Carleton University, Systems and Computer Engineering

SYSC 4405: Example Test – 120 minutes, November 2, 2007

Background: You're building a portable music recorder and playback system. The system has recorded a sample sound, and you with to upsample this sound for playback. *Input:* The input $x(t)$ signal plays for 1 second and is recorded at a microphone:

$$x(t) = \begin{cases} 0 \text{ mV} & \text{if } t \leq 0\ s \\ 10\sin(600\pi t) + 5\sin(4100\pi t) \text{ mV} & \text{if } 0\ s < t < 1\ s \\ 0 \text{ mV} & \text{if } t \geq 1\ s \end{cases} \tag{1}$$

1. (5 points) Without using any type of anti-aliasing filter, the signal is sampled at $1000 \frac{\text{samples}}{\text{second}}$, giving the sampled sequence $x[n]$ (here $x[0]$ corresponds to $x(0)$). **Calculate and sketch $x[n]$ for the range $-5 \leq n \leq 5$.**

   ANSWER:

$$x[-5] = x[-4] = x[-3] = x[-2] = x[-1] = x[0] = 0$$

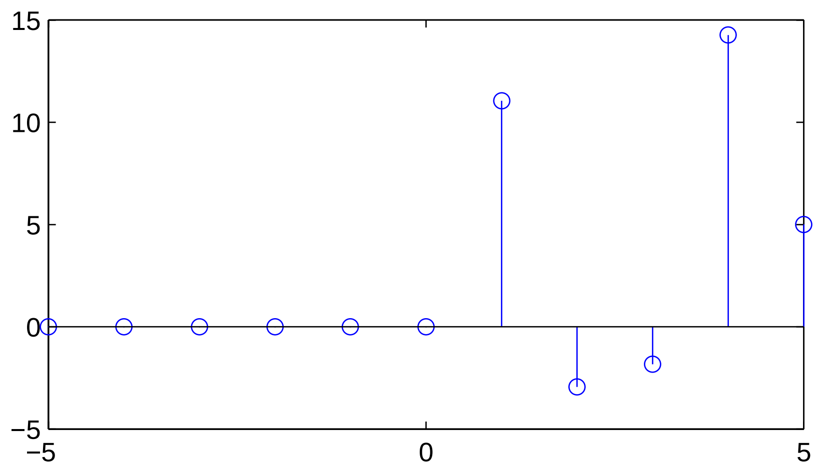
$$x[1] = 10\sin\left(\frac{3}{5}\pi t\right) + 5\sin\left(\frac{1}{10}\pi t\right) \text{ mV} \simeq 11.0557 \text{ mV}$$

$$x[2] = 10\sin\left(\frac{6}{5}\pi t\right) + 5\sin\left(\frac{1}{5}\pi t\right) \text{ mV} \simeq -2.9389 \text{ mV}$$

$$x[3] = 10\sin\left(\frac{9}{5}\pi t\right) + 5\sin\left(\frac{3}{10}\pi t\right) \text{ mV} \simeq -1.8328 \text{ mV}$$

$$x[4] = 10\sin\left(\frac{2}{5}\pi t\right) + 5\sin\left(\frac{2}{5}\pi t\right) \text{ mV} \simeq 14.2658 \text{ mV}$$

$$x[5] = 10\sin(\pi t) + 5\sin\left(\frac{1}{2}\pi t\right) \text{ mV} = 5 \text{ mV}$$



2. (5 points) It turns out that this sampling frequency isn't adequate for this signal. Sketch the process of sampling this signal in the Fourier domain. **Sketch $FT\{x(t)\}$, the sampling function $FT\{s(t)\}$, and the sampled signal $FT\{x(t)s(t)\}$.** Show how aliasing happens in these sketches.

ANSWER:

3. (5 points) **Calculate the Nyquist frequency for this sampling rate, and calculate at what frequency the aliased representation of** $\sin(4100\pi t)$ **will appear in the sampled signal**.

   ANSWER:
   The sampling frequency is 1000 Hz. The Nyquist frequency is half of that, namely 500 Hz. That is, anything above 500 Hz is aliased.

$$\sin\left(2\pi\frac{2050n}{1000}\right) = \sin\left(2\pi\frac{50n}{1000}\right)$$

   The frequency 2050 Hz is aliased down to 50 Hz.

4. (5 points) We wish to upsample (interpolate) the signal by $L = 3$ to approximate a sampling rate of $3000\frac{\text{samples}}{\text{second}}$. Let $y[n]$ be the upsampled signal. **Draw a block diagram of a DSP interpolator.** Briefly, explain the function of each block. **Is upsampling a signal the same as sampling it at the original higher rate? Why or why not?**

   ANSWER:
   An interpolator consists of an upsampler, which places reproduces the values of the input at every I outputs, and outputs zero everywhere else. It is followed by a low-pass filter with cut-off frequency$\frac{\pi}{I}$ and gain I, in order to interpolate the missing values.

   An undersampled signal necessarily loses information, and cannot therefore be recovered by upsampling.

5. (5 points) A filter is required in the interpolator in the previous question. What is the frequency response $H(\omega)$ of the filter? **Calculate the impulse reponse** $h[n]$ **of the filter.**

   ANSWER:

$$H(\omega) = \begin{cases} 3 & \text{if } |\omega| \leq \frac{\pi}{3} \\ 0 & \text{if } |\omega| > \frac{\pi}{3} \end{cases}$$

$$h[n] = \begin{cases} 3\frac{\sin\frac{\pi}{3}n}{\pi n} & \text{if } n \neq 0 \\ 1 & \text{if } n = 0 \end{cases}$$

6. (5 points) Assume that $x[n]$ has the following values, and is zero elsewhere. **Calculate** $y[601]$, assuming the filter from the previous question.

n	199	200	201	202
$x[n]$	-11.06	-0.00	11.06	-2.94

   ANSWER:

$$\begin{aligned} y[601] &\simeq x[199]h[4] + x[200]h[1] + x[201]h[-2] + x[202]h[-5] \\ &= -\frac{3\sqrt{3}}{8\pi}(-11.06) + 0 + \frac{3\sqrt{3}}{4\pi}(11.06) - \frac{3\sqrt{3}}{10\pi}(-2.94) \\ &= 7.346 \end{aligned}$$

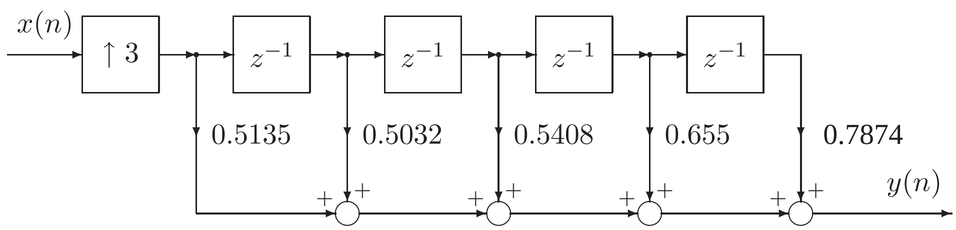
7. (5 points) Unfortunately, it isn't possible to create an ideal filter with an LCCDE system. Instead, we implement a linear phase filter with the following four zeros: $z = 0.40 \pm j1.05$, $z = -0.89 \pm j0.65$. **Calculate $H(z)$ for this filter.**

   ANSWER:

$$\begin{aligned}
H(z) &= G\left(1 - z^{-1}(0.4 + j1.05)\right)\left(1 - z^{-1}(0.4 - j1.05)\right) \\
&\times \left(1 - z^{-1}(-0.89 + j0.65)\right)\left(1 - z^{-1}(-0.89 - j0.65)\right) \\
&= G\left(1 - z^{-1}0.8 + z^{-2}1.2625\right)\left(1 - z^{-1}(-1.78) + z^{-2}(1.2146)\right) \\
&= G(1 + 0.98z^{-1} + 1.0531z^{-2} + 1.2756z^{-3} + 1.5334z^{-4}) \\
H(z=1) &= 3 = G(1 + 0.98 + 1.053 + 1.2756 + 1.5334) = 5.8421G \Rightarrow G = 0.5135 \\
H(z) &= 0.5135 + 0.5032z^{-1} + 0.5408z^{-2} + 0.6550z^{-3} + 0.7874z^{-4} \\
h[n] &= [0.5135 \quad 0.5032 \quad 0.5408 \quad 0.6550 \quad 0.7874]
\end{aligned}$$

8. (5 points) **Design and sketch an LCCDE system to implement $H(z)$. Is this an FIR or IIR filter?**

   ANSWER:



   This filter has no poles, hence it is FIR.

9. (5 points) **Calculate $y[601]$ using this nonideal filter.** Use the values of $x[n]$ from question 6.

   ANSWER:

$$y[601] = h[1]x[200] + h[4]x[199] = 0.5032 \times 0 + 0.7874 \times (-11.06) = -8.7086$$

10. (5 points) Characterize the inverse filter $G(z) = H^{-1}(z)$ according to the properties:

    (a) linear
    (b) memoryless
    (c) shift-invariant
    (d) stable
    (e) invertible

    ANSWER:

    (a) linear: Any system described by a transfer function is LTI, therefore it is linear.
    (b) memoryless: Any non-trivial LTI system has memory.
    (c) shift-invariant: Any system described by a transfer function is LTI, therefore it is time-invariant.
    (d) stable: The poles will be located outside the unit circle, therefore the system is unstable.
    (e) invertible: It is invertible: its inverse is $G(z)$.