given only the signal $x_q[n]$, **describe a way to get an indication of whether clipping has occured.**

ANSWER:
A) There is no clipping in this signal ($|V| < V_{max}$)
B) If a quatized signal equals the A/D limits, then clipping should be suspected.

5. (5 points) **Calculate the magnitude of quantization noise in $x_q[n]$, in units of V^2.**

   ANSWER:
   Quantization Noise = $\frac{\Delta^2}{12}$ = 1.8330×10^{-6} V^2

6. (5 points) **For each of the two sinusoidal components in the input signal, calculate their representation in $x[n]$. Label each component as to whether it is a) Aliased or not, and b) Folded or not.**

   ANSWER:
   Nyquist frequency is $\frac{1}{2}1.0$ kHz. A) Not aliased ($2\pi(110$ Hz$) < \frac{1}{2}1.0$ kHz). B) Aliased ($2\pi(1110$ Hz$) > \frac{1}{2}1.0$ kHz). (Folded freqs are, F, s.t. $\frac{1}{2}1.0 < F < 1.0$ kHz). Thus, B is not folded.

7. (5 points) In order to detect the start of the sound, the input signal is processed as follows:

   $$x(t) \rightarrow \boxed{\text{Absolute Value}} \rightarrow \boxed{\text{Low Pass Filter}} \rightarrow y(t)$$

   after this process, the signal is compared to a threshold (which we don't consider here). **Is this system: a) Linear, b) Shift Invariant, c) Memoryless, d) LSI, e) Causal, f) Stable?**

   ANSWER:
   a) Linear = NO b) Shift Invariant = YES c) Memoryless = NO d) LSI = NO e) Causal = YES f) Stable = YES

8. (5 points) For the system in Q#7, **sketch the expected output for the input sound signal,** for the period $1.2 \leq t \leq 1.6$ s. (values do not need to be exact)

   ANSWER:
   Output starts at zero; after start of sine waves, the LPF output lowly increases to a saturation level.

9. (5 points) For the system in Q#7, the low pass filter

   $$y[n] = 0.98y[n-1] + (1 - 0.98)(x[n-10] + x[n-11])$$

   is used. **Sketch the system block diagram**, (using Delay$\times X$) to indicate longer delays, if needed) and **calculate $y[n]$ for n corresponding to $1.498 \leq t \leq 1.502$ s, assuming no initial conditions.**

   ANSWER:
   Block diagram has a) feedforward direction: a delay of 10 (then gain of 0.02) and another delay (to a gain of 0.02. On the feedback path, there is a single delay (to a gain of 0.98.

10. (10 points) For the LPF in Q#9
    A) **calculate the impulse response, $h[n]$.**
    B) **calculate the DTFT, $H(e^{j\omega})$.**

    ANSWER:
    A)
    $$h[n] = 0.02\left(0.98^{n-10}u[n-10] + 0.98^{n-11}u[n-11]\right). \quad (1)$$

B)

$$\begin{aligned}
H(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} h[n]e^{-jn\omega} && (2)\\
&= 0.02\left(\sum_{n=10}^{\infty} 0.98^{n-10}e^{-jn\omega} + \sum_{n=11}^{\infty} 0.98^{n-11}e^{-jn\omega}\right) && (3)\\
&= 0.02\left(\sum_{k=0}^{\infty} 0.98^{k}e^{-j(k+10)\omega} + \sum_{h=0}^{\infty} 0.98^{h}e^{-j(h+11)\omega}\right) && (4)\\
&= 0.02\left(e^{-10j\omega}\sum_{k=10}^{\infty}\left(0.98e^{-j\omega}\right)^{k} + e^{-11j\omega}\sum_{h=10}^{\infty}\left(0.98e^{-j\omega}\right)^{h}\right) && (5)\\
&= 0.02\left(\frac{e^{-10j\omega}}{1-0.98e^{-j\omega}} + \frac{e^{-11j\omega}}{1-0.98e^{-j\omega}}\right) && (6)\\
&= 0.02\left(\frac{e^{-10j\omega}+e^{-11j\omega}}{1-0.98e^{-j\omega}}\right) && (7)
\end{aligned}$$

where we set $k = n - 10$ in the first, and $h = n - 11$ in the second sum.