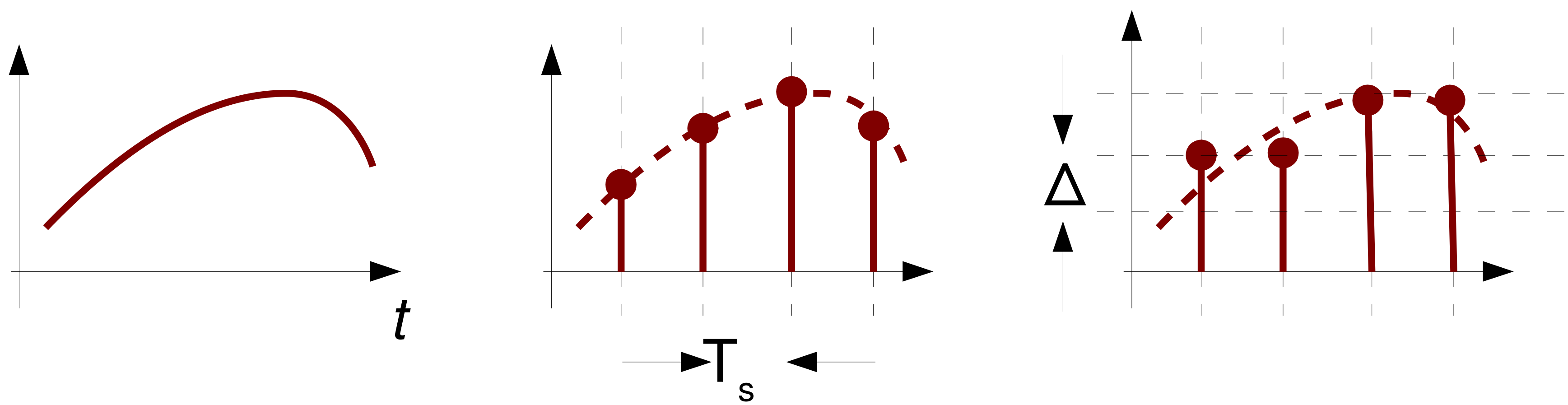
Spectrum

Slide 8C.1

Discrete Time Signals

Continuous-Time signals x(t) → Discrete-Time signals $x[n]$ → Quantized signals $x_q[n]$



Spectrum

Sum of Sinusoids

A signal *x*(*t*) can be is represented as a sum of sinusoids

$$x(t) = \sum_{k=1}^{N} A_k \cos(\Omega_k t + \phi_k)$$

where each sinusoid can have

- its own frequency Ω_k
- its own amplitude A_k
- its own phase ϕ_k

To put the sinusoid in the spectrum representation, we need to expand each sinusoid into *rotating phasors*.

Spectrum: Phasor Representation

Frequency spectrum representation => *Euler's formula*.

For cosine: $$\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2}$$

For sine: $$\sin(\theta)=\frac{e^{j\theta}-e^{-j\theta}}{2j}=\left\{\frac{e^{j\theta}-e^{-j\theta}}{2}\right\}e^{-j\pi/2}$$

For our general sinusoid it is decomposed as:

$$A\cos(\Omega t+\phi)=\frac{A}{2}\left\{e^{j(\Omega t+\phi)}+e^{-j(\Omega t+\phi)}\right\}$$

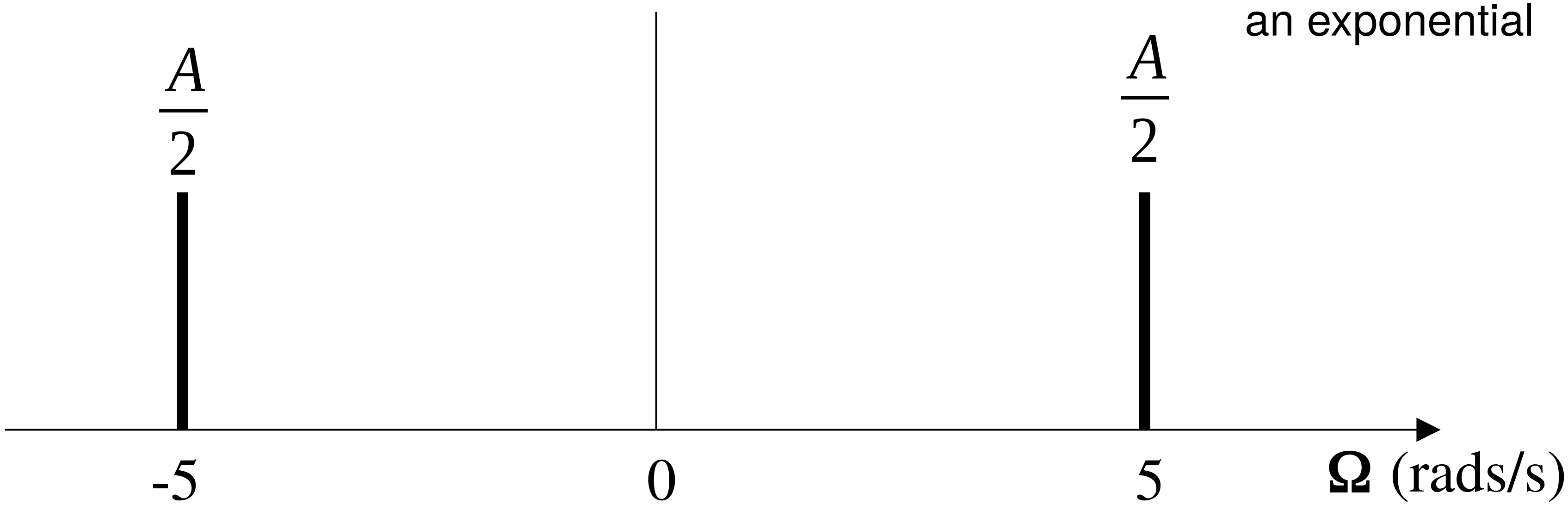
Spectrum

Graphical Representation

We can represent the following sinusoid graphically in a frequency spectrum as depicted below.

$$A\cos(5t) = \frac{A}{2}e^{j5t} + \frac{A}{2}e^{-j5t}$$

Each bar represents an exponential



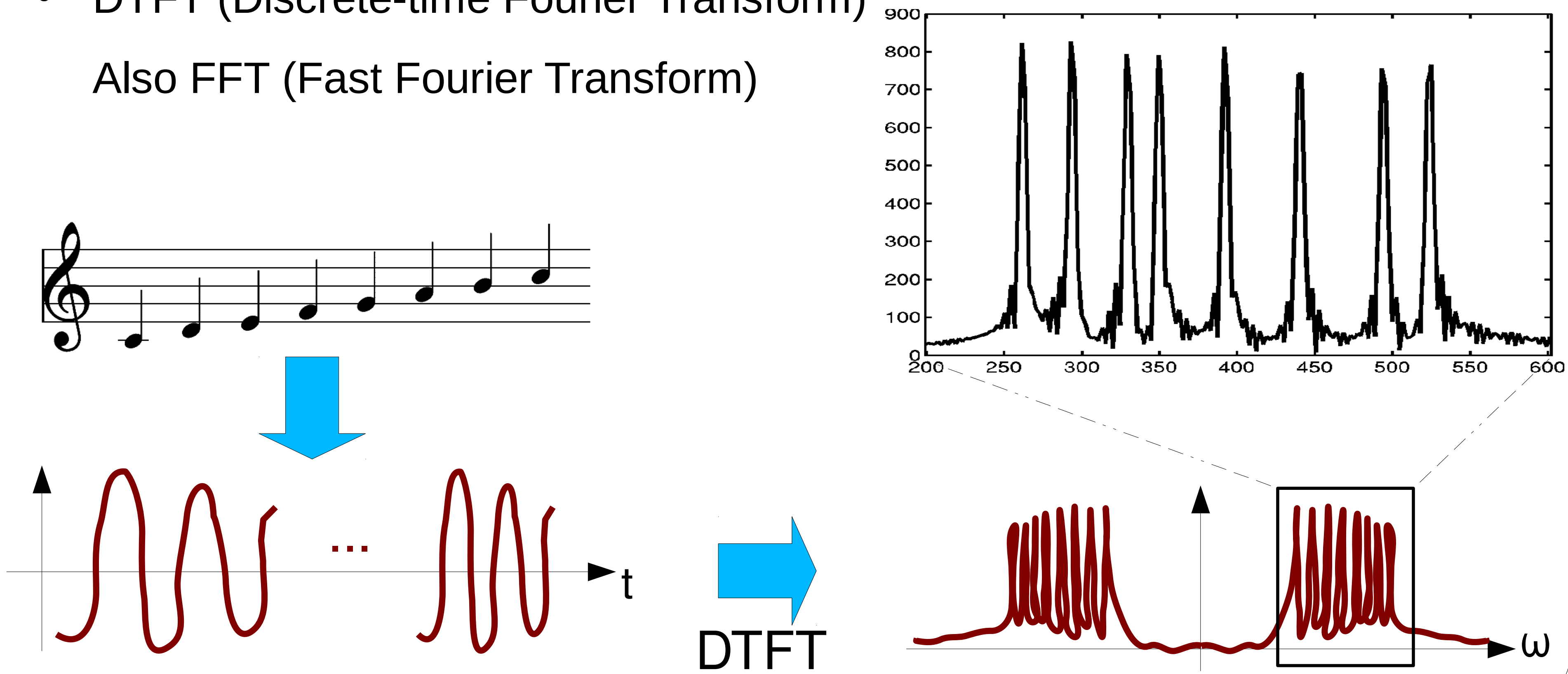
Amplitude, *phase* and *frequency* are represented.

Spectrum
Fourier Analysis

Fourier analysis: represent signal as a spectrum of phasors

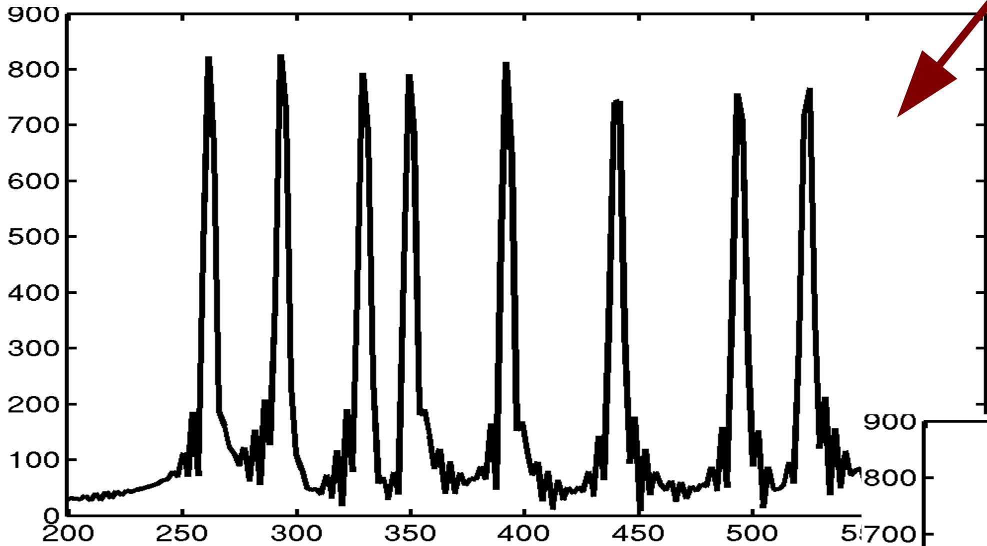
- DTFT (Discrete-time Fourier Transform)

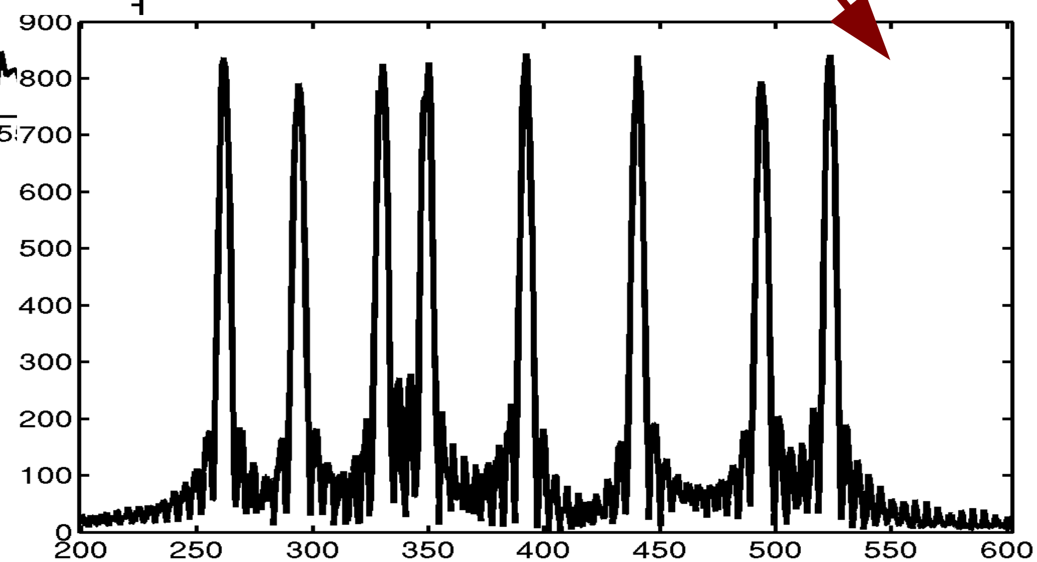
  Also FFT (Fast Fourier Transform)



Spectrum Analysis

Compare two signals $x_1(t)$ and $x_2(t)$





Signal Analysis
When in Time?

From the frequency spectrum of $x_1(t)$ and $x_2(t)$ we know they contain roughly the same frequency content.

When in time did these frequencies occur?

Possible answers:

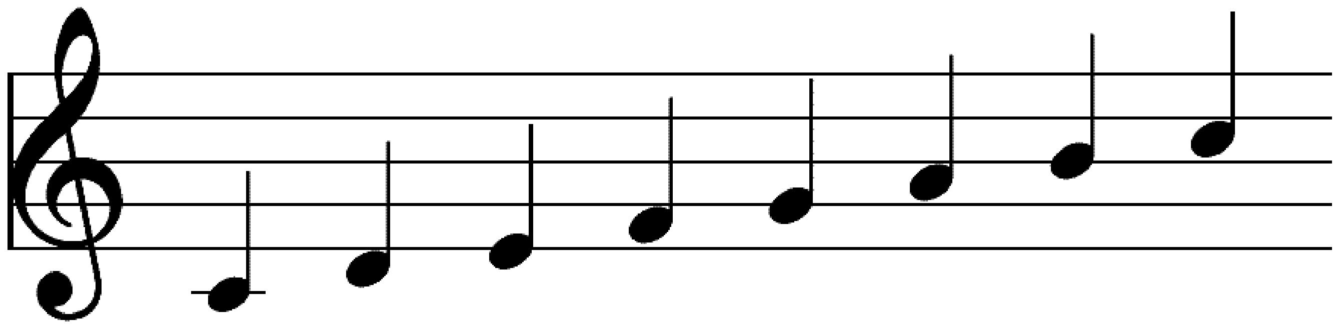
- Over all samples n for all time
- Each frequency at a different point in time
- Each frequency at a various points in time, possibly overlapping, possible repeating

Problem: From spectrum alone, we do not know *when* in time a particular frequency component has occurred

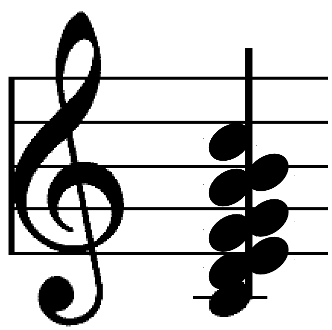
Signal Analysis

$x_1[n]$ and $x_2[n]$

$x_1(t)$: formed from a C-major scale.



$x_2(t)$: formed by playing notes simultaneously.



Spectrograms

Time-Varying Signals

How can we analyze a *time-varying signal* such where the frequency content changes over time?

We can calculate the DTFT over partitions of $x[n]$ and plot a 2D image where

- x-axis is time (specific partition)
- y-axis is frequency (index k for $X[k]$)
- colour/height is magnitude-squared of $X[k]$

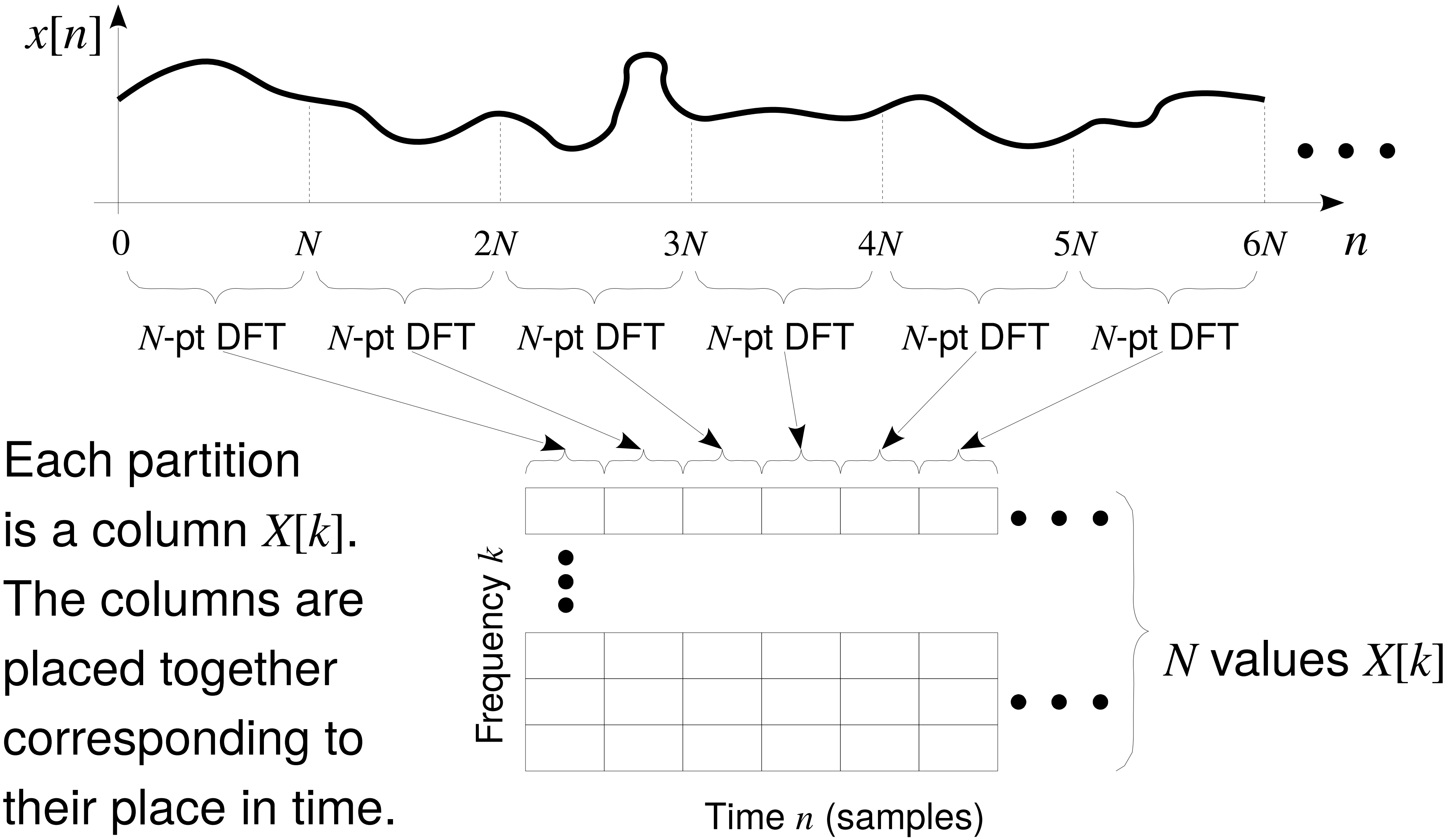
This is known as a ***spectrogram***.

Time-Frequency

Slide 8C.10

Spectrograms

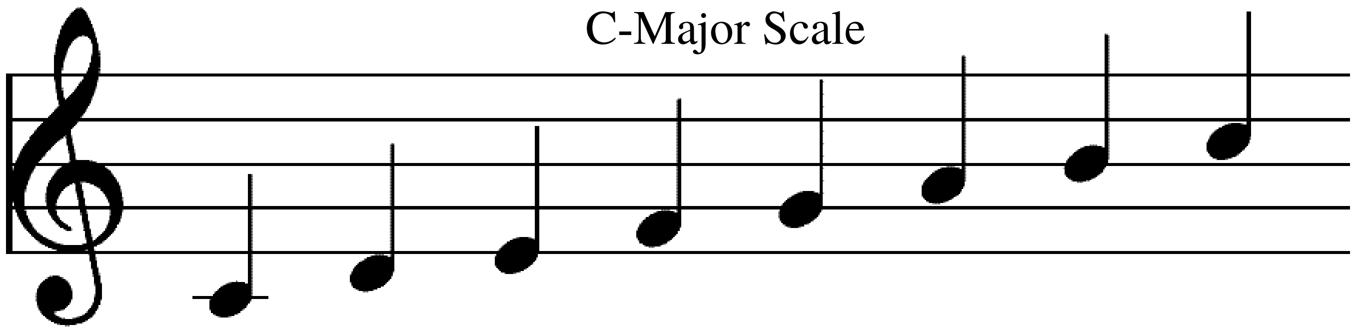
Partitioning Time



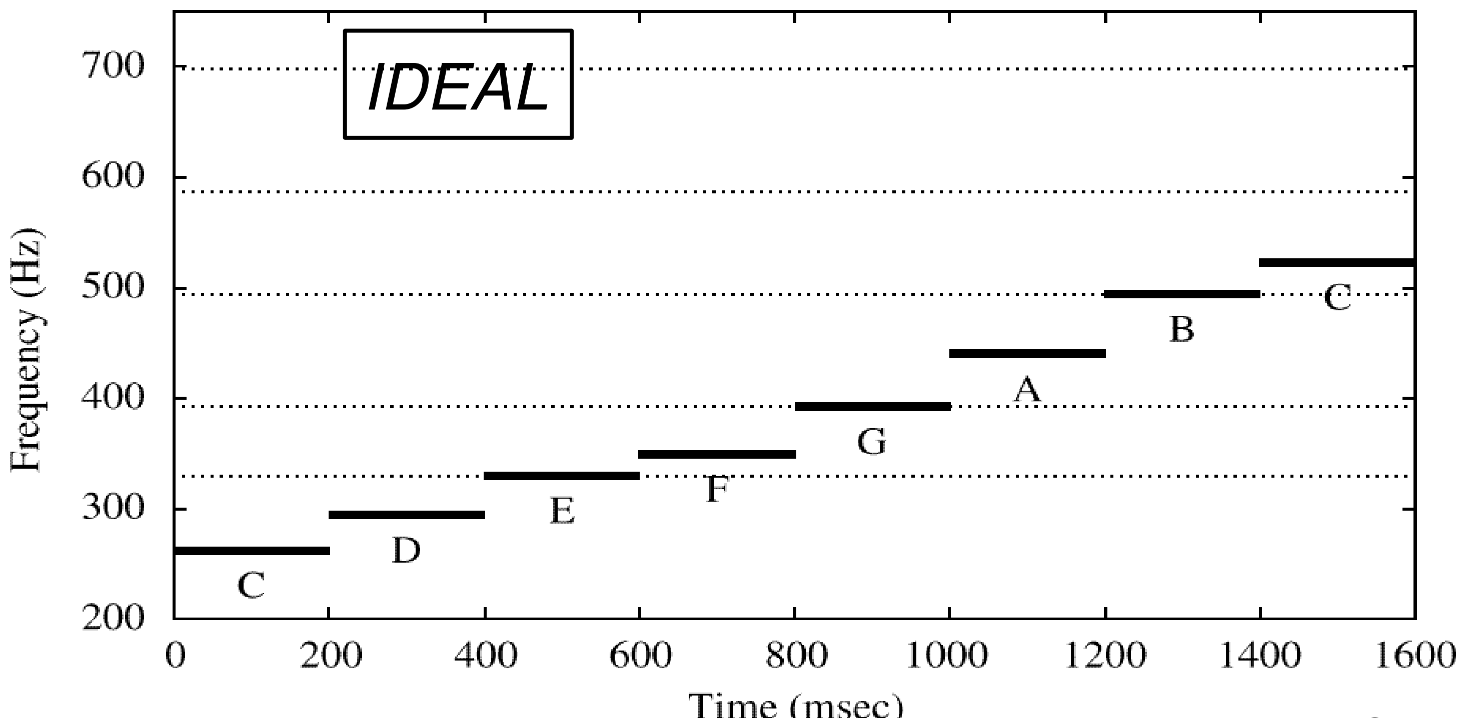
Each partition is a column $X[k]$. The columns are placed together corresponding to their place in time.

Spectrograms
Ideal C-Major Scale

C-Major Scale

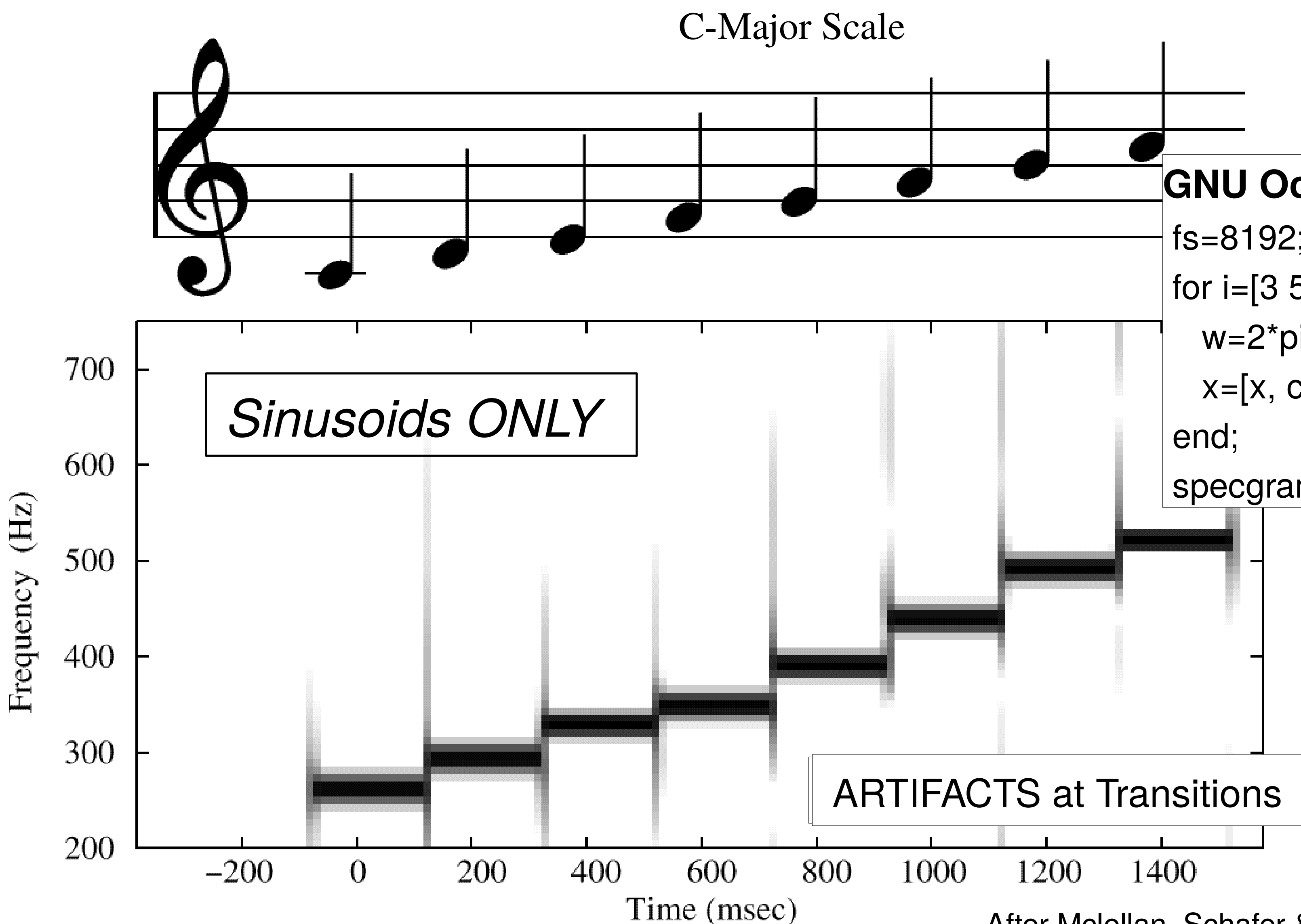


Frequencies of C-Major Scale



After Mclellan, Schafer & Yoder, *DSP First*

Spectrograms
C-Major Scale



GNU Octave/MATLAB:

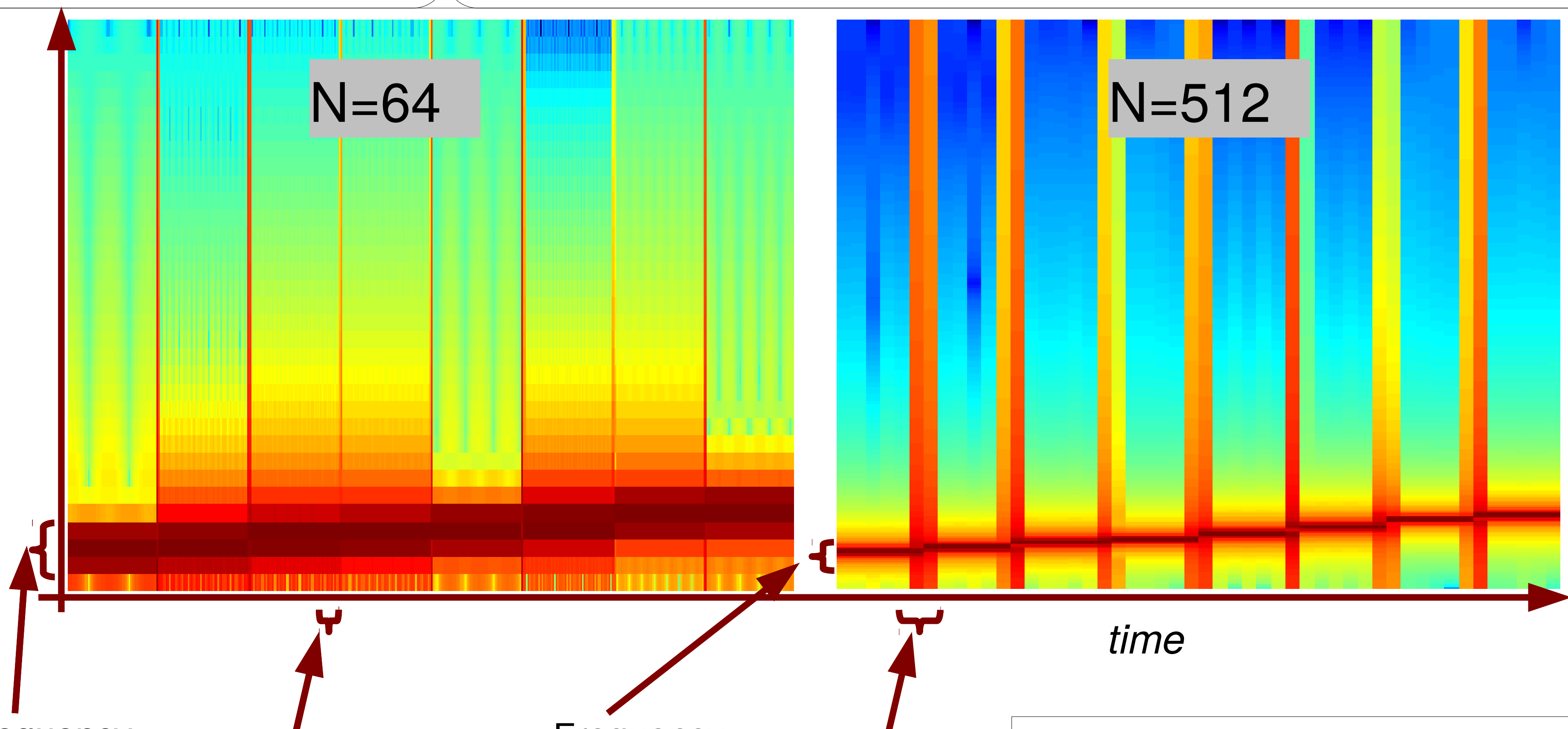
```
fs=8192;t=0:1/fs:0.2;x=[];
for i=[3 5 7 8 10 12 14 15]
   w=2*pi*(220*2^(i/12));
   x=[x, cos(w*t)];
end;
specgram(x,256,fs);
```

After Mclellan, Schafer & Yoder, *DSP First*

Time-Frequency

Slide 8C.13

Spectrogram Window Width (N)



GNU Octave/MATLAB:

```
fs=8192;t=0:1/fs:0.2;x=[];
for i=[3 5 7 8 10 12 14 15]
    w=2*pi*(220*2^(i/12));
    x=[x cos(w*t)];
end;
specgram(x,N,fs);
```

This effect is the *time-frequency* resolution trade-off.
It is similar to the Heisenberg uncertainty principle

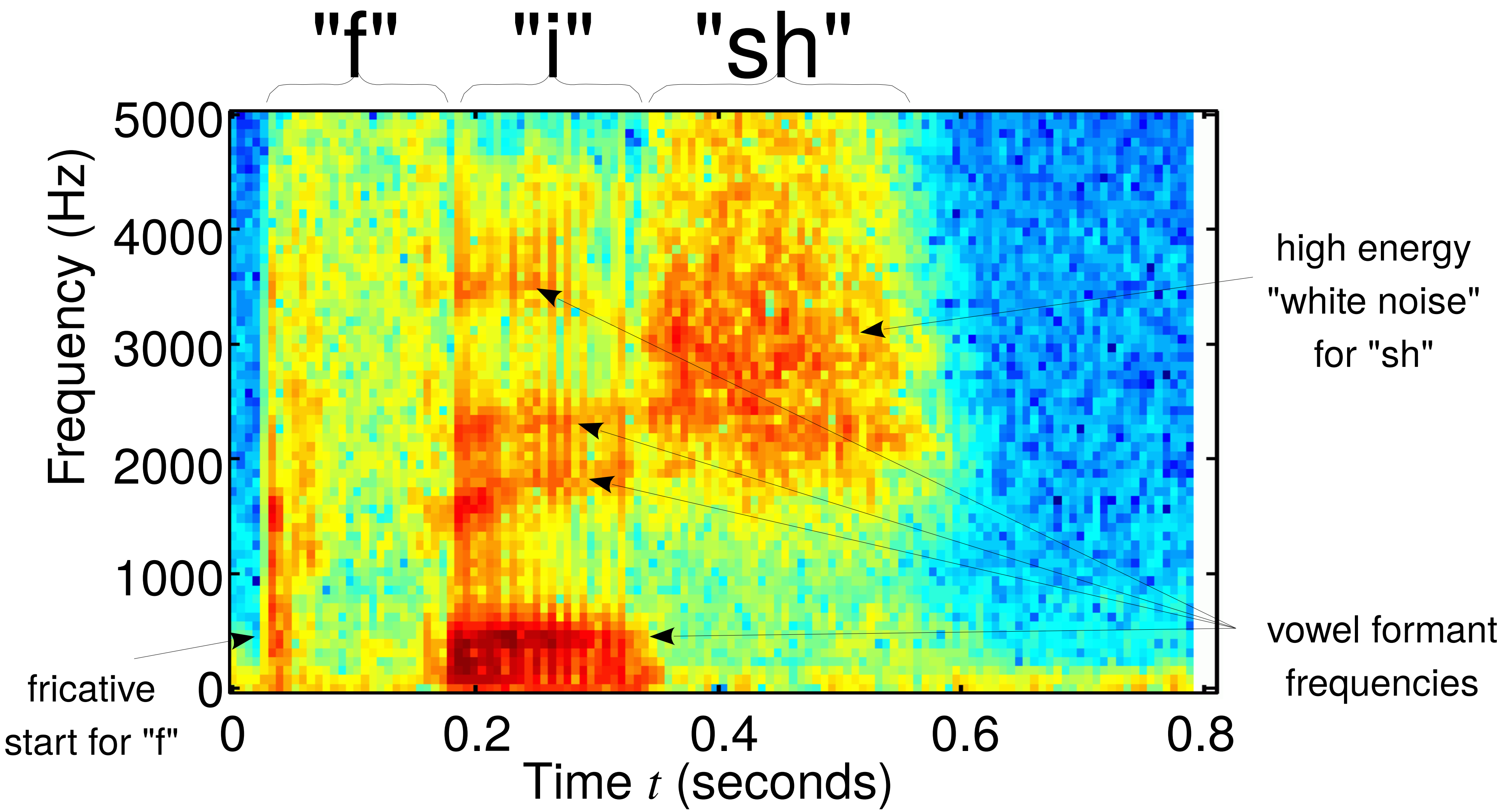
Time-Frequency

Slide 8C.14

Spectrograms

Speech

Spectrogram of male speaker saying "fish".



Questions

- What kind of situations is time frequency analysis useful. Comment on analysis of:
  - Speech
  - Music
  - EMG signals
- What is the origin of the artefacts at the frequency transitions?
- If F_s=10kHz and N=1024, what time frame does spectrogram window #9 represent?
- What about the phase in the phasor? Where is that shown in the spectrogram?