Quiz 4 Answer book

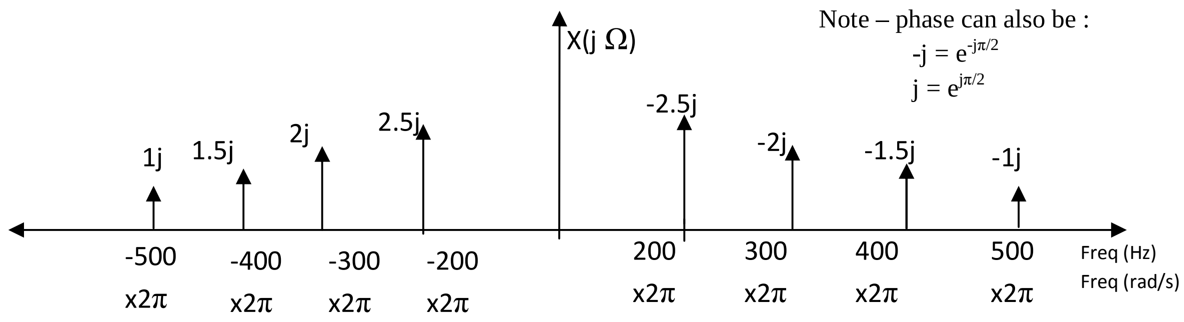
Consider the signal, $x(t)$,

$$x(t)= 5\ \sin(2\pi 200t) + 4\ \sin(2\pi 300t) + 3\ \sin(2\pi 400t) + 2\ \sin(2\pi 500t)\ \text{mV}$$

A. **Show the Fourier Transform phasor plot of $x(t)$**

Answer: $x(t)= 5\ \sin(2\pi 200t) + 4\ \sin(2\pi 300t) + 3\ \sin(2\pi 400t) + 2\ \sin(2\pi 500t)$ mV

$$= -2.5j\ (e^{j(200\ (2\pi t)} - e^{-j(200\ (2\pi t)}) + -2.0j\ (e^{j(300\ (2\pi t)} - e^{-j(300\ (2\pi t)})$$
$$-1.5j\ (e^{j(400\ (2\pi t)} - e^{-j(400\ (2\pi t)}) + -1.0j\ (e^{j(500\ (2\pi t)} - e^{-j(500\ (2\pi t)})\ \ \text{mV}$$




B. Initially we sample $x(t)$ at 700Hz. **Calculate $x[n]$. Is the signal aliased?**

Answer: Note when this was first posted the sampling rate was 400Hz so watch for students that did not get correction.

Sampling rate = 700Hz → Nyquist is 350Hz– so 400 Hz and 500 Hz are aliased.

$x(t) = 5\sin(2\pi 200t) + 4\ \sin(2\pi 300t) + 3\ \sin(2\pi 400t) + 2\ \sin(2\pi 500t)$ mV

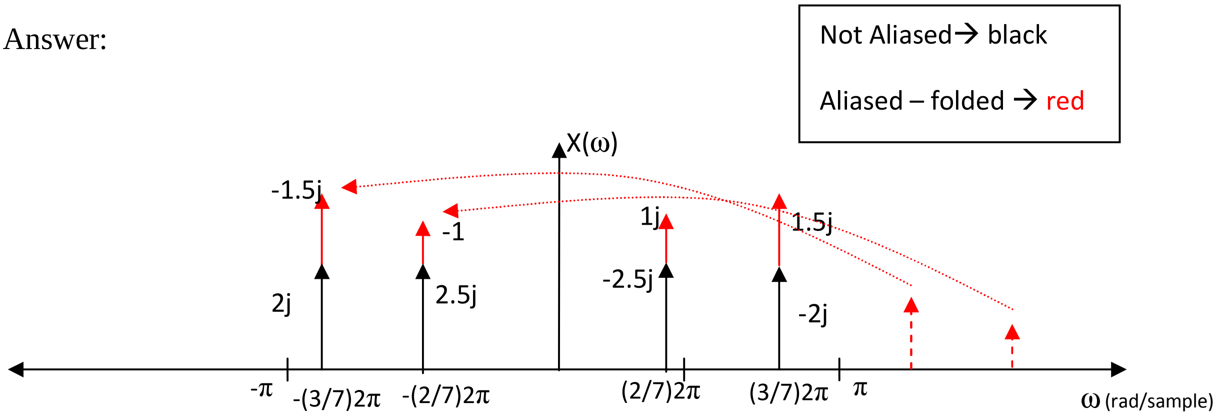
$x[n] = 5\sin(2\pi 200n/700) + 4\sin(2\pi 300n/700) + 3\sin(2\pi 400n/700) + 2\sin(2\pi 500n/700)$ mV

$= 5\sin(2\pi 200n/700) + 4\sin(2\pi 300n/700) + 3\sin(2\pi(400\text{-}700)n/700)$
$+ 2\sin(2\pi(500\text{-}700n/700)$ mV

$= 5\sin(2\pi 2n/7) + 4\sin(2\pi 3n/7) + 3\sin(-2\pi(3)n/7)$
$+ 2\sin(-2\pi(2n/7)$ mV

C. **Show the Fourier Transform phasor plot of *x*[*n*]. Label each aliased component as "Folded" or "Non-folding".**

Answer:




D. If we consider the aliased components to be noise, **What is the signal to noise ratio?** (power is proportional to the sum of Fourier Transform phasor amplitude squared)?

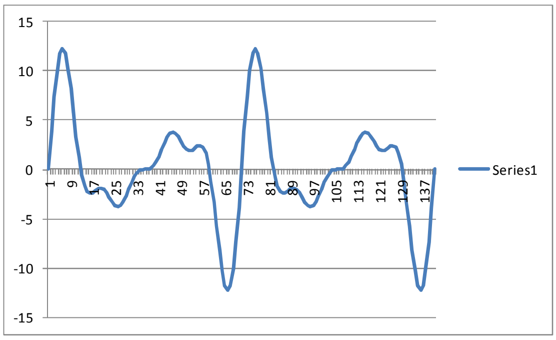
Answer: Signal energy = $(5^2 + 4^{2)}/2 = 20.5\text{x}10^{-6}$ W
(divide by 2 for true power from peaks, assume 1 Ohm load)

Noise energy = $(3^2 + 2^2)/2 = 6.5 \text{ x}10^{-6}$ W

SNR = 20.5/6.5 = 3.154 ➔ $10\log_{10}(41/13) = 4.99$dB

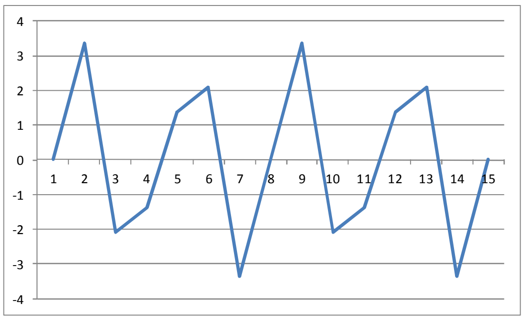
E. We wish to sample the signal with an ADC. **What is the maximum and minimum signal amplitude (give units)?**

Answer:




x[n] = 5sin(2π2n/7) + 4sin(2π3n/7) +
+3sin(2π4n/7) + 2sin(2π5n/7) mV

Base answer - Max value of signal +14mV Min value of signal -14mV
But that assumes signals get to their peaks / valleys all in together but above signals do not...




Going a step beyond the expected
….. Note that close inspection of the signal shows that the signal is periodic and the sin peaks never align. It peaks at +/- 12.3mV. But when sampling alignment considered the Max/Min values are greatly different
Max/Max = +/- 3.35mV

F. We use a 10 bit ADC with $X_{max} = -X_{min} = 1V$. **What is Δ? What is the amplitude of quantization noise?**

Answer:

$\Delta = (1 – (-1)) / (2^{10} -1)$ V = (2) / (1023)

= 1.955 mV (peak to peak) ← noise amplitude

G. **Is the noise level due to aliasing greater than the noise level due to quantization noise?**

Answer:

$\Delta^2/12 = .319 \times 10^{-6}$ W

Sampling alias noise >>> Quantization noise

H. **Calculate the DTFT, *X*(*ω*), of:** $x[n]= u[n](0.1)^n$

Answer:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (0.1)^n\, e^{-j\omega n}$$

Because of u[n ]

$$= \sum_{n=0}^{\infty} (0.1\, e^{-j\omega})^n$$

$$= 1 / (1 - 0.1\, e^{-j\omega})$$

I. **Calculate the DTFT, $X(\omega)$, of:** $x[n] = u[n](0.1)^n \cos(0.1n)$

Answer:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (0.1)^n \cos(0.1n) e^{-j\omega n}$$

Because of u[n ]

$$= \tfrac{1}{2}\sum_{n=0}^{\infty} (0.1)^n (e^{j0.1n} + e^{-j0.1n}) e^{-j\omega n}$$

$$= \tfrac{1}{2}\sum_{n=0}^{\infty} (0.1(e^{j0.1} + e^{-j0.1}) e^{-j\omega})^n$$

$$= \tfrac{1}{2} \left( \sum_{n=0}^{\infty} (0.1(e^{j0.1} e^{-j\omega})^n + \sum_{n=0}^{\infty} (0.1(e^{-j0.1} e^{-j\omega})^n \right)$$

$$= \tfrac{1}{2} (1 / (1 - 0.1\, e^{j0.1} e^{-j\omega}) + 1 / (1 - 0.1\, e^{-j0.1} e^{-j\omega}))$$