

Question 1 (1 point): What is your name and student number?

Name: _____ Lab Section: _____

Student Number: _____

Question 2 (4 points): Calculate the transfer function - H[z] for the difference equation below, then find the inverse Z-transform of H[z] with its ROC.

$$y[n] = 5 x[n] - 0.2 y[n-1] + 0.24 y[n-2]$$

Solution

$$y[n] + 0.2 y[n-1] - 0.24 y[n-2] = 5 x[n]$$

$$Y(z) (1 + 0.2z^{-1} - 0.24z^{-2}) = 5 X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{5}{(1 + 0.2z^{-1} - 0.24z^{-2})} = H(z)$$

$$H(z) = \frac{5}{(1 + 0.6z^{-1})(1 - 0.4z^{-1})} = \frac{A_1}{(1 + 0.6z^{-1})} + \frac{A_2}{(1 - 0.4z^{-1})}$$

poles $z = -0.6, 0.4$ or $z^{-1} = -5/3, 2.5$

$$A_1 = \frac{5}{(1 - 0.4z^{-1})} \Big|_{z=-0.6} = \frac{5}{(1 - 0.4/-0.6)} = \frac{5}{(5/3)} = 3$$

$$A_2 = \frac{5}{(1 + 0.6z^{-1})} \Big|_{z=0.4} = \frac{5}{(1 + 0.6/0.4)} = \frac{5}{(5/2)} = 2$$

$$H(z) = \frac{3}{(1 + 0.6z^{-1})} + \frac{2}{(1 - 0.4z^{-1})}$$

$$h[n] = 3 (-0.6)^n u[n] + 2 (0.4)^n u[n] \quad \text{ROC } |z| > 0.6$$

some useful formulas

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$H(z) = \sum_{k=1}^N \frac{A_k}{1 - d_k z^{-1}}$$

$$A_k = (1 - d_k z^{-1}) H(z) \Big|_{z=d_k}$$

$$Z^{-1}\{X(z)\} = x[n]$$

Time-domain \Leftrightarrow z-domain

$$\delta[n] \Leftrightarrow 1$$

$$\delta[n - n_d] \Leftrightarrow z^{-n_d}$$

$$f[n - n_d] \Leftrightarrow z^{-n_d} F(z)$$

$$A a^n u[n] \Leftrightarrow \frac{A}{1 - a z^{-1}} \quad \text{ROC: } |z| > |a|$$

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$$y[n] = 9x[n] + 0.5y[n-1] + 0.14y[n-2]$$

Solution

$$y[n] - 0.5y[n-1] - 0.14y[n-2] = 9x[n]$$

$$Y(z)(1 - 0.5z^{-1} - 0.14z^{-2}) = 9X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{9}{(1 - 0.5z^{-1} - 0.14z^{-2})} = H(z)$$

$$H(z) = \frac{9}{(1 + 0.2z^{-1})(1 - 0.7z^{-1})} = \frac{A_1}{(1 + 0.2z^{-1})} + \frac{A_2}{(1 - 0.7z^{-1})}$$

poles $z = -0.2, 0.7$ or $z^{-1} = -5, 1/7$

$$A_1 = \frac{9}{(1 - 0.7z^{-1})} \Bigg|_{z=-0.2} = \frac{9}{(1 - .7/-.2)} = \frac{9}{(9/2)} = 2$$

$$A_2 = \frac{9}{(1 + 0.2z^{-1})} \Bigg|_{z=0.7} = \frac{9}{(1 + .2/.7)} = \frac{9}{(9/7)} = 7$$

$$H(z) = \frac{2}{(1 + 0.2z^{-1})} + \frac{7}{(1 - 0.7z^{-1})}$$

$$h[n] = 2(-0.2)^n u[n] + 7(0.7)^n u[n] \quad \text{ROC } |z| > 0.7$$

some useful formulas

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$H(z) = \sum_{k=1}^N \frac{A_k}{1 - d_k z^{-1}}$$

$$A_k = (1 - d_k z^{-1}) H(z) \Big|_{z=d_k}$$

$$Z^{-1}\{X(z)\} = x[n]$$

Time-domain \Leftrightarrow z-domain

$$\delta[n] \Leftrightarrow 1$$

$$\delta[n - n_d] \Leftrightarrow z^{-n_d}$$

$$f[n - n_d] \Leftrightarrow z^{-n_d} F(z)$$

$$Aa^n u[n] \Leftrightarrow \frac{A}{1 - az^{-1}} \quad \text{ROC: } |z| > |a|$$