Quiz 3 Answer book

Background: You're building a portable music recorder and playback system. The system has recorded a sample sound for playback. The input, $x(t)$, at the microphone is:

$$x(t)= 10 \sin(600\pi t) + 5 \sin(4100\pi t) \text{ mV}$$

A. Show the Fourier transform, $X(\Omega)$, as a phasor plot.

$\sin(wt) = (e^{jwt}-e^{-jwt}) /2j$

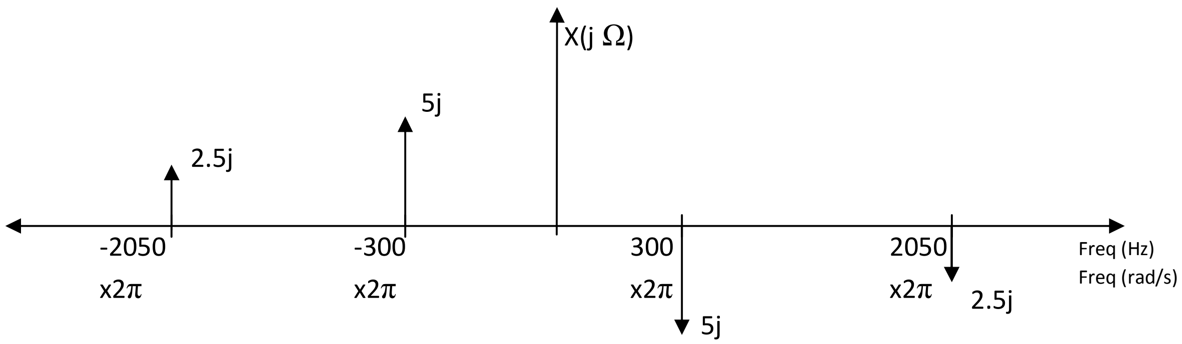
$= -(e^{jwt}-e^{-jwt})j/2$

Answer: $x(t)= 10 \sin(600\pi t) + 5 \sin(4100\pi t)$ mV

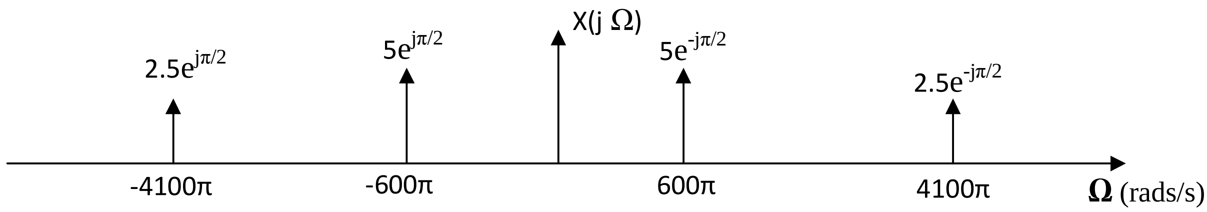
$= 10 \sin(300 (2\pi t)) + 5\sin(2050(2\pi t))$ mV

$= -5j\, e^{j(300 (2\pi t)} + 5j\, e^{-j(300 (2\pi t)} - 2.5je^{j(2050(2\pi t))} + 2.5je^{-j(2050(2\pi t))}$mV

$= 5e^{j600\pi t} (e^{-j\pi/2}) + 5e^{-j600\pi t} (e^{j\pi/2}) + 2.5e^{j4100\pi t} (e^{-j\pi/2}) + 2.5e^{-j4100\pi t} (e^{j\pi/2})$ mV



OR



B. Without using any type of anti-aliasing filter, the signal is sampled at 1000 samples/s, giving a sampled sequence $x[n]$. **Calculate $x[n]$** showing each term in it's lowest frequency form.

Answer: $x(t)= 10 \sin(600\pi t) + 5 \sin(4100\pi t)$ mV

Fs = 1000 Hz

$x[n] = x(n/Fs) = 10 \sin[600\pi n/1000] + 5 \sin[4100\pi n/1000]$ mV

$= 10 \sin[(0.3)\ 2\pi n] + 5 \sin[(0.05)\ 2\pi n + 4\pi n]$ mV

$= 10 \sin[(0.3)\ 2\pi n] + 5 \sin[(0.05)\ 2\pi n]$ mV

C. **Calculate the Nyquist frequency for this sampling rate**, and calculate at what frequency the aliased representation of $\sin(4100\pi t)$ will appear in the sampled signal. **Is this signal aliased?**

Answer: Fs = 1000Hz → Nyquist frequency = Fs/2 = 500Hz

Yes the signal is aliased
The 300Hz component is fine (not aliased)
The 2050Hz component is aliased to 50Hz

D. **Calculate the value of $x[n]$ for $n = 0 \ldots 3$.**

Answer: $x[n]= 10 \sin[0.2*2\pi n] + 5 \sin[0.05*2\pi n]$ mV

$x[0]= 10 \sin[0.2*2\pi 0] + 5 \sin[0.05*2\pi 0]$ mV = 0 mV

$x[1]= 10 \sin[0.2*2\pi 1] + 5 \sin[0.05*2\pi 1]$ mV = 11.0557 mV

$x[2]= 10 \sin[0.2*2\pi 2] + 5 \sin[0.05*2\pi 2]$ mV = -2.9389 mV

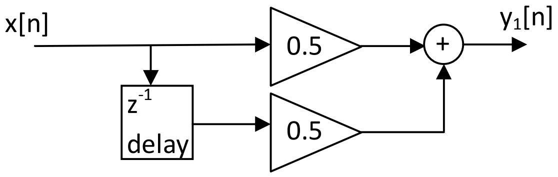
$x[3]= 10 \sin[0.2*2\pi 3] + 5 \sin[0.05*2\pi 3]$ mV = -1.8328 mV

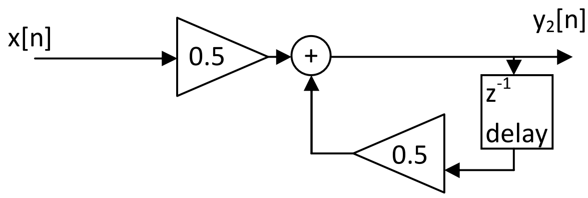
E. Input $x[n]$ is sent into two filters:

Filter 1: $y = f_1(x)$: $\quad y_1[n] = ½(x[n] + x[n-1])$
Filter 2: $y = f_2(y)$: $\quad y_2[n] = ½(x[n] + y[n-1])$

Show the block diagram for each filter. Calculate $y_1[n]$ and $y_2[n]$ for n = 0 … 3, and $x[n] = \delta[n]$. Assume initial conditions are zero.

Answer:





n	x[n] = δ[n]	$y_1[n]$	$y_2[n-1]$	$y_2[n]$
-1	0	0	0	0
0	1	½	0	½
0	0	½	½	¼
0	0	0	¼	1/8
0	0	0	1/8	1/16

F. **Calculate the impulse response *$h_1[n]$* and *$h_2[n]$*, for each filter**

Answer:

$$h_1[n] = 0.5\ \delta[n] + 0.5\ \delta[n-1]$$

$$h_2[n] = 0.5\ (0.5)^n\ u[n]$$

G. The filters f_1 and f_2 are combined in various ways. Calculate the impulse response of the following combined filters

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i)   x --->| f1 |-->| f2 | ---> y
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Answer: ➔ $h_1[n] * h_2[n]$ (convolution)

$$= (0.5\ \delta[n] + 0.5\ \delta[n-1]) * 0.5\ (0.5)^n\ u[n]$$

$$= 0.5\ \delta[n] * 0.5\ (0.5)^n\ u[n] + 0.5\ \delta[n-1] * 0.5\ (0.5)^n\ u[n]$$

$$= 0.5^2\ (0.5)^n\ u[n] + 0.5^2\ (0.5)^{n-1}\ u[n-1]$$

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ii)  x --->| f2 |-->| f1 | ---> y
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Answer: ➔ $h_2[n] * h_1[n]$ (convolution)

$$= h_1[n] * h_2[n]$$

$$= 0.5^2\ (0.5)^n\ u[n] + 0.5^2\ (0.5)^{n-1}\ u[n-1]$$

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iii)x -+->| f2 |-->|+| ---> y
       |->| f1 |----^
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Answer: ➔ $h_2[n] + h_1[n]$

$$= (0.5\ \delta[n] + 0.5\ \delta[n-1]) + 0.5\ (0.5)^n\ u[n]$$