Carleton University,
Systems and Computer Engineering
SYSC 4405: Final Exam, December 10, 2007
Exam Number: 1

Instructions:

- This exam has **2** pages and **15** questions. Answer all questions.
- You have **180 minutes** to complete this exam.
- This is a closed book exam; however, you are permitted to bring one (1) 8.5" $\times$ 11" sheet of notes into the exam. You are permitted to use a non-programmable calculator. You may not communicate with anyone during the exam except the instructor.
- Write your answers on an examination booklet. *You may take this examination paper with you*

Background: You are designing a DSP system to demodulate and detect the signal from a Morse code system which transmits an FSK signal using amplitude modulation in the shortwave at $f_C = 3210$ kHz (*Note that you do not need to understand either FSK or AM to be able to answer these question*). The analog input stage has already been designed. It amplifies the input level to about 100 mV. The Morse code signal is a sequence of **on/off** codes, encoded into the input signal, $x(t)$, as:

$$x(t) = \begin{cases} 100cos(2\pi f_C t)cos(2\pi 811 t)\ mV & \text{Send: } \textbf{on} \\ 100cos(2\pi f_C t)cos(2\pi 523 t)\ mV & \text{Send: } \textbf{off} \end{cases}$$

Thus, **on** and **off** are encoded with 811 Hz and 523 Hz AM modulated tones, respectively.

1. (1 point) Your exam is exam number **1**. Write down this number.

2. (5 points) We wish to demodulate this signal and detect the **on/off** codes using an all DSP system (ie with no analog demodulation). As mentioned, the antenna and amplifier have already been built. **Sketch the DSP system components as a block diagram. On your diagram, label the: a) Sample/Hold, b) Analog Low pass filter,** and **c) Clock.**

3. (5 points) An 8-bit analog to digital converter is used with a sampling rate of 10 MHz. (Such high speed ADCs are now readily available; for example, the AD775 8-Bit 20 MSPS is about US$1.50 in 1000 piece quantity). The sampled $x(t)$ is $x[n]$. At $t = 0.5$ s, the signal is turned to **on** from **off**. **Sketch** $x[4\ 999\ 995] \ldots x[5\ 000\ 005]$**. Calculate the value of** $x[n]$ **at four points on your sketch.**

4. (5 points) To demodulate this filter, we wish to filter it with a bandpass filter at the carrier frequency, 3210 kHz. A (realizable) filter is designed with two poles at $z_p = 0.99e^{\pm j\omega_0}$ and two zeros at $z_z = 1.2e^{\pm j\omega_0}$. **Calculate the value of** ω_0 **which corresponds to** f_C**. Sketch the poles and zeros of** $H(z)$**, showing the unit circle and the region of convergence.**

5. (5 points) The gain of the filter $H(z)$ is set so that the magnitude response at f_C, $|H(e^{j\omega_0})|$ is 1.0. **Calculate the transfer function,** $H(z)$**, for this filter.**

6. (5 points) **Sketch an LCCDE system to implement** $H(z)$.

7. (5 points) Given the following costs for operations (for real or complex numbers) **calculate the time required per sample processed by the filter. Can this DSP filter algorithm work in real time?**

operation	delay	add/subtract	multiply	other operations
time	2 ns	3 ns	10 ns	0

8. (5 points) **Is** $H(z)$ **minimum phase?** If not **calculate a filter** $H'(z)$ **with the same magnitude frequency response as** $H(z)$ **but which is minimum phase.**

9. (5 points) We create a filter, $F(z)$, with the same zeros and multiplicative constant as $H(z)$, but with a single pole at $z = -0.9$. **Calculate the impulse response,** $f[n]$**.**

10. (5 points) Another approach that is considered to demodulate the signal is using the DFT using block convolution with the overlap-add method. Using a filter $h_2[n]$ of length $L = 51$, **select** M **and** N **so that the DFT can be implemented with a radix-2 FFT algorithm. Draw a sketch to illustrate the operation of this filter.** Show the output, $y[n]$, and the amount of signal output at each step.

11. (5 points) Use the operation times of Q#7, **calculate the time required per sample processed by this filter. Can this DSP filter algorithm work in real time?**

12. (5 points) The filter $H_2(z)$ is required to be a band-pass filter with a center frequency f_C and bandwidth 5 kHz. **Sketch the ideal filter** $H_2(\omega)$**, and calculate** $h_2[n]$**.**

13. (5 points) The Hamming window has the form:
$$w[n + \tfrac{L-1}{2}] = rect(\tfrac{n}{L})\big(0.54 + 0.46\, cos(\tfrac{2\pi n}{L})\big)$$
It has side-lobe amplitude of -41 dB, stopband attenuation of -53 dB, and transition width of $3.3/L$. If we use a Hamming window for $H_2(z)$, with length specified in Q#10, **calculate the properties of the filter passband, stopband, and transition band. Sketch the filter specification showing** δ_p**,** δ_s**,** ω_p**, and** ω_s**.**

14. (5 points) **Characterize the windowed filter** $H_2(z)$ **for the following properties.**

linear	memoryless
shift-invariant	stable
causal	LSI
realizable	linear phase
minimum phase	all pass

15. (5 points) Since the signal is bandlimited, it is not necessary to sample this signal at $\geq 2f_C$. Using a sketch of $X(f)$, **show how a sampling frequency less than** $2f_C$ **can still result in a sampled signal,** $x[n]$**, without aliasing.**