SYSC 4405: Final Exam, #1
December 21, 2011
Systems and Computer Engineering
Carleton University,

Instructions:

- This exam has **3** pages and **16** questions. Answer all questions.
- You have **180 minutes** to complete this exam.
- This is a closed book exam; however, you are permitted to bring one 8.5" $\times$ 11" sheet of notes.
- You are permitted to use a non-programmable calculator.
- Write your answers on an examination booklet. You may take this examination paper with you.
- An ideal low pass filter with cutoff frequency ω_c has impulse response: $h_{LP}[n] = \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c}{\pi} n\right)$
- The FFT requires $Nlog_2(N)$ complex additions and $1 - N + \frac{1}{2}Nlog_2(N)$ complex multiplications.
- Filter windows have the form: $w[n] = a_0 + a_1 cos(\pi\frac{n-L}{L}) + a_2 cos(2\pi\frac{n-L}{L}) + a_3 cos(3\pi\frac{n-L}{L})$, where

Window Name	Atten. (dB)	TBW (/L)	a_0	a_1	a_2	a_3
Rectangular	20.8	0.46	1	0	0	0
Hann	43.9	1.56	0.5	0.5	0	0
Hamming	53.9	1.90	0.53836	0.46164	0	0
Blackman	75.3	2.79	0.42	0.5	0.08	0
Blackman-Nutall	112.7	4.09	0.363582	0.489178	0.136510	0.010641

Background: After graduation, you get a job working for an internet telephony company, producing a product designed to compete with skype.com. Like all internet telephony products, the user's voice is recorded from a microphone, processed to remove noise, assembled into packets and then sent through the internet to the receiver. You are assigned to work on a team which filters the users voice. The key challenges are that the characteristics of the user and microphone are unknown, and many microphones have several resonances. Also, a users voice will echo, especially if they are not close to the microphone.

The designed system should work for older computers, and it has been specified that the minimum CPU speed for the software is 0.5 GHz. For calculations based on this chip you may assume that: 1) multiplications of complex numbers require 3 clock cycles; 2) additions of complex numbers require 1 clock cycle; and 3) delays and other operations take no time.

The voice signal you record has frequency content in the range 20–20 kHz. It is filtered with an analog anti-aliasing low pass filter with a cut-off frequency of 3 kHz and then sampled at a sampling frequency of 8.0 kHz, using an 12 bit A/D converter.

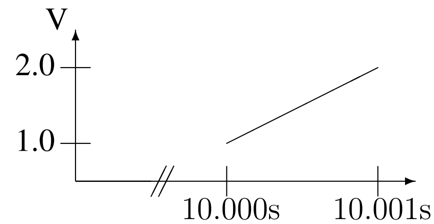
If you do not have the answer to a previous question, state your assumptions at the beginning of the answer. Full marks will be given for any question in which assumed values are used correctly.

1. (1 point) Your exam is exam number **1**. Write down this number.

2. (5 points) The voice input from the microphone is $x(t)$ which is digitized to get $x[n]$. $x[n]$ is then filtered using a DSP filter to get $y[n]$. **Sketch the system components as a block diagram.** On your

diagram, label $x[n]$, $y[n]$, $x(t)$, and the a) DSP filter, b) Analog to digital converter, c) Sample/Hold, d) Clock, and e) Analog Low pass filter.

3. (5 points) Consider a short segment of voice input, $x(t)$, during the time $10.0\ s \leq t \leq 10.001\ s$ where the input is rising linearly from $x(10.000 \text{ s}) = 1.0$ V to $x(10.001 \text{ s}) = 2.0$ V. **Sketch the signal** $x[n]$ during this period (assume you can ignore the antialiasing filter).



4. (5 points) For the first four samples of $x[n]$ starting at $t = 10.0$ s, **calculate the output** $y[n]$ if $x[n]$ is passed through a filter $H(z)$, where

$$H(z) = \frac{1 + z^{-1} + 2z^{-2}}{1 - z^{-1} - 2z^{-2}}.$$

Assume samples of $y[n]$ before $t = 10.0s$ are zero.

5. (5 points) The A/D converter has an input range covering -5 V to 5 V. **Calculate the value of the first two quantized samples,** $x_q[n]$, starting at $t = 10.0$ s. **What is the quantization error for each sample?**

6. (5 points) Another member of your team has built some software which identifies resonances in the microphone, which your software will need to filter. The software identifies a resonance at $f_r = 1.3$ kHz. **What normalized frequency $\hat{\omega}_r$ corresponds to this resonance?** To filter this resonance, a filter with poles, $p_{1,2}$, and zeros, $z_{1,2}$.

$$p_{1,2} = 0.7e^{\pm j\hat{\omega}_r}, z_{1,2} = 1.1e^{\pm j\hat{\omega}_r}$$

Sketch the location of the poles and zeros in the z-plane.

7. (5 points) **Calculate the filter** $H_r(z)$ which has the poles and zeros in the previous question with a gain (b_0) of 10.0. **What is** $|H_r(e^{j\hat{\omega}})|$ **at DC** (ie. $\hat{\omega} = 0$?)

8. (5 points) **What is the ROC of** $H_r(z)$**?** Is the filter: a) stable? b) linear? c) memoryless? d) shift-invariant? e) causal?

9. (5 points) **Sketch the magnitude of the frequency response** of $H_r(z)$, labelling the frequency of any maximum or minimum values.

10. (5 points) **Sketch the DSP block diagram to implement the filter** $h_r[n]$**,** corresponding to $H_r(z)$. **How many clock cycles are required for each sample processed by** $h_r[n]$**?**

11. (5 points) In this question, we choose to implement resonance cancellation in an FIR filter (rather than the IIR filter in Q6). We require frequencies $1.2 \leq f \leq 1.4$ kHz to have a gain of 2.0 ± 0.1, while frequencies $f \leq 1.0$ kHz and $f \geq 1.6$ kHz must have a gain of 10.0 ± 0.1. **Sketch the filter requirements.**

12. (5 points) The filter in Q11 is equivalent to a filter passing all frequencies (at a gain 10.0) minus a band pass filter (with a gain of 8.0). **Write an expression for an ideal (ie. non-causal) filter** $h_{ideal}[n]$ which meets the requirements of Q11.

13. (5 points) Design a realizable (ie. causal, finite-length) FIR filter, $h_{FIR}[n]$, which meets the filter requirements of Q11. **Choose an appropriate filtering window function $w[n]$; calculate the FIR filter length, M; and, write an expression for $h_{FIR}[n]$**

14. (5 points) We use overlap-add block processing to implement this filter, using an FFT size of $N = 1024$. **Calculate the size, B, of signal $x_B[n]$ processed per block. What time is required to process each block on the minimum specified CPU speed?**

15. (5 points) The latency is the delay between recording the sound and it arriving at the destination computer. Assuming the network takes 10 ms to transmit each packet (each containing one block of size N). **What is the maximum latency?** For calculation of the latency, include the time for a) sampling, b) DSP calculations (previous question) and c) network transmission.

16. (5 points) **Estimate the minimum length of FIR filter required to do echo cancellation**, if we care about echos from objects 34 cm from both the speaker and the microphone (and the speed of sound in air is ≈ 340 m/s).