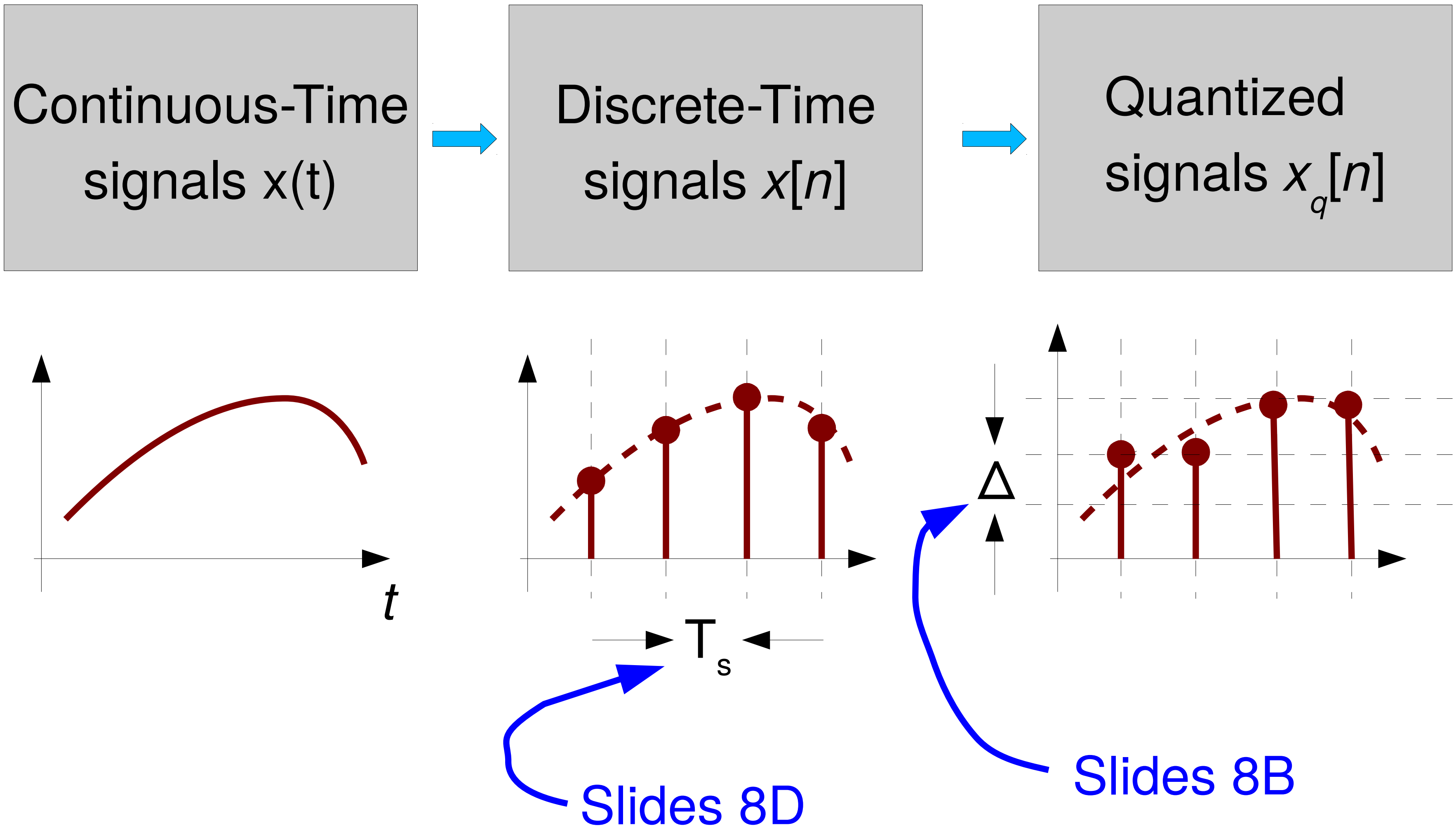
Sampling Rate



Shannon/Nyquist sampling theorem

How often/fast does $x(t)$ need to be sampled for reconstruction?

Shannon Sampling Theorem (Nyquist Sampling Theorem)

A continuous-time signal $x(t)$ with frequencies no higher than F_{max} can be reconstructed **exactly** from its discrete-time samples $x[n] = x(nT_s)$, if the samples are taken at a rate $F_s = 1/T_s$ that is greater than $2F_{max}$ (the Nyquist rate).

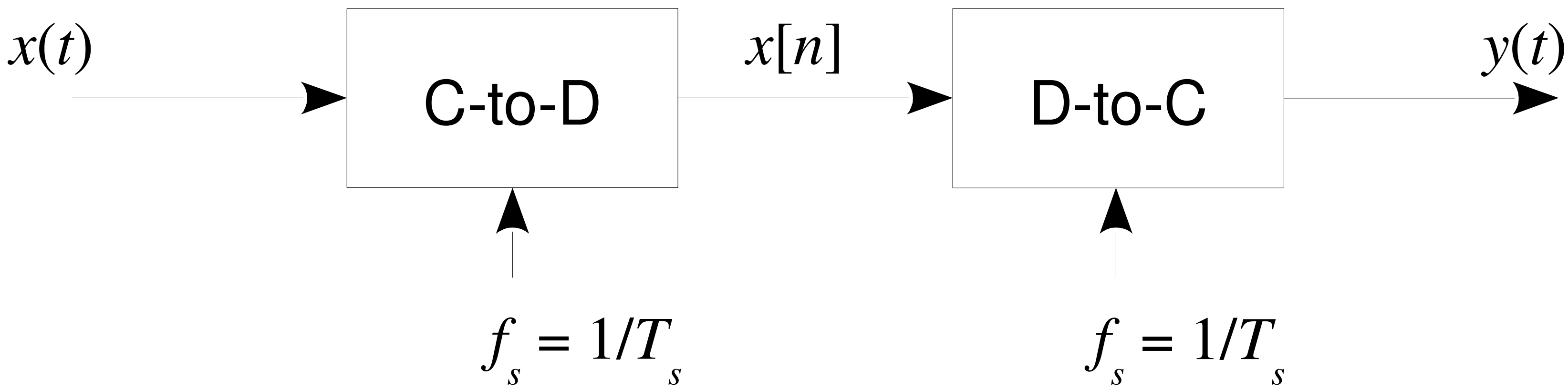
Note: We can also say that $F_{max} = B$ which is the bandwidth of $x(t)$.

Sampling/Aliasing

Slide 8D.3

Shannon/Nyquist Sampling Theorem

When does $y(t) = x(t)$?



$$y(t) = x(t) \quad \text{if } F_s > 2F_{max}$$

Note: $2.0 \times F_{max}$ is maximum frequency of $x(t)$.

$x(t)$ must be bandlimited (maximum frequency F_{max})

for $x(t)$ to be perfectly reconstructed .

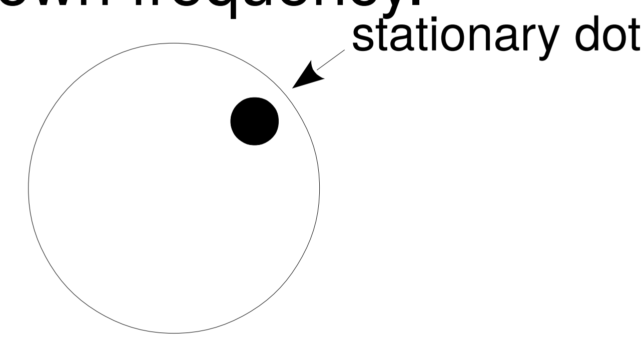
Rotating Phasor

Sampling Theorem

Shannon's Sampling Theorem can be justified by considering the rotating phasor (represented with a dot in a circle below).

Experiment setup: Disk spinning at unknown frequency.

Possible result: Stationary dot



If stationary dot occurs for $f_s = 10$ Hz,

then what is the rotation frequency

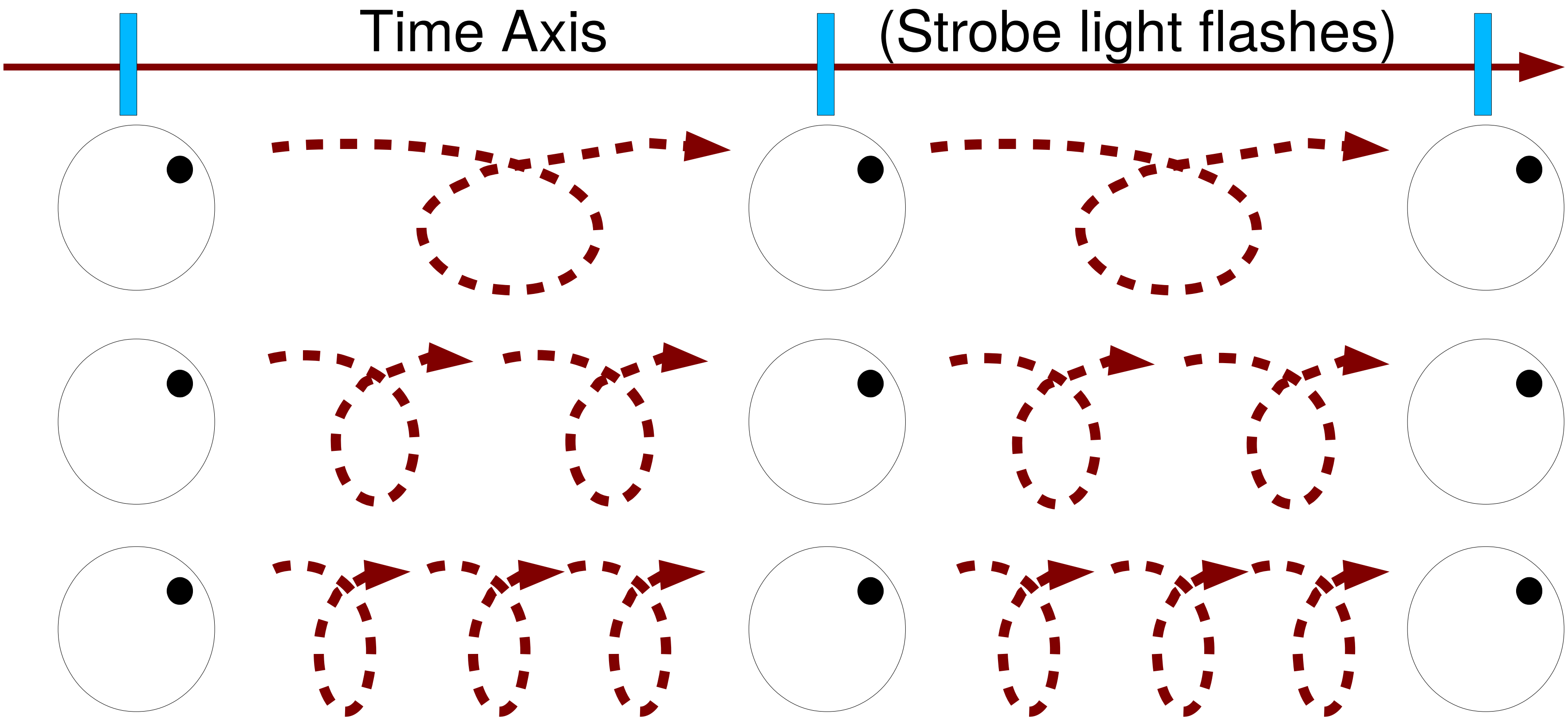
f_0 of the disk?

Answer: Any integer multiple of sampling period f_s.

i.e., $f_0 = 10$ Hz, 20 Hz, 30 Hz, 40 Hz, ...

So, one dot not enough to determine rotation frequency f

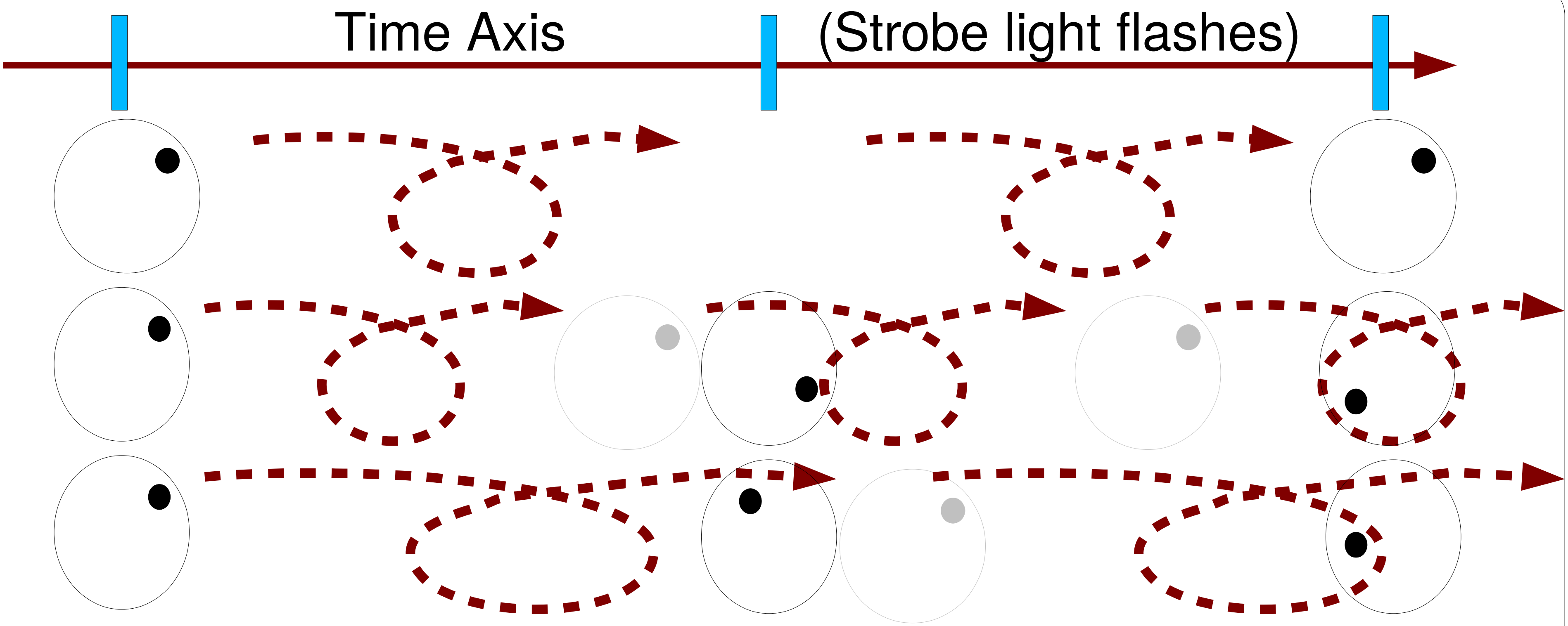
Examples of rotating phasors



Based on seeing the rotating wheel, we can't tell whether it is rotating one, twice or more between each flash of the strobe light.

We see this effect looking at wheels of nearby cars at night (neon lights flash at twice the power line frequency ~ 2×60Hz = 120Hz

Examples of rotating phasors Aliasing



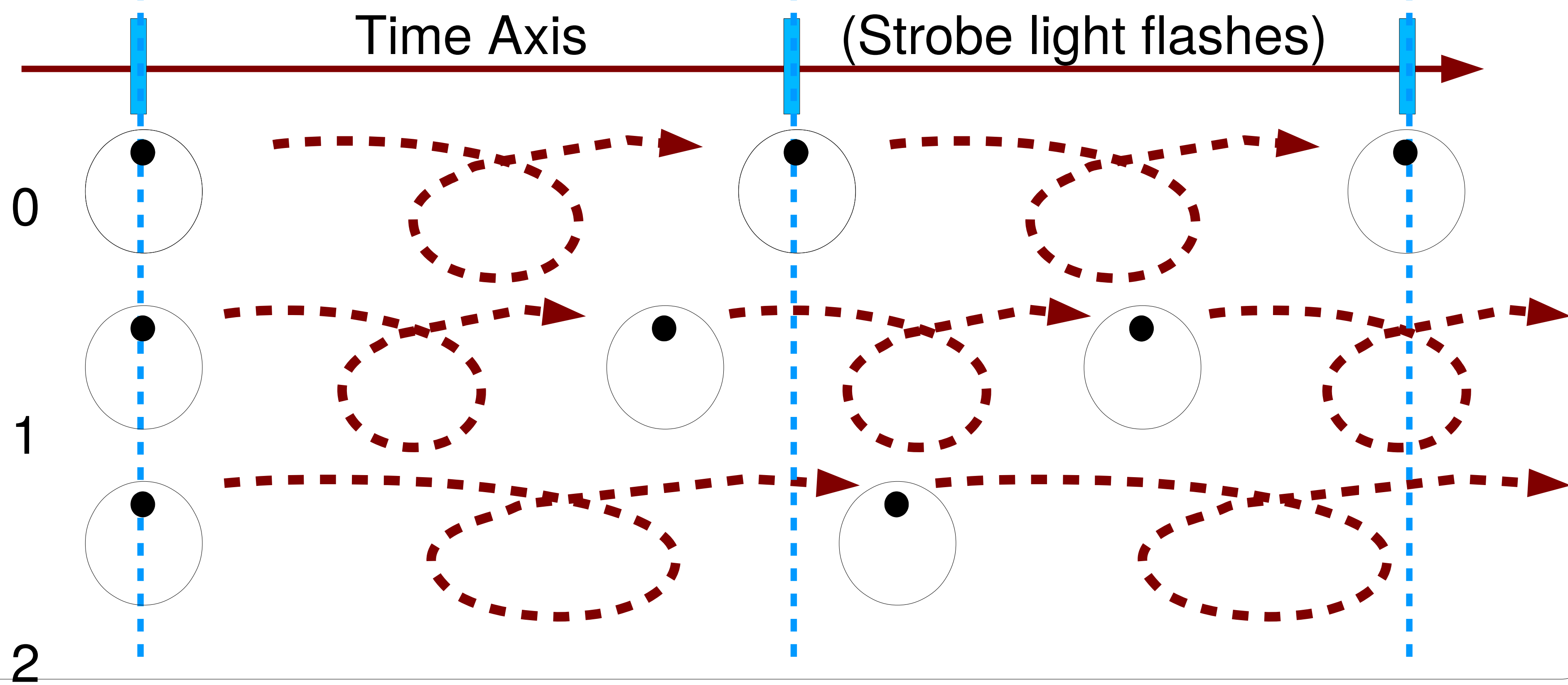
If the rotational frequency is slightly different (slower or faster) than the strobe time (sampling frequency), then we see the wheel slowly rotating forward or backward.

This effect is called aliasing (a frequency shows up in a “disguise” as another frequency)

Aliasing example

Example

- Strobe flashes at (Power Freq)×2 ≈ 100Hz
- Wheel 0 Rotates at 100 Hz ⇒ Period = 1/100Hz ≈ 10.0ms
- Wheel 1 Rotates at 101 Hz ⇒ Period = 1/101Hz ≈ 9.9ms
- Wheel 2 Rotates at 99 Hz ⇒ Period = 1/99Hz ≈ 10.1ms



Aliasing example

Example

- Strobe flashes at (Power Freq)×2 ≈ 100Hz
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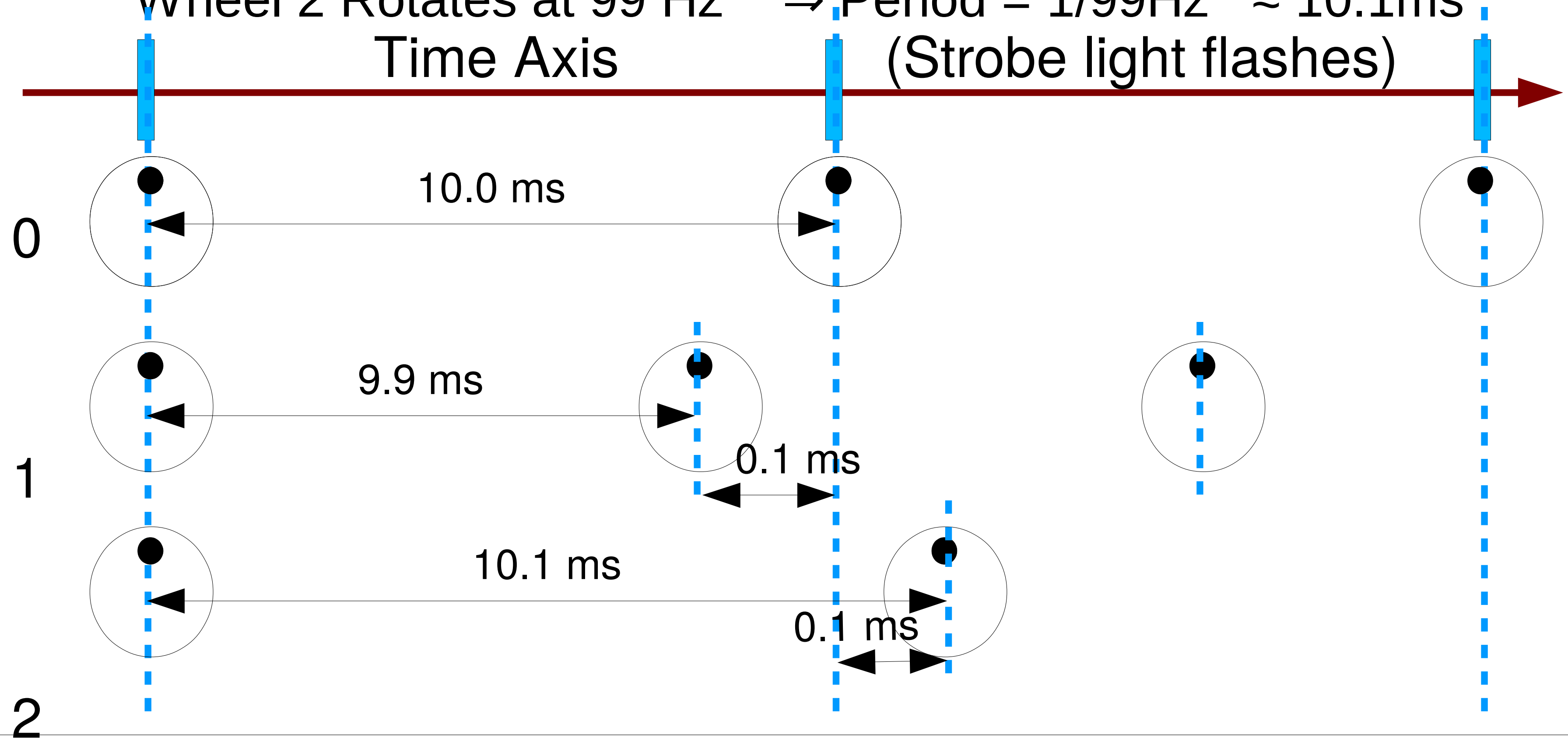
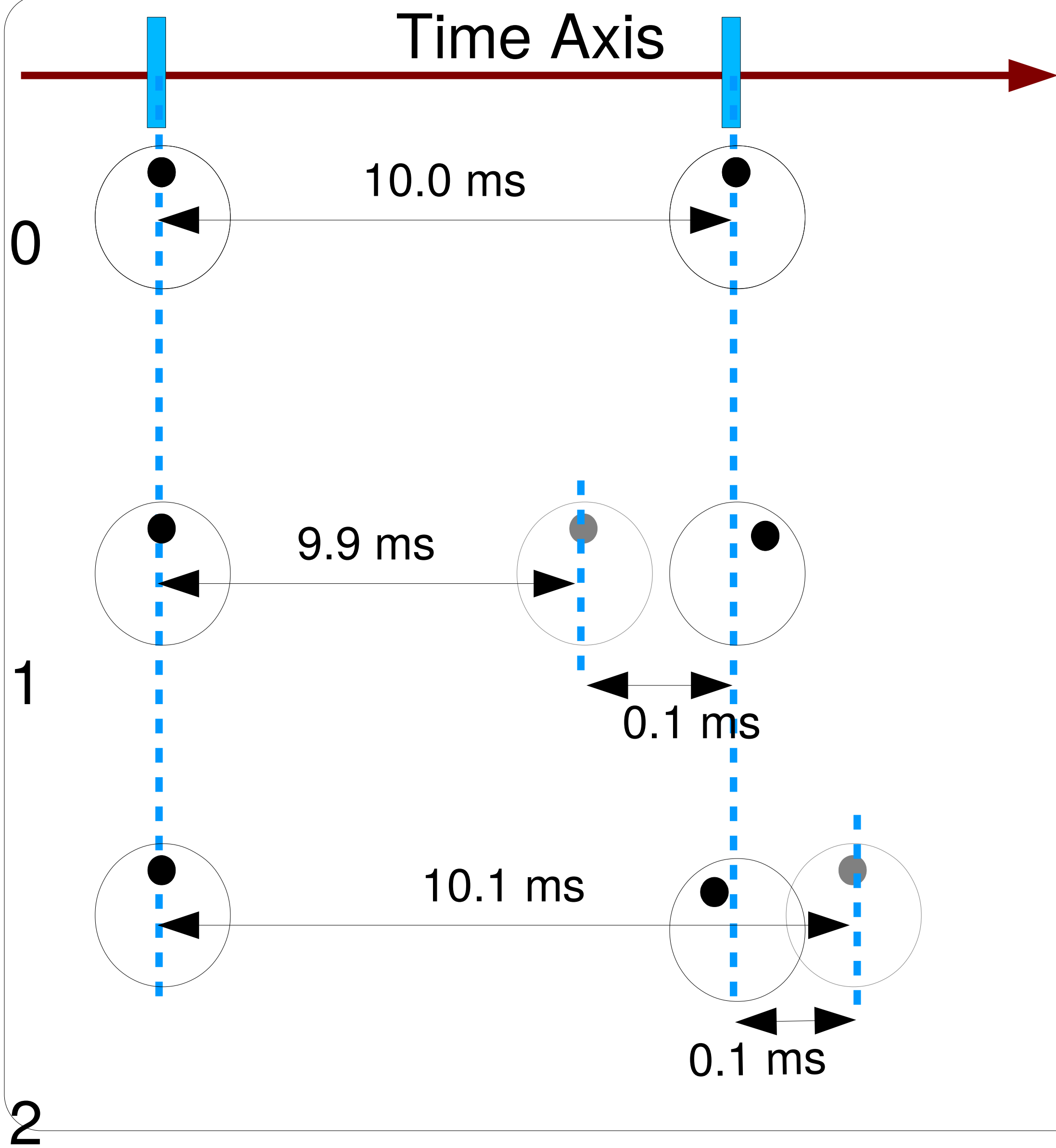


Sampling/Aliasing

Slide 8D.9

Apparent rotation



Wheel 0:
- Rotates an extra 0s
- Fraction = 0s/10ms = 0 cycles
- Freq = 0 cyc/0.01s = 0 Hz

Wheel 1:
- Rotates an extra 0.1s
- Frac = 0.1s/9.9s = .01 cycles
- Freq = 0.01 cyc/0.01s = 1 Hz

Wheel 2:
- Rotates an extra -0.1s ms
- Frac = -0.1s/10.1s = -0.01 cycles
- Freq = -0.01 cyc/0.01s = -1 Hz

Sampling/Aliasing

Slide 8D.10

Origin of aliased signals

Step 1: (C-T)

Continuous time signal

(here, most signal at one freq)

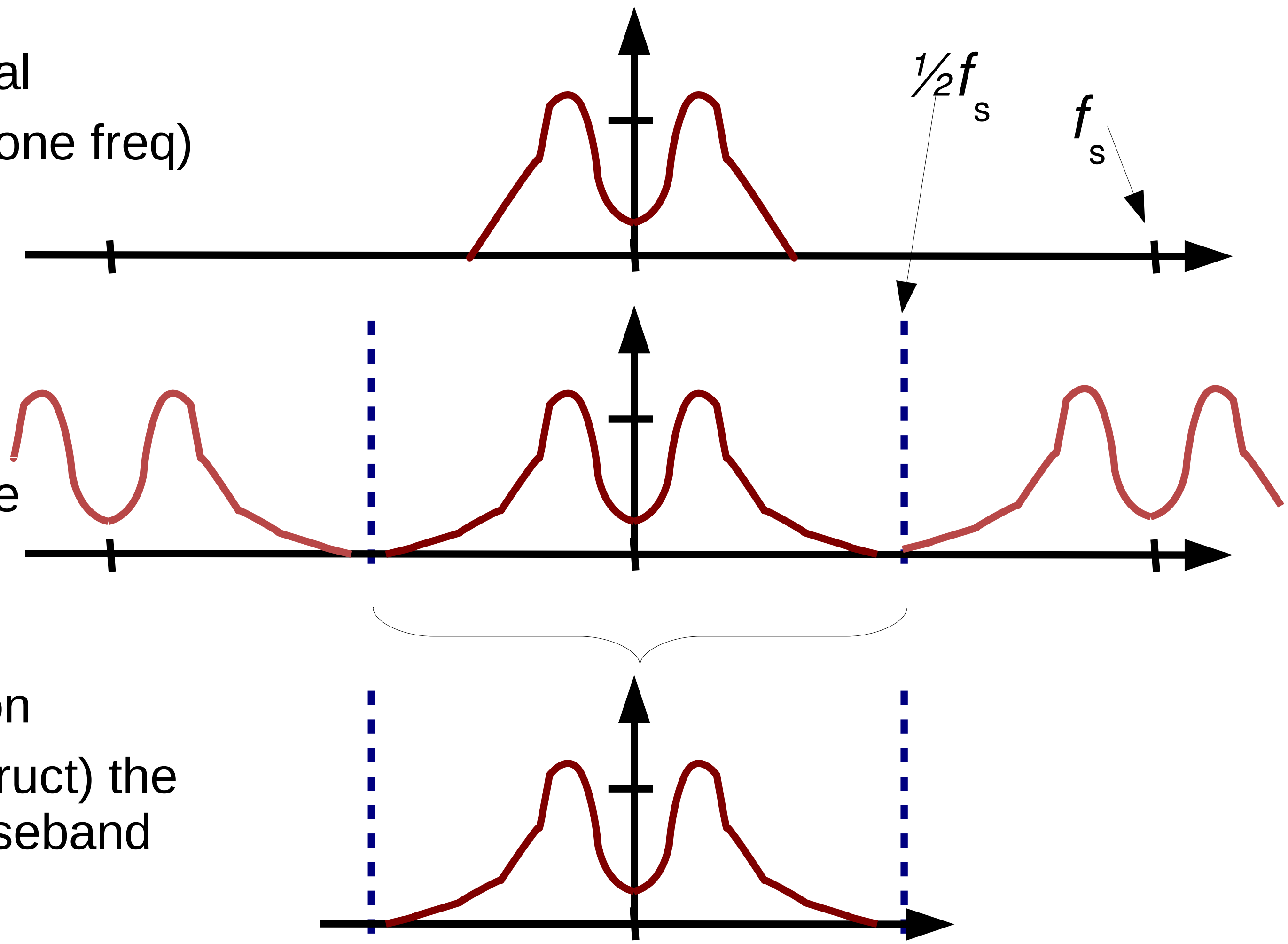
Step 2: (D-T)

Signal is sampled.

x[*n*] could originate from any replica of the spectrum

Step 3: Reconstruction

We interpret (reconstruct) the signal from the baseband spectral replica



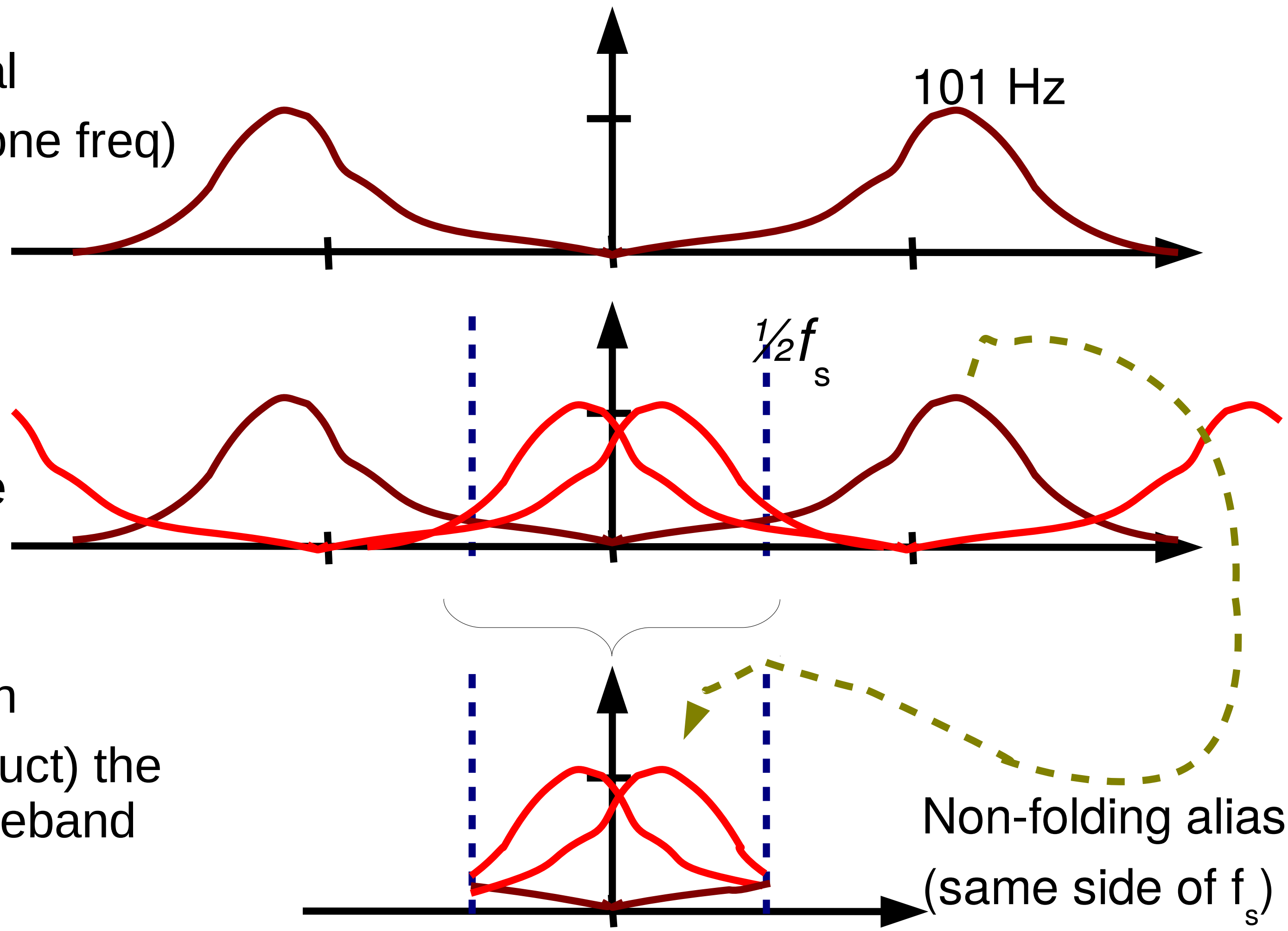
Perfect Reconstruction because $f_{max} < ½ f_s$

Origin of aliased signals: non-folding case

Step 1: (C-T)
Continuous time signal
(here, most signal at one freq)

Step 2: (D-T)
Signal is sampled.
$x[n]$ could originate from any replica of the spectrum

Step 3: Reconstruction
We interpret (reconstruct) the signal from the baseband spectral replica



Aliased Reconstruction because $f_{max} > ½ f_s$

Sampling/Aliasing

Slide 8D.12

Origin of aliased signals: folding case

Step 1: (C-T)

Continuous time signal

(here, most signal at one freq)

Step 2: (D-T)

Signal is sampled.

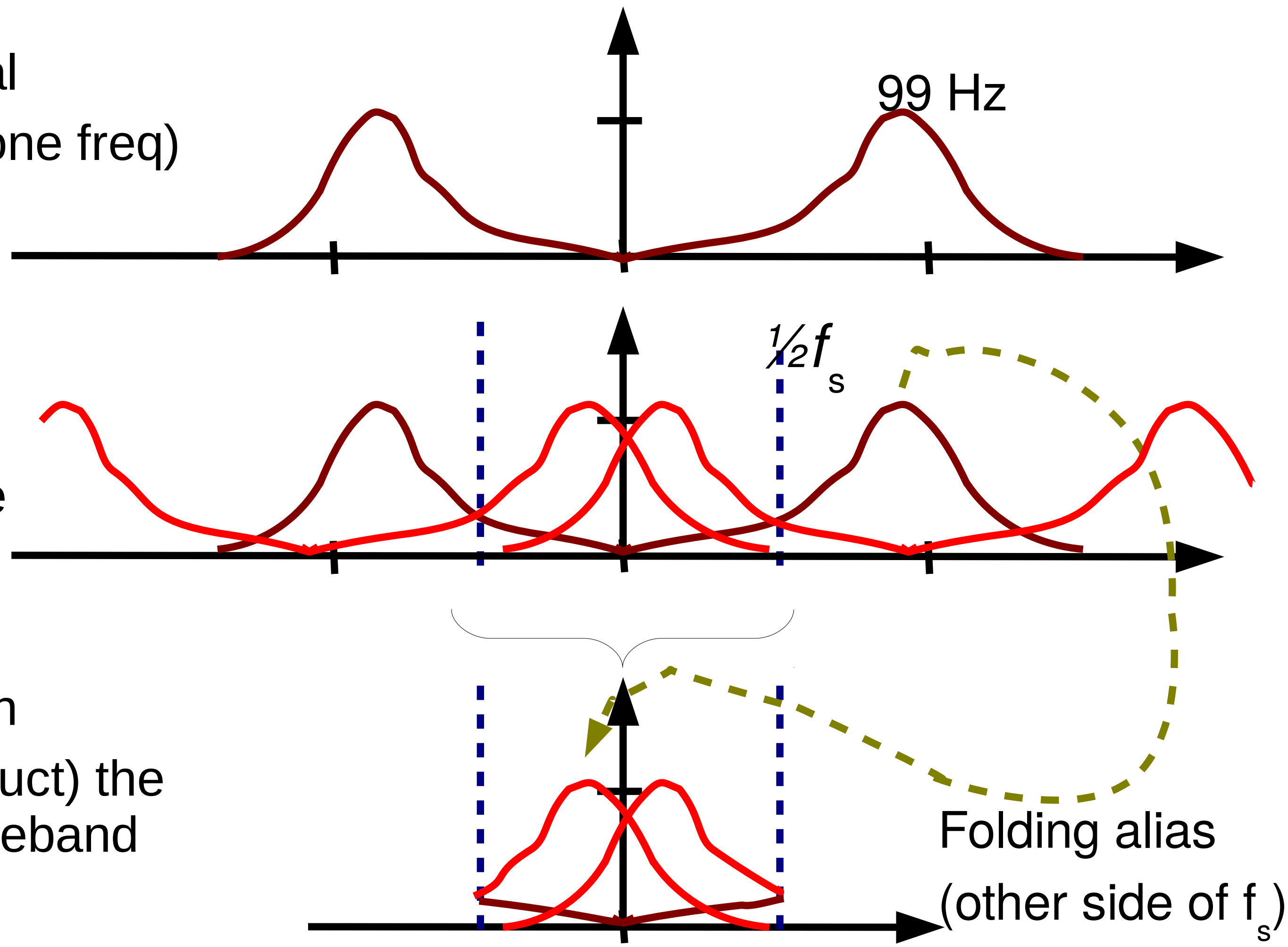
$x[n]$ could originate

from any replica of the

spectrum

Step 3: Reconstruction

We interpret (reconstruct) the signal from the baseband spectral replica



Aliased Reconstruction because $f_{max} > ½ f_s$

Folding vs Non-Folding

Non-Folding case:

– $x_1[n] = A\cos(\,2\pi(101)t + \theta\,)$

$= A\cos(\,2\pi(101)(0.01)n + \theta\,)$

$= A\cos(\,2\pi(1.01)n + \theta\,)$

$= A\cos(\,(2\pi + 0.02\pi)n + \theta\,)$

$= A\cos(\,2\pi(0.01)n + \theta\,)$

Non-folding:

Frequency is wrong

Direction is right

Folding case:

– $x_2[n] = A\cos(\,2\pi(99)t + \theta\,)$

$= A\cos(\,2\pi(99)(0.01)n + \theta\,)$

$= A\cos(\,2\pi(0.99)n + \theta)$

$= A\cos(\,(2\pi - 0.02\pi)n + \theta\,)$

$= A\cos(\,-2\pi(0.01)n + \theta\,)$

$= A\cos(\,2\pi(0.01)n - \theta\,)$

Folding:

Frequency is wrong

Direction is wrong

Questions

- EMG signal has maximum content at a f=5kHz.
  What sampling frequency required? What T_s?
- Assume uniform frequency content in EMG. Filter with a 4^{th} order Chebychev 0.5dB LPF with f_c of 1kHz. Sample frequency is f_s=10 kSamples/s.
  Is there aliasing? Estimate maximum amplitude of aliasing contribution?
- Sample signal with ADC. What resolution required so the quantization error is less than the aliasing contribution?
- Range of ADC is –2 V to +2V. EMG signal is –100mV to +100mV. Specify the ADC in bits (B) and f_s.