Question 1: Voltage dividers

With Reference to Figure 1, write down the expressions for the voltages across R1 and R2.

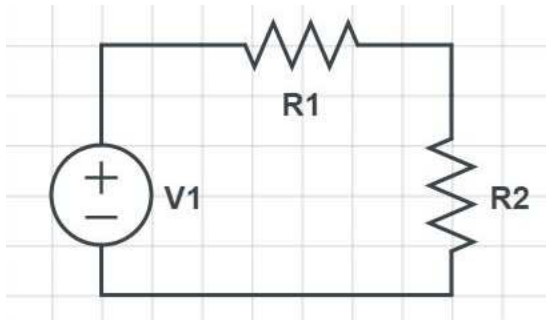


Figure 1

If V1 = 11V, R1 = 220Ω and R2 = 330Ω

a) Calculate the voltage across R1

$$V1\frac{R1}{R1+R2} = 11\text{V}\frac{220}{550} = 4.4\text{V}$$

b) Calculate the current supplied by the source

$$I = \frac{V1}{R1+R2} = \frac{11\text{V}}{550\Omega} = 20\text{ mA}$$

c) If we wish to replace R1 and R2 with a single resistor, what should its value be?

$$R = R1 + R2 = 550\Omega$$

Question 2: Kirchhoff's current laws and Resistors in parallel

Referring to Figure 2 and using V1 = 11V, R1 = 220Ω and R2 = 330Ω

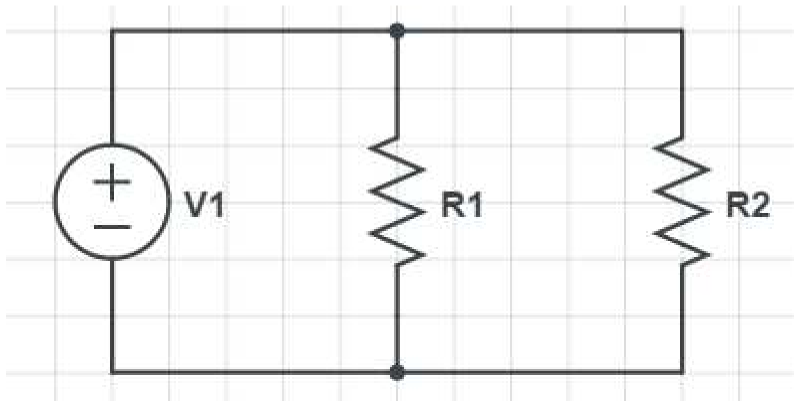


Figure 2

a) What is the total current supplied by the source?

$$I = I1 + I2 = \frac{V1}{R1} + \frac{V1}{R2} = \frac{11\text{V}}{220\Omega} + \frac{11\text{V}}{330\Omega} = 83.3\text{ mA}$$

b) What is the voltage across R2?

$$V1 = 11\text{V}$$

c) If we wish to replace R1 and R2 with a single resistor, what should its value be?

$$\frac{1}{R} = \frac{1}{220\Omega} + \frac{1}{330\Omega} \Rightarrow R = 132\ \Omega$$

Question 3: Series-parallel circuit

In Figure 3 below find I_o in the circuit below using basic circuit theory.

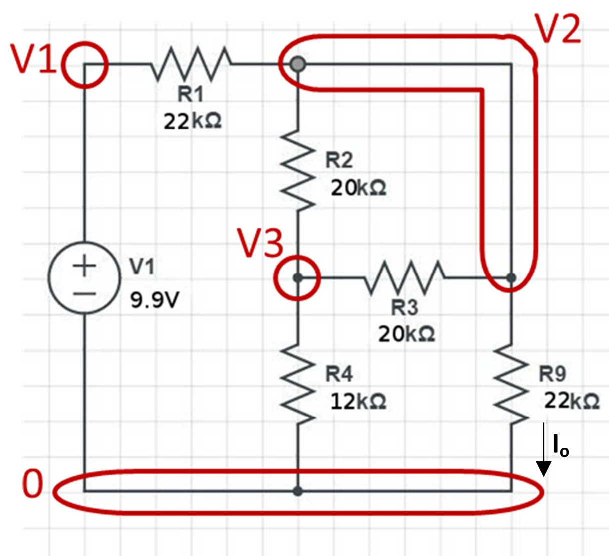


Figure 3

Using formal nodal analysis:

Node 2: $$\frac{\boldsymbol{V_1 - V_2}}{\boldsymbol{R_1}} - \frac{\boldsymbol{V_2 - V_3}}{\boldsymbol{R_2}} - \frac{\boldsymbol{V_2 - V_3}}{\boldsymbol{R_3}} - \frac{\boldsymbol{V_2 - 0}}{\boldsymbol{R_9}} = \boldsymbol{0}$$

Node 3: $$\frac{\boldsymbol{V_2 - V_3}}{\boldsymbol{R_2}} - \frac{\boldsymbol{V_3 - V_2}}{\boldsymbol{R_3}} - \frac{\boldsymbol{V_3 - 0}}{\boldsymbol{R_4}} = \boldsymbol{0}$$

Collecting terms:

$$\frac{\boldsymbol{V_1}}{\boldsymbol{R_1}} - \boldsymbol{V_2}\left(\frac{1}{\boldsymbol{R_1}} + \frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}} + \frac{1}{\boldsymbol{R_9}}\right) + \boldsymbol{V_3}\left(\frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}}\right) = 0$$

$$\boldsymbol{V_2}\left(\frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}}\right) - \boldsymbol{V_3}\left(\frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}} + \frac{1}{\boldsymbol{R_4}}\right) = \boldsymbol{0}$$

Eliminating $\boldsymbol{V_3}$:

$$\boldsymbol{V_2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_9}\right) - \boldsymbol{V_2}\frac{\left(\frac{1}{R_2} + \frac{1}{R_3}\right)^2}{\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)} = \frac{\boldsymbol{V_1}}{\boldsymbol{R_1}}$$

$$\boldsymbol{I_0} = \frac{\boldsymbol{V_2}}{\boldsymbol{R_9}} = \frac{\boldsymbol{V_1}}{\boldsymbol{R_1 R_9}}\left[\frac{1}{\boldsymbol{R_1}} + \frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}} + \frac{1}{\boldsymbol{R_9}} - \frac{\left(\frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}}\right)^2}{\left(\frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}} + \frac{1}{\boldsymbol{R_4}}\right)}\right]^{-1}$$

$$= \frac{\boldsymbol{V_1}}{\boldsymbol{R_1 R_9}}\left[\frac{1}{\boldsymbol{R_1}} + \frac{1}{\boldsymbol{R_9}} + \left(\frac{1}{\boldsymbol{R_2}} + \frac{1}{\boldsymbol{R_3}}\right)\left[1 - \frac{1}{1 + \frac{1/\boldsymbol{R_4}}{1/\boldsymbol{R_2} + 1/\boldsymbol{R_3}}}\right]\right]^{-1}$$

$$= \frac{9.9\text{V}}{22\text{k}.22\text{k}}\left[\frac{1}{22\text{k}} + \frac{1}{22\text{k}} + \left(\frac{1}{20\text{k}} + \frac{1}{20\text{k}}\right)\left[1 - \frac{1}{1 + \frac{1/12}{1/20 + 1/20}}\right]\right]^{-1}$$

$$= \frac{9.9\text{V}}{22\text{k}.22\text{k}}\left[\frac{1}{22\text{k}} + \frac{1}{22\text{k}} + \frac{1}{10\text{k}}\left[1 - \frac{1}{1 + \frac{10}{12}}\right]\right]^{-1} = \frac{9.9\text{V}}{22\text{k}.22\text{k}}\left[\frac{3}{22\text{k}}\right]^{-1}$$

finally giving

$$\boldsymbol{I_0} = \frac{9.9\text{V}}{22\text{k}.22\text{k}}\frac{22\text{k}}{3} = 0.15 \text{ mA}$$

(ugh!).

By inspection:

The RHS of the circuit simplifies to

$$(\boldsymbol{R_2} \| \boldsymbol{R_3} + \boldsymbol{R_4}) \| \boldsymbol{R_9} = (20\text{k} \| 20\text{k} + 12\text{k}) \| 22\text{k} = 22\text{k} \| 22\text{k} = 11\text{k}$$

Hence the total current from the source is

$$\frac{\boldsymbol{V_1}}{\boldsymbol{R_1} + (\boldsymbol{R_2} \| \boldsymbol{R_3} + \boldsymbol{R_4}) \| \boldsymbol{R_9}} = \frac{9.9V}{22\text{k} + 11\text{k}} = 0.3\text{mA}$$

of which exactly half flows through $\boldsymbol{R_9}$.

Hence $\boldsymbol{I_0} = \mathbf{0.15mA}$ as before (much easier!)

Question 4: Capacitors

In Figure 4, a capacitor of value C1 = 33nF and C2 = 220nF are charged to voltage V1 = 4V

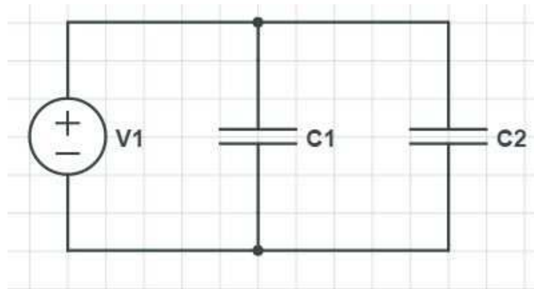


Figure 4

a) What is the charge on each capacitor?

$$Q1 = C1V1 = (33\text{nF})(4\text{V}) = 1.32 \times 10^{-7}\ \text{coulomb}$$

$$Q2 = C2V1 = (220\text{nF})(4\text{V}) = 8.8 \times 10^{-7}\ \text{coulomb}$$

b) Write down an expression's for the net capacitance of a number of capacitors in series and a number of capacitors in parallel.

Series: $\frac{1}{C} = \frac{1}{C1} + \frac{1}{C2} + \cdots + \frac{1}{Cn}$

Parallel: $C = C1 + C2 + \cdots + Cn$

Question 5: Charging Capacitor

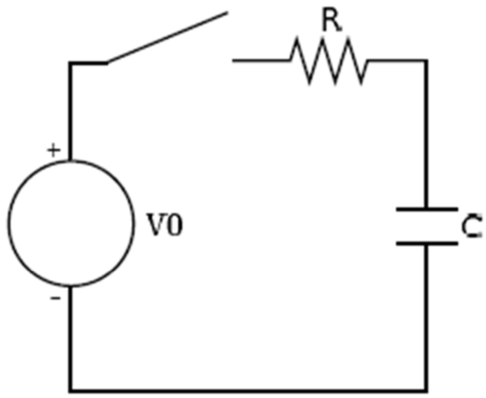
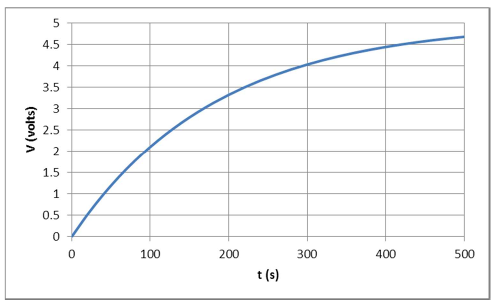


Figure 5

a) At time t = 0 a switch is closed, connecting a voltage source V0=5V through resistor R = 47k to a capacitor C = 3900µF. Sketch the voltage across the capacitor as a function of time. Show approximates times on the horizontal axis.



b) What will the voltage across the Resistor be after 100 milliseconds?

Since $100\text{ ms} \ll \tau$, the capacitor will be essentially uncharged at this time i.e. to a very good approximation, $V_c = 0\text{V}$ and $V_R = 5\text{V}$. We could calculate it exactly using

$$V_R = 5\text{-}V_c = 5e^{-t/RC}$$

with $RC = (47\text{k})(3900\mu\text{F}) = 183.3\text{ s}$, $V_R = 5e^{-(0.1/183.3)} = 4.997\text{V}$

c) Calculate the 10%-90% rise time.

$$0.1V_0 = V_0\left(1 - e^{-t_1/RC}\right)$$
$$0.9V_0 = V_0\left(1 - e^{-t_2/RC}\right)$$

Dividing by V_0 and rearranging,

$$0.9 = e^{-t_1/RC}$$
$$0.1 = e^{-t_2/RC}$$

or (dividing top by bottom)

$$t_2 - t_1 = RC\ln 9$$

With $RC = 183.3\text{ s}$, $t_2 - t_1 = 402.75\text{ s}$