Quiz 5a Answer book

A. **Calculate the DFT of the sequence x[n]= {0,0,0,0,4,0,0,0}**

Answer: $W = e^{-2\pi j/N}$ Where N = 8

$W = e^{-\pi j/4}$ = (1-j) / root(2)

$W^2 = -j$ $W^4 = -1$ $W^6 = j$ $W^8 = 1$

W^3 = (-1-j) / root(2) W^5 = (1+j) / root(2) W^7 = (1+j) / root(2)

$$X[k] = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \\ 1 & W^2 & W^4 & W^6 & W^8 & W^{10} & W^{12} & W^{14} \\ 1 & W^3 & W^6 & W^9 & W^{12} & W^{15} & W^{18} & W^{21} \\ 1 & W^4 & W^8 & W^{12} & W^{16} & W^{20} & W^{24} & W^{28} \\ 1 & W^5 & W^{10} & W^{15} & W^{20} & W^{25} & W^{30} & W^{35} \\ 1 & W^6 & W^{12} & W^{18} & W^{24} & W^{30} & W^{36} & W^{42} \\ 1 & W^7 & W^{14} & W^{21} & W^{28} & W^{35} & W^{42} & W^{49} \end{pmatrix} [\mathbf{0,0,0,0,4,0,0,0}]^{\mathbf{T}}$$

$$= 4*[1 \quad W^4 \quad W^8 \quad W^{12} \quad W^{16} \quad W^{20} \quad W^{24} \quad W^{28}]^{\mathbf{T}}$$
$$= 4*[1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1]^{\mathbf{T}}$$
$$= [4 \quad -4 \quad 4 \quad -4 \quad 4 \quad -4 \quad 4 \quad -4]^{\mathbf{T}}$$

B. Calculate the IDFT of the sequence X[k]= {8,0,0,0,8,0,0,0}

Answer: $W = e^{-2\pi j/N}$ Where N = 8

IDFT Matrix is Conjugate of the DFT Matrix

$W = e^{-\pi j/4}$ = (1-j) / root(2)

$W^2 = -j$ $W^4 = -1$ $W^6 = j$ $W^8 = 1$
W^3 = (-1-j) / root(2) W^5 = (1+j) / root(2) W^7 = (1+j) / root(2)

$$x[n] = \frac{1}{8} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \\ 1 & W^2 & W^4 & W^6 & W^8 & W^{10} & W^{12} & W^{14} \\ 1 & W^3 & W^6 & W^9 & W^{12} & W^{15} & W^{18} & W^{21} \\ 1 & W^4 & W^8 & W^{12} & W^{16} & W^{20} & W^{24} & W^{28} \\ 1 & W^5 & W^{10} & W^{15} & W^{20} & W^{25} & W^{30} & W^{35} \\ 1 & W^6 & W^{12} & W^{18} & W^{24} & W^{30} & W^{36} & W^{42} \\ 1 & W^7 & W^{14} & W^{21} & W^{28} & W^{35} & W^{42} & W^{49} \end{pmatrix}^{*} [8,0,0,0,8,0,0,0]^{\mathbf{T}}$$

Note '*' means complex conjugate

$= [\ 8+8 \quad 8+8W^4 \quad 8+8W^8 \quad 8+8W^{12} \quad 8+8W^{16} \quad 8+8W^{20} \quad 8+8W^{24} \quad 8+8W^{28}]^{\mathbf{T}}/8$
$= [\ 1+1 \quad 1-1 \quad 1+1 \quad 1-1 \quad 1+1 \quad 1-1 \quad 1+1 \quad 1-1]^{\mathbf{T}}$
$= [\ 2 \quad 0 \quad 2 \quad 0 \quad 2 \quad 0 \quad 2 \quad 0]^{\mathbf{T}}$

C. We wish to calculate the convolution ($y[n]= h[n] * x[n]$) where

$x[n]= \{2,4,6,8,10,12,14,18\}$
$h[n]= ½\{1,1\}$

i. Using linear convolution, **calculate** $y[0]$ to $y[5]$

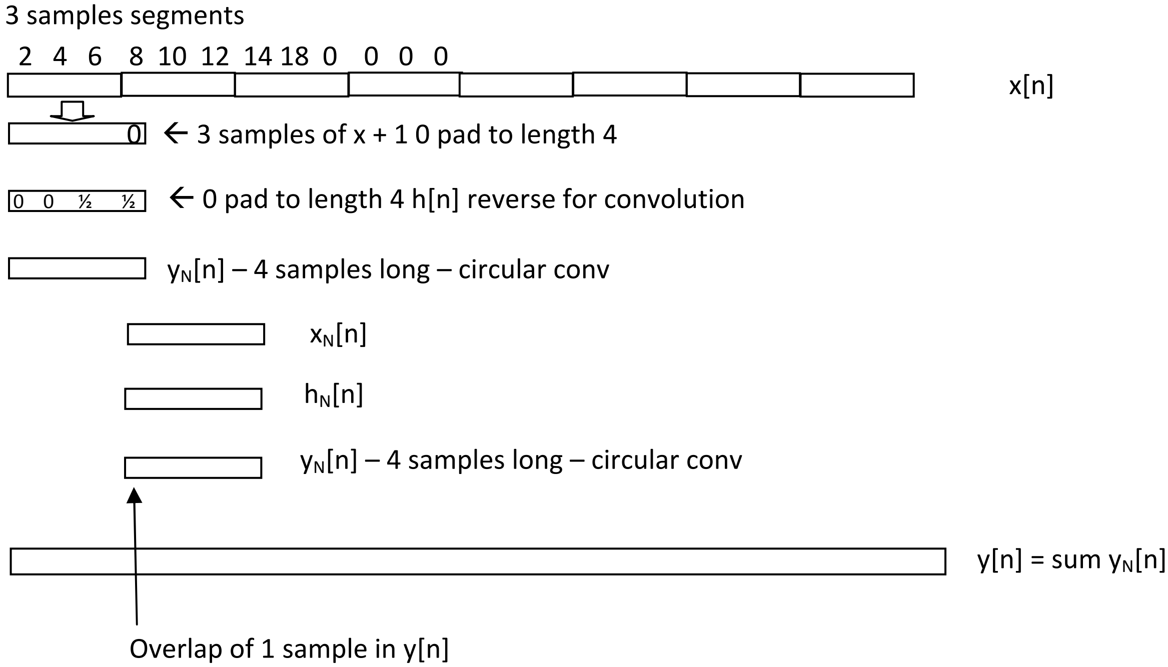
Answer:

n	-1	0	1	2	3	4	5	6	7	8	
x	0	2	4	6	8	10	12	14	18	0	
h	½	½									n=0
		½	½								n=1
			½	½							n=2
				½	½						n=3
					½	½					n=4
						½	½				n=5
y		1	3	5	7	9	11				

ii. **Sketch the operation of the overlap-add method** using N=4, M=2, and L=3.

Answer:

N = 4
M = 2 filter length
L = N-M+1 = 3 ← Max block size of X



iii. **Calculate** $y[0]$ to $y[5]$ using overlap-add with these parameters. Implement circular convolution using the DFT and IDFT of length N=4.

Answer:

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} ½ \\ ½ \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ (1-j)/2 \\ 0 \\ (1+j)/2 \end{bmatrix}$$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 2-4j-6 \\ 2-4+6 \\ 2+4j-6 \end{bmatrix} = \begin{bmatrix} 12 \\ -4-4j \\ 4 \\ -4+4j \end{bmatrix}$$

position n=0,1,2

$$X[k] \,.\!*\, H[k] = \begin{bmatrix} 12 \\ -4(1+j)(1-j)/2 \\ 0 \\ -4(1-j)(1+j)/2 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 0 \\ -4 \end{bmatrix}$$

$$y[n] = \tfrac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 12 \\ -4 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

position n=0,1,2

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 8 \\ 10 \\ 12 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 \\ 8-10j-12 \\ 8-10+12 \\ 8+10j-12 \end{bmatrix} = \begin{bmatrix} 12 \\ -4-10j \\ 10 \\ -4+10j \end{bmatrix}$$

position n=3,4,5

$$X[k] \,.\!*\, H[k] = \begin{bmatrix} 30 \\ (-4-10j)(1-j)/2 \\ 0 \\ (-4+10j)(1+j)/2 \end{bmatrix} = \begin{bmatrix} 30 \\ -7-3j \\ 0 \\ -7+3j \end{bmatrix}$$

$$y[n] = \tfrac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 30 \\ -7-3j \\ 0 \\ -7+3j \end{bmatrix} = \begin{bmatrix} (30-14)/4 \\ (30+3+3)/4 \\ (30+7+7)/4 \\ (30-3-3)/4 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 11 \\ 6 \end{bmatrix}$$

position n=3,4,5

n 0 1 2 3 4 5

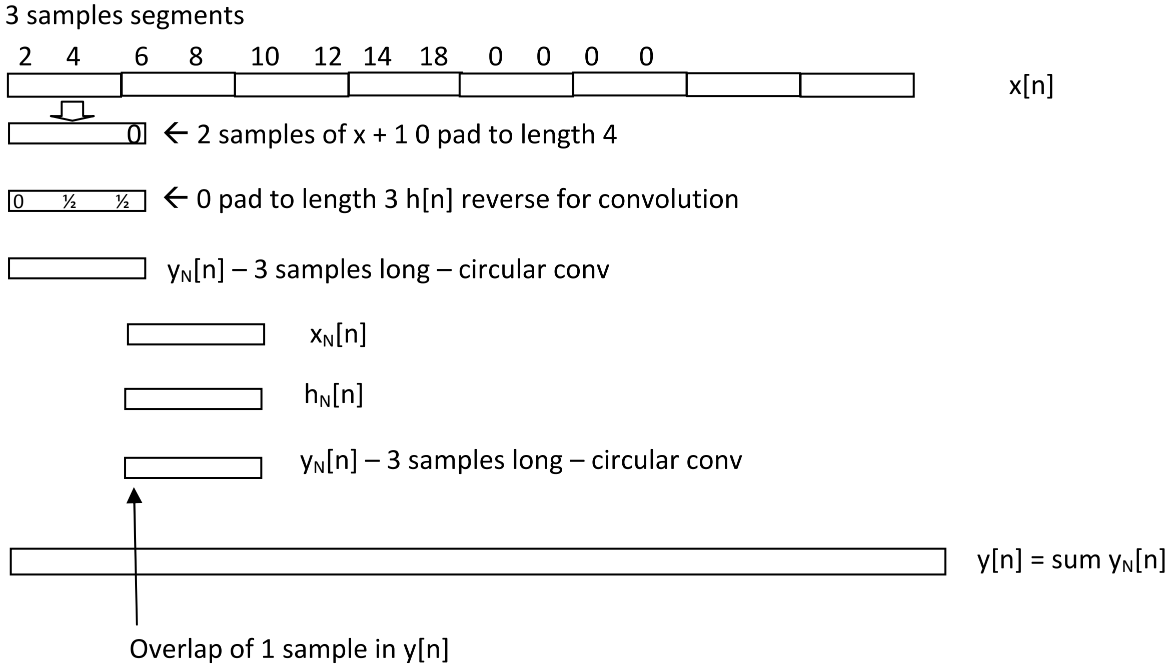
y[n] = {1 3 5 3} +
{4 9 11 6}

= {1 3 5 7 9 11 } n=0 – 5 Note y[6] does NOT = 6 (need next calculation)

iv. **Sketch the operation of the overlap-add method** using N=3, M=2, and L=2.

Answer:

N = 3
M = 2 filter length
L = N-M+1 = 2 ← Max block size of X



v. **Calculate** $y[0]$ to $y[3]$ using overlap-add with these parameters. Implement circular convolution directly for each step.

Answer:

$y_{01}[n] = h \circledast x_{01}[n]$ ← circular convolution!

n	-3	-2	-1	0	1	2	3	
x_{01}	2	4	0	2	4	0	2	
h		0	½	½				n=0
h			0	½	½			n=0
h				0	½	½		n=0
y_{01}				1	3	2		

$y_{23}[n] = h \circledast x_{23}[n]$ ← circular convolution!

n	-1	0	1	2	3	4	5	
x_{23}	6	8	0	6	8	0	6	
h		0	½	½				n=0
h			0	½	½			n=0
h				0	½	½		n=0
y_{23}				3	7	4		

Y[n] = {1 3 2} +

{3 7 4}

= {1 3 5 7 …} ref Note on y[4] does NOT = 4 (need next calculation)

FYI – Here is the answer done using DFT and IDFTs….

$W = e^{-2\pi j/3} = -0.5000 - 0.8660j$
$W^2 = -0.5000 + 0.8660j$
$W^4 = -0.5000 - 0.8660j$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W & W^2 \\ 1 & W^2 & W^4 \end{bmatrix} \begin{bmatrix} ½ \\ ½ \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.2500 - 0.4330j \\ 0.2500 + 0.4330j \end{bmatrix}$$

See R3DFT Matrix in Matlab figure for matrix values

$$X_{01}[k] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W & W^2 \\ 1 & W^2 & W^4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 6.0000 \\ 0.0000 - 3.4641j \\ 0.0000 - 3.4641j \end{bmatrix} \qquad n=0,1$$

$$X_{23}[k] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W & W^2 \\ 1 & W^2 & W^4 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 14.0000 \\ 2.0000 - 6.9282j \\ 2.0000 + 6.9282j \end{bmatrix} \qquad n=2,3$$

$Y_{01}[k] = X_{01}[k] .* H[k] = \{6 \ \ -1.5000-0.8660j \ \ -1.5000+0.8660i\}^T$

$Y_{23}[k] = X_{23}[k] .* H[k] = \{14 \ \ -2.5000-2.5981j \ \ -2.5000+2.5981j\}^T$

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R3DFT =

   1.0000             1.0000             1.0000
   1.0000            -0.5000 - 0.8660i  -0.5000 + 0.8660i
   1.0000            -0.5000 + 0.8660i  -0.5000 - 0.8660i

>> R3IDFT

R3IDFT =

   1.0000             1.0000             1.0000
   1.0000            -0.5000 + 0.8660i  -0.5000 - 0.8660i
   1.0000            -0.5000 - 0.8660i  -0.5000 + 0.8660i
```

$$y_{01}[n] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & W & W^2 \\ 1 & W^2 & W^4 \end{bmatrix}^* Y_{01}[k] = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \qquad n=0,1$$

See R3IDFT Matrix in Matlab figure for matrix values
Note '*' means complex conjugate

$$y_{01}[n] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & W & W^2 \\ 1 & W^2 & W^4 \end{bmatrix}^* Y_{23}[k] = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} \qquad n=0,1$$

n 0 1 2 3

Y[n] = {1 3 2} +

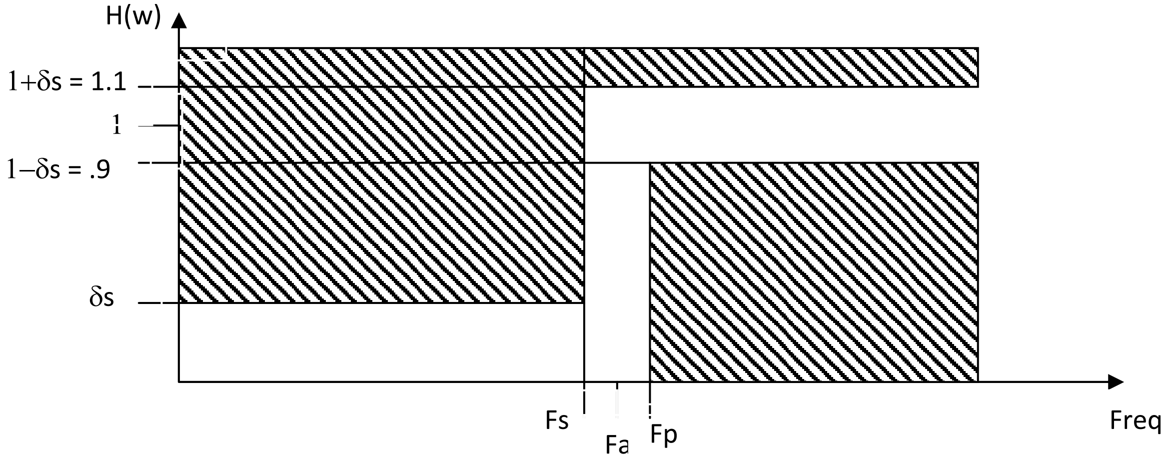
{3 7 4}

= {1 3 5 7 …} ref Note on y[4] does NOT = 4 (need next calculation)

D. Given a DSP system with T_s=1ms, we need a high pass FIR filter, $h_{HP}[n]$, which will 1) Accept frequencies above 100Hz (to within 10%) 2) Reject frequencies below 60Hz (by at least 40 dB)

i. Calculate the center frequency and sketch the filter requirements

Answer:



$Fs = 1/Ts = 1/10^{-3} = 1kHz$

$Fp = 100Hz$ → normalized to 0.1

$Fs = 60Hz$ → normalized to 0.06

$Fa = (Fp+Fs)/2 = 80Hz$ → normalized to 0.08 (center acceptance)

Stop attenuation 40dB = $-20\log10(\delta s) = .01$

$\delta p = .1$

ii. Calculate the ideal filter $h_{ideal}[n]$.

Answer:

$h_{HPIDEAL}[n] = (-1)^n (\omega_c/\pi) \text{ sinc } (n (\omega_c/\pi))$
$w_c = \pi - 2\pi * 0.08 = .84\pi$ Since ω_a needs to be translated to LP equivalent

$h_{HPIDEAL}[n] = (-1)^n (.84) \text{ sinc } (.84n))$

iii. Calculate a window $w[n]$ to meet the requirements.

Answer:

For a stop band of 40dB per slide 22.14

- ➔ Hann is acceptable Window (Hamming, Blackman, Blackman-Nutall also meet)
- ➔

window $[n+L] = a_0 + a_1 \cos(\pi\, n/L)$

$= 0.5 + 0.5 \cos(\pi\, n/L)$

Transition Bandwidth (for Hann) = 1.56 / L

L = 1.56 / (.1-.06) = 1.56/.04 = 39

Filter length = N = 2L+1 = 79

window[n] = 0.5 + 0.5 cos (π (n-39)/39)

iv. What is the FIR filter $h_{HP}[n]$.

Answer:

h_{HP} = window .* $h_{HPIDEAL}$

$(0.5 + 0.5 \cos(\pi (n-39)/39)).* (-1)^{n-39} (.84) \text{sinc}(.84(n-39))$ For n= 0:78