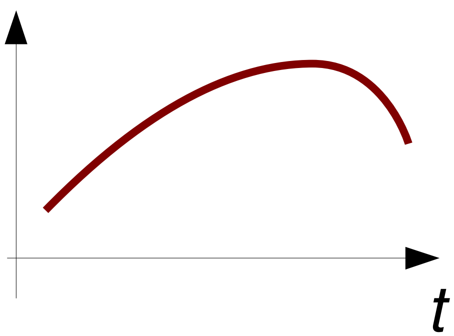
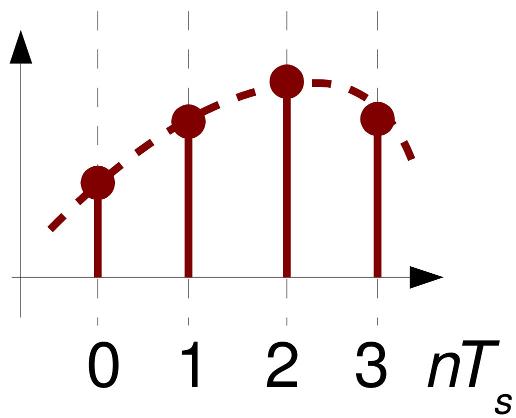
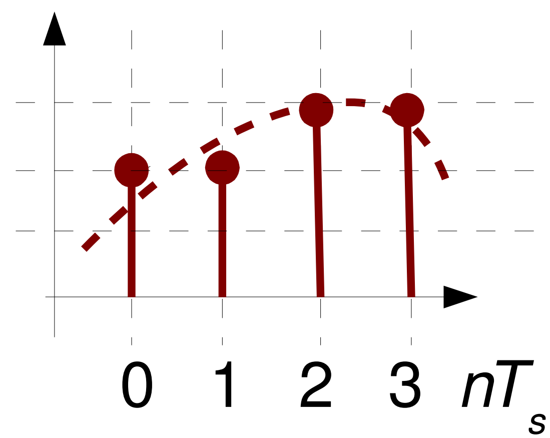
Quantization

Continuous-Time signals x(t) → Discrete-Time signals $x[n]$ → Quantized signals $x_q[n]$







Quantization: approximating a continuous range of values by a small set of discrete values.

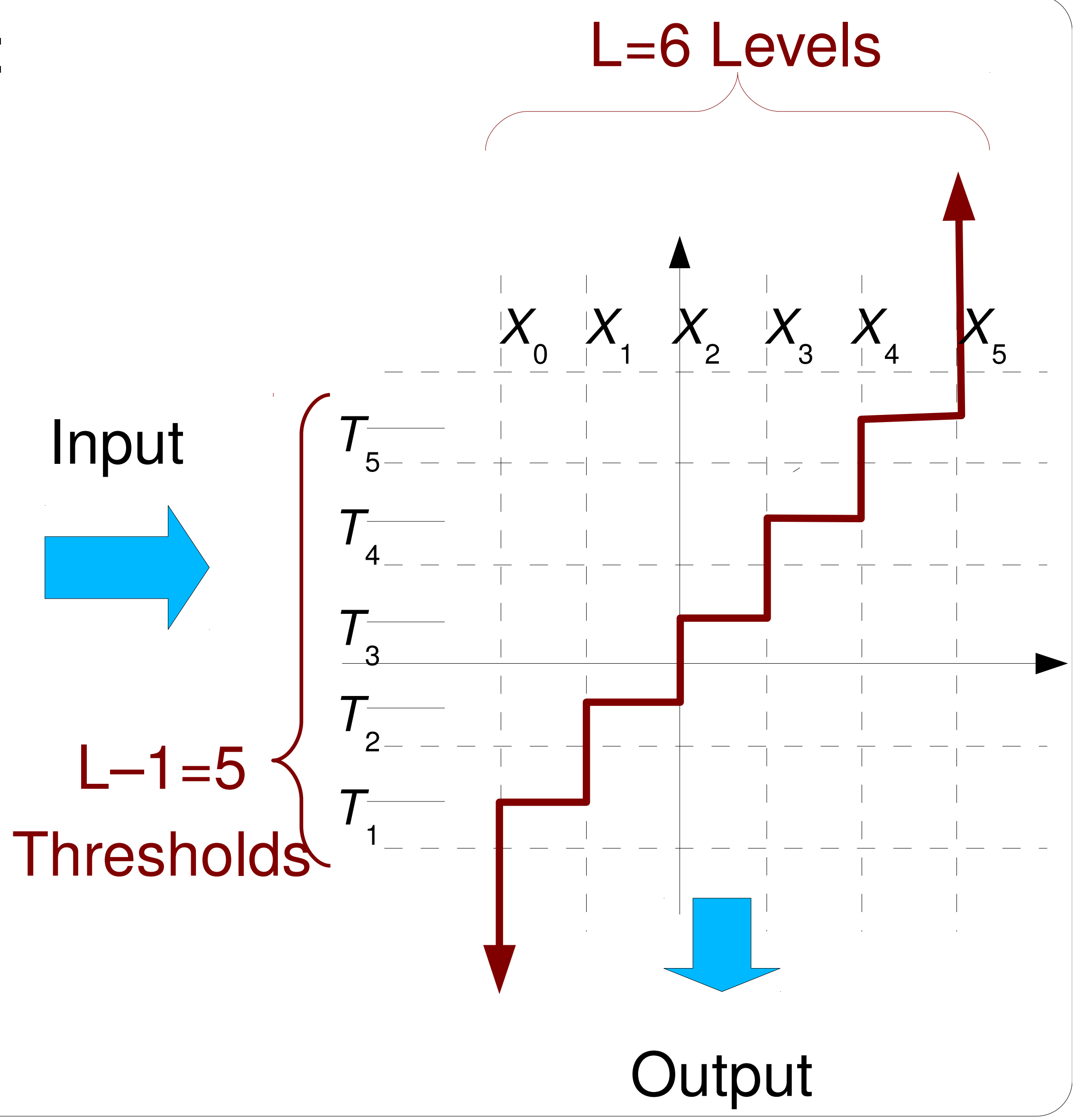
Quantization function

Quantizer represented by:

$$x_q[n] = Q(\ x[n]\ )$$

where

$$x_q[n] = \begin{cases} X_L & \text{if } T_L < x < +\infty \\ X_{L-1} & \text{if } T_{L-1} < x < T_L \\ & \vdots \\ X_1 & \text{if } T_1 < x < T_2 \\ X_0 & \text{if } -\infty < x < T_1 \end{cases}$$



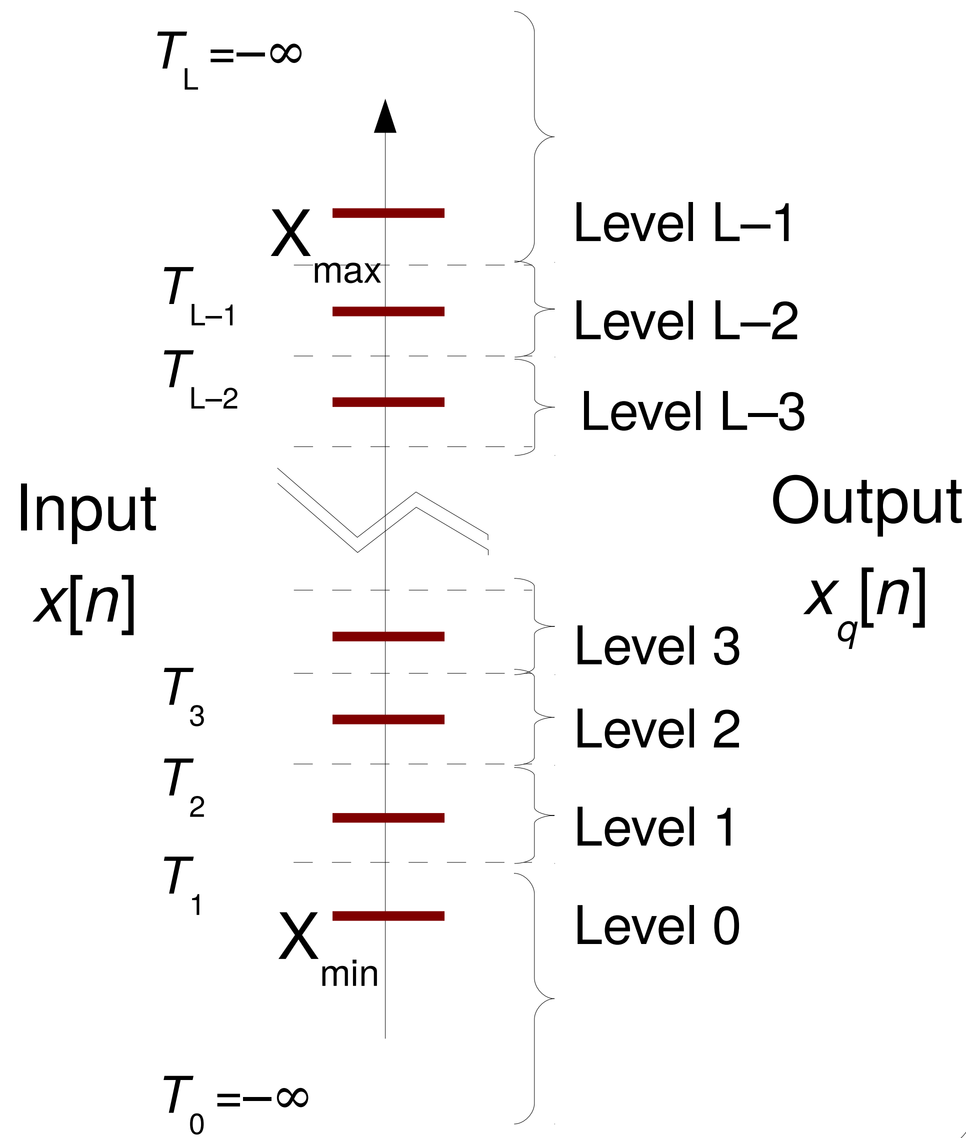
Quantizer

Our quantizer has

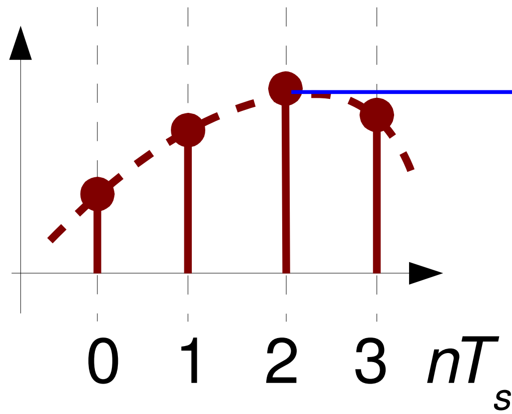
- L levels
- covers the range from X_{min} to X_{max}
- Level L_k has thresholds
  - Upper Threshold T_k
  - Lower Threshold T_{k-1}

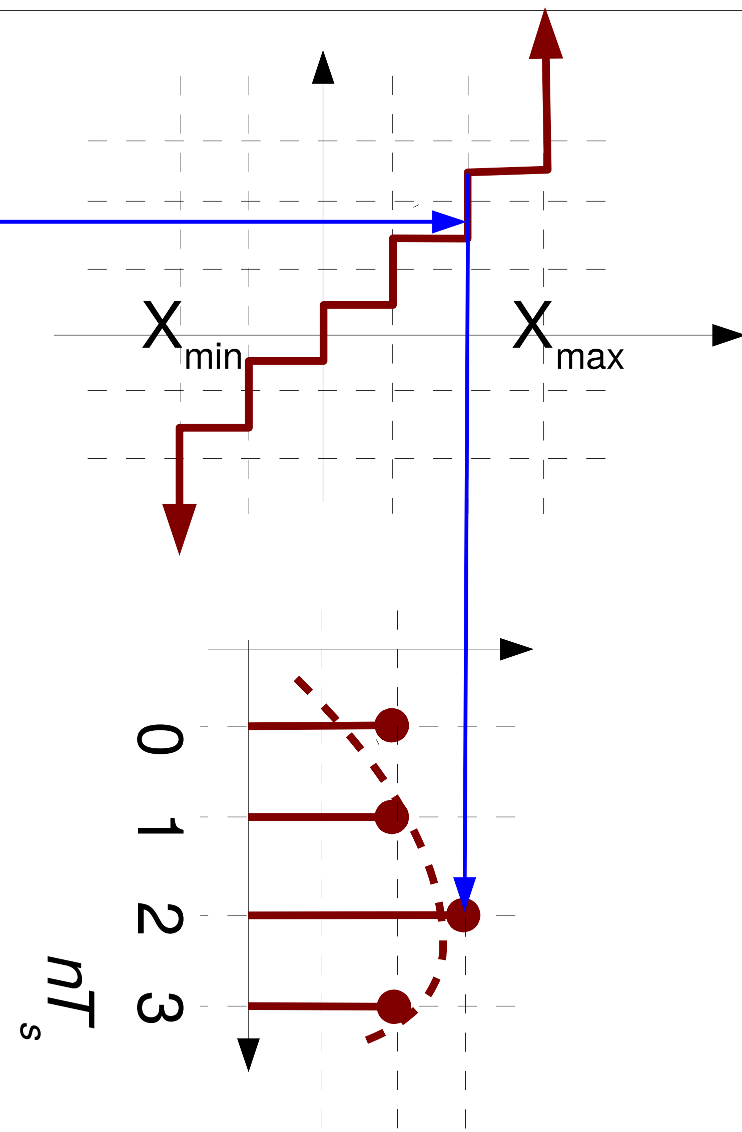
Quantizers can be

- Uniform
  - $T_k - T_{k-1} = \Delta =$ constant
  - Δ is the **quantization step**
- Non-uniform
  - $T_{k-1} - T_k$ varies
- Adaptive
  - Δ is changes with time



Uniform Quantizer





Given X_{min} and X_{max}, we calculate the quantization step, Δ, for a uniform quantizer

$$\Delta = \frac{X_{max} - X_{min}}{L-1} = \frac{Range}{L}$$

Quantization

Slide 8B.5

Range and Resolution

- Resolution = Quantization step = Δ

  $\Delta=(X_{max} - X_{min})/(L-1)$
- X_{max} is Δ below top of range

  $\Delta = \text{Range} / L$

Examples:

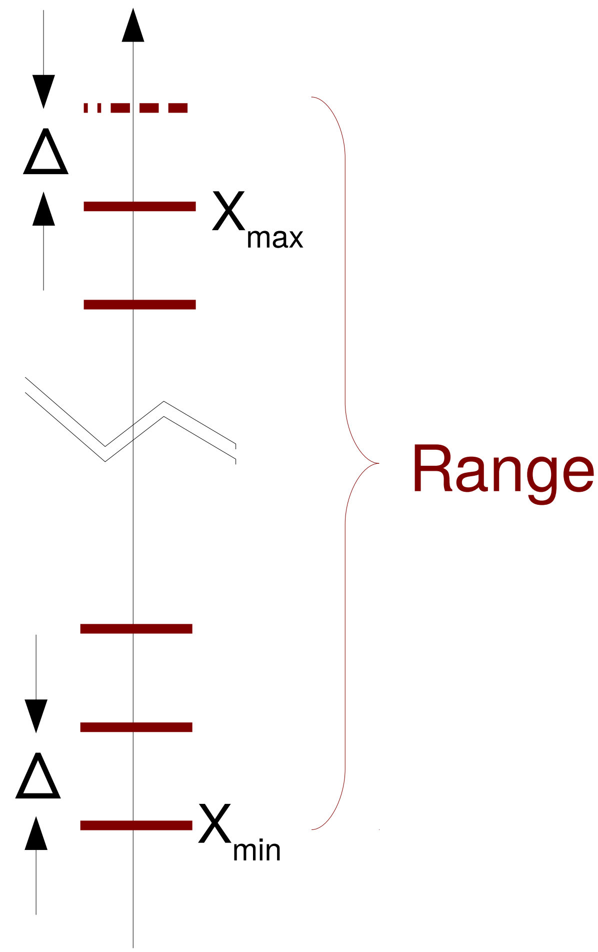
- 8-bit converter: Range is 0 to 4V

  $\Delta = \text{Range} / L = (4 - 0)/2^8 = 15.6\text{mV}$

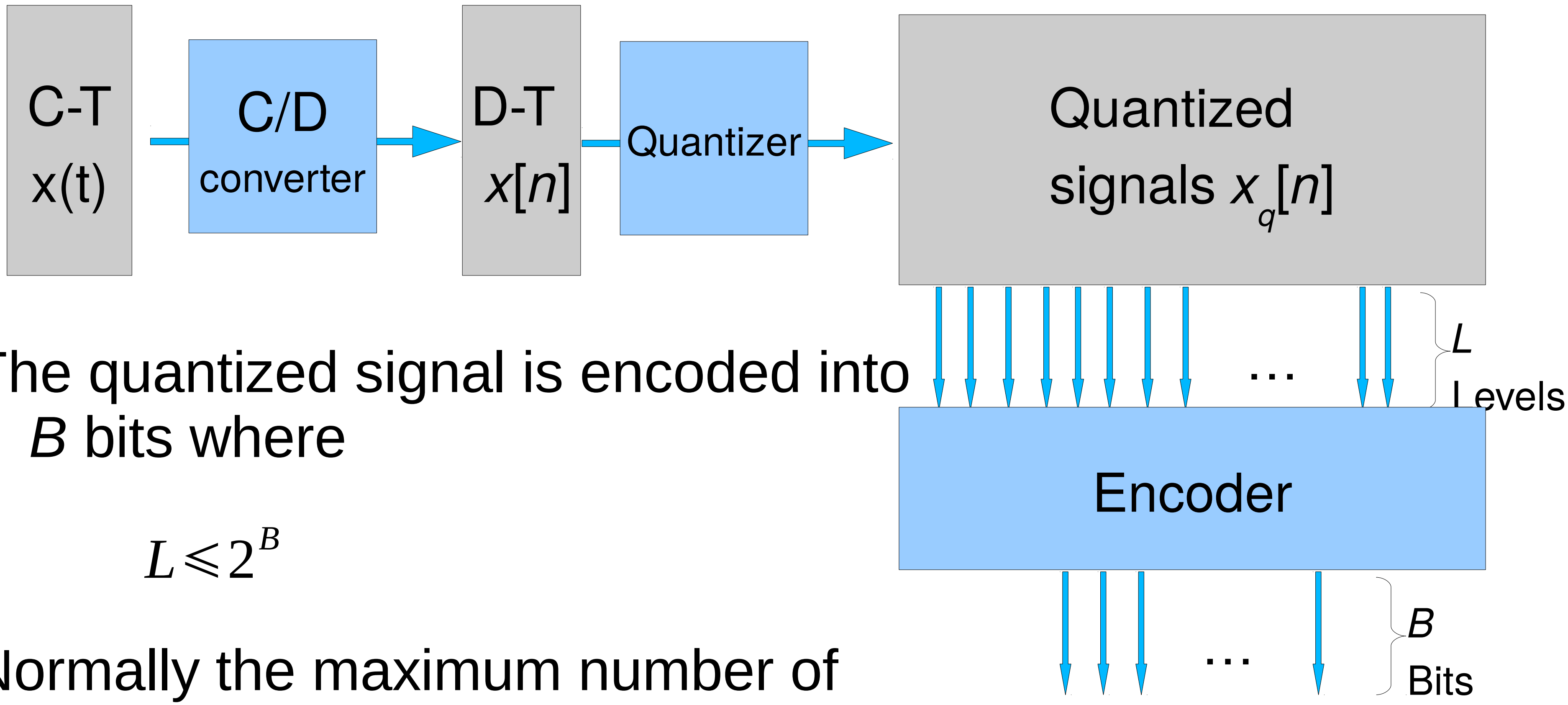
  $X_{max} = 4\text{V} - 15.6\text{mV}$
- 12-bit converter: Range is –1 to 1V

  $\Delta = \text{Range} / L = (1 - -1)/2^{12} = 0.49\text{mV}$

  $X_{max} = 1\text{V} - 0.49\text{mV}$



Encoding



The quantized signal is encoded into *B* bits where

$$L \leqslant 2^B$$

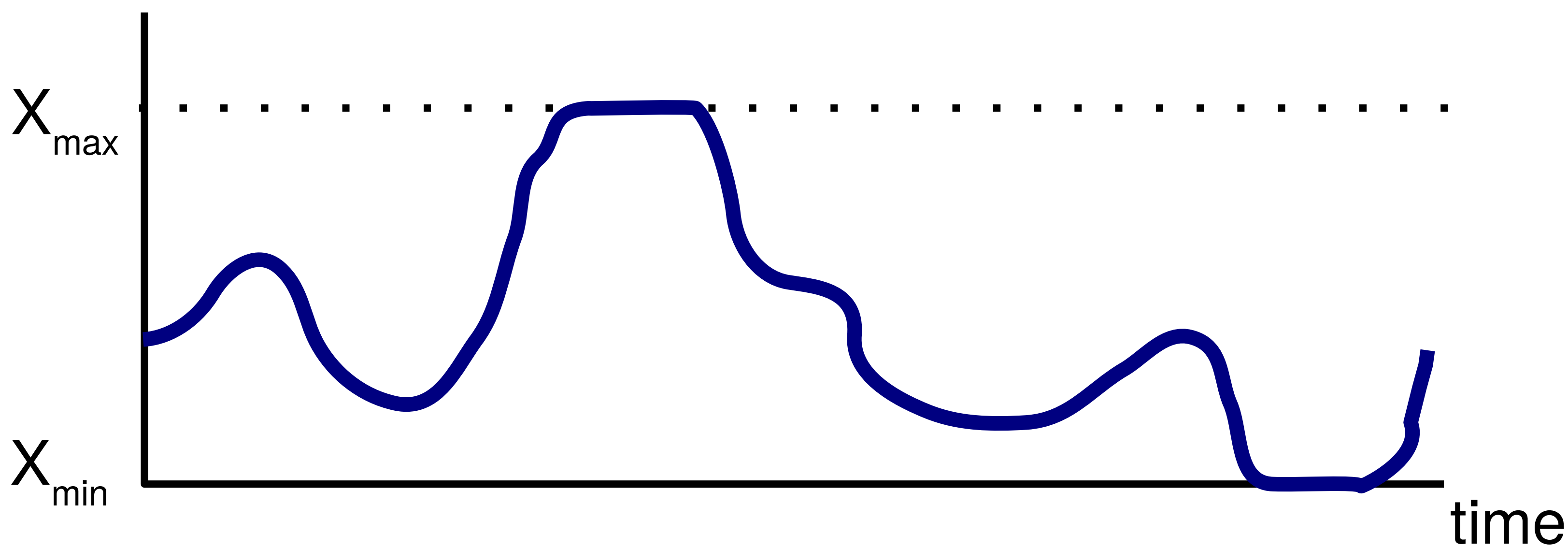
Normally the maximum number of levels are chosen for the number of bits. Common values are 8, 12, 16

Questions

- What is the role of
  - Sample and Hold?
  - Quantizer?
  - Encoder?
- An 8-bit ADC has a range of 0V – 1V
  - What is the resolution
  - What is the sensitivity?
  - To what voltages do the maximum and minimum output values correspond?
- What happens if the signal x(t) is outside the range X_{min} to X_{maxX}?

Question

- You are processing data, and you see values of X_{min} or X_{max} in the recorded data
- What does that mean?
- How should you process it?



Analog to Digital Converters

Encoding: 8, 12, 16 bit ADCs exist (also called bit depth)

Technologies

- Direct conversion (Flash conversion) – fastest, least bits
- Successive-approximation
- Sigma-Delta ADC
- Integrating ADC – slowest, largest bits

Terminology

- Range: $X_{max} - X_{min}$
  - Single Supply: $X_{min} = 0$
  - Differential: $X_{min} = -X_{max}$
- Resolution: Number of discrete values (= *L*)
  - Resolution in Volts is $(X_{max} - X_{min})/(L-1)$
- Sampling Frequency

ADC Example (Unipolar / Single Supply)

- 3-bit ADC => 2^3 = 8 Levels
- Input Range: Example 0 – 2V
- Δ = (Input Range)/Levels = 2V / 8 = 0.250V
- X_{max} = Max input range – Δ = 1.750

Output Level	Code	Min	Max	Level (V)	Min (V)	Max (V)
0	000	-∞	T_1	0.000	-∞	0.125
1	001	T_1	T_2	0.250	0.125	0.375
2	010	T_2	T_3	0.500	0.375	0.625
3	011	T_3	T_4	0.750	0.625	0.875
4	100	T_4	T_5	1.000	0.875	1.125
5	101	T_5	T_6	1.250	1.125	1.375
6	110	T_6	T_7	1.500	1.375	1.625
7	111	T_7	+∞	1.750	1.625	+∞

ADC Examples (Unipolar / Single Supply)

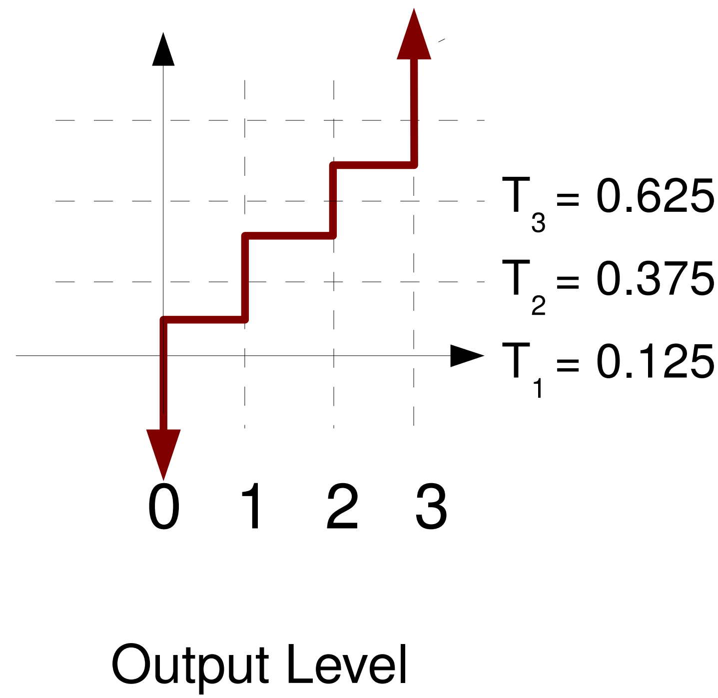
Single Supply / Unipolar ADC

- Range: 0 to 1 V
- B = 2
- $L = 2^2 = 4$
- $\Delta = (1 – 0) / 4 = 0.25$
- $X_{min} = 0$
- $X_{max} = 1V – \Delta = 0.75V$
- Thresholds, $T_1 \ldots T_3$

$T_1 = L_0 + (1 – ½)\times\Delta = 0.125$

$T_2 = L_0 + (2 – ½)\times\Delta = 0.375$

$T_3 = L_0 + (3 – ½)\times\Delta = 0.625$



ADC Examples (Bipolar)

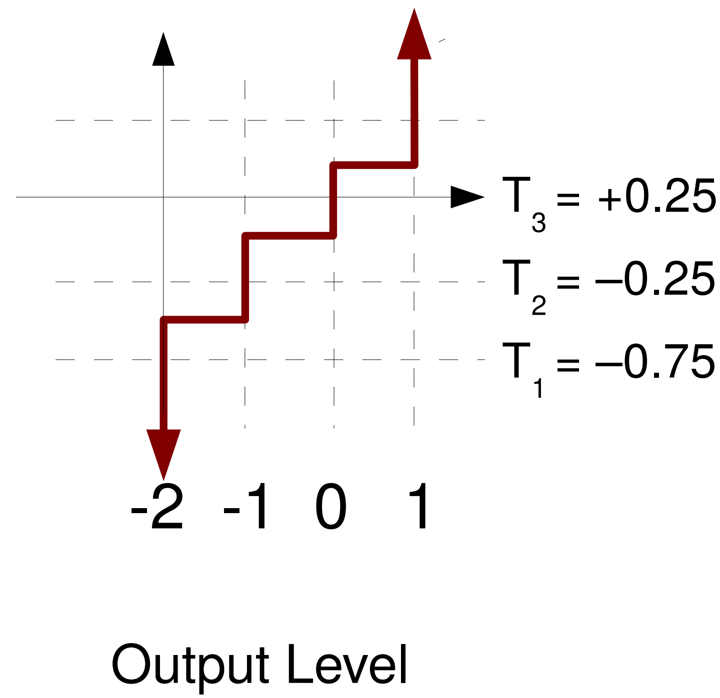
Single Supply / Unipolar ADC

- Range: –1 to +1 V
- $B = 2$
- $L = 2^2 = 4$
- $\Delta = (1 - -1) / 4 = 0.5$
- $X_{min} = -1V$
- $X_{max} = 1V - \Delta = 0.5V$
- Thresholds, $T_1 \ldots T_3$

$T_1 = L_0 + (1 - ½)\times\Delta = -0.75$

$T_2 = L_0 + (2 - ½)\times\Delta = -0.25$

$T_3 = L_0 + (3 - ½)\times\Delta = +0.25$

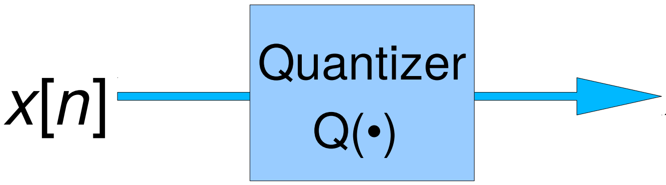


ADC Example (Bipolar / Dual Supply)

- 3-bit ADC => 2^3 = 8 Levels
- Input Range: Example –2V to +2V
- Δ = (Input Range)/Levels = 4V / 8 = 0.50V
- X_{max} = Max input range – Δ = 1.50

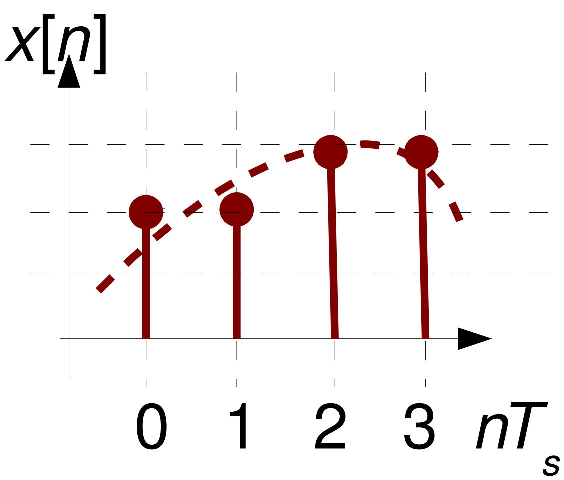
Output Level	Code	Min	Max	Level (V)	Min (V)	Max (V)
0	100	-∞	T_1	–2.00	-∞	–1.75
1	101	T_1	T_2	–1.50	–1.75	–1.25
2	110	T_2	T_3	–1.00	–1.25	–0.75
3	111	T_3	T_4	–0.50	–0.75	–0.25
4	000	T_4	T_5	0.00	–0.25	+0.25
5	001	T_5	T_6	+0.50	+0.25	+0.75
6	010	T_6	T_7	+1.00	+0.75	+1.25
7	011	T_7	+∞	+1.50	+1.25	+∞

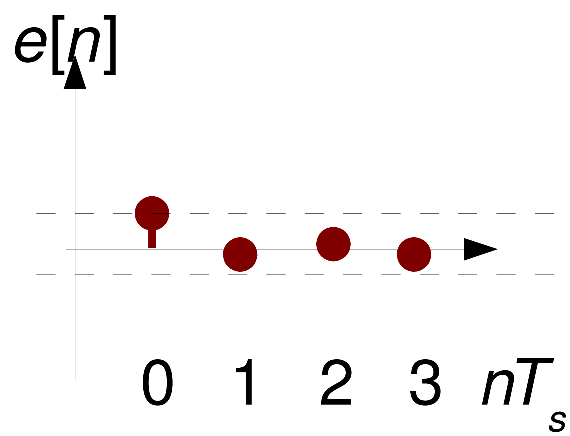
Quantization Error



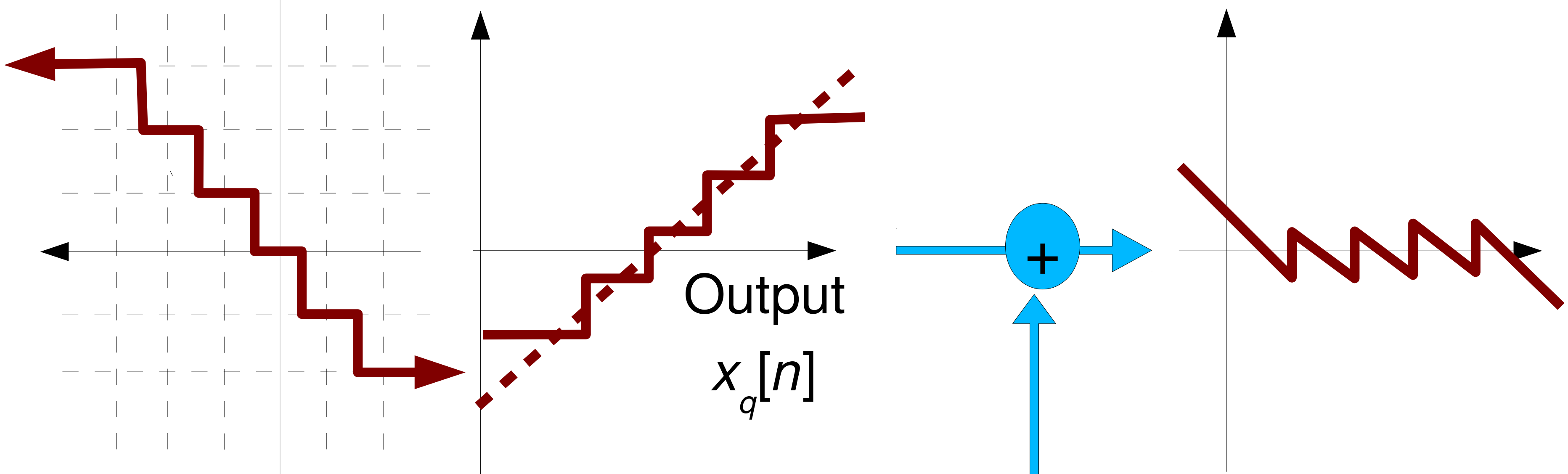
$x_q[n] = Q(\ x[n]\ )$

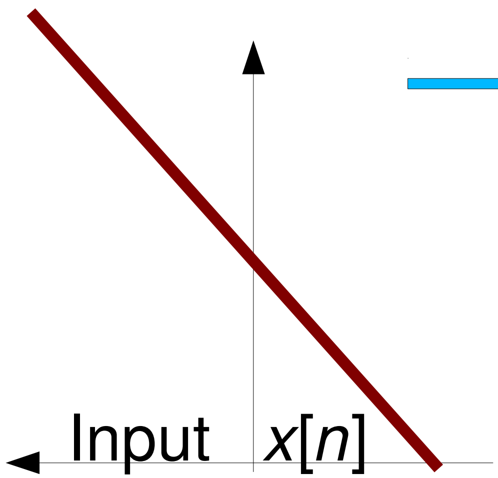
The difference $x_q[n] - x[n] = e[n]$ is called the Quantization Error





Quantization Error Limits





$e[n]$ is always less than ½(Δ) if

$(X_{min} - ½Δ) < x[n] < (X_{max} + ½Δ)$

We can consider e[n] to be random noise, if

- Statistics of e[n] do not change with time
- e[n] is uncorrelated with input x[n]

Questions

- What happens to the quantization error if $X_{min} > x[n]$
- If L=100, how many bits B are required for the encoder.
- If L is even and one level is 0, how do we assign levels for a double sided ADC?
- If the measurement noise at the sensor results in a SNR of 40 dB, and the input range of the ADC is 0-100°C.

  We want the quantization noise to be lower than the measurement noise. How many quantization bits, B, should the ADC have?

  (You may assume the temperature is uniformly distributed between 10-50°C, and the ADC input voltage is linearly related to temperature)

Question (cont ...)

An 8-bit ADC has an input range of +1 V to −1 V (corresponding to output codes of −128 to 127).

You use this ADC to design a circuit to measure EMG signals with a range of ±50 mV.

- What is the range of digital output values from the ADC we expect from these EMG signals?
- What is the resolution (in Volts)?
- What is the maximum quantization error?
- When you look at the data, there are a number of recorded values of −128 in the data stream. What does this mean?