For a decreasing exponential with time constant $\tau$, which is offset by $V(\infty)$, we have $\Delta V(t) = V(t) - V(\infty)$,

$$\Delta V(t) = \Delta V(0)e^{-t/\tau} \rightarrow V(t) - V(\infty) = (V(0) - V(\infty))e^{-t/\tau}$$

and

$$\frac{V(t) - V(\infty)}{V(0) - V(\infty)} = e^{-t/\tau}$$

For a square-wave of frequency $f$, period is $1/f = 2T$, as below

### Low-pass RC filter

- $V(\infty) = +5 V$.
- When stable $V(0) = -V(T)$.

$$V(T) - V(\infty) = (V(0) - V(\infty))e^{-T/\tau}$$

$$-V(0) - 5V = (V(0) - 5V)e^{-T/\tau}$$

$$-V(0) - V(0)e^{-T/\tau} = -5Ve^{-T/\tau} + 5V$$

$$-V(0)(1 + e^{-T/\tau}) = +5V(1 - e^{-T/\tau})$$

$$V(0) = -5V\left(\frac{1 - e^{-T/\tau}}{1 + e^{-T/\tau}}\right)$$

As $-T/\tau \rightarrow 0$,

$$V(0) = -5V\left(\frac{1}{1+1}\right) = 0$$

As $-T/\tau \rightarrow \infty$,

$$V(0) = -5V\left(\frac{1-0}{1+0}\right) = -5V$$

### High-pass RC filter

- $V(\infty) = 0 V$.
- When stable $V(T) = 10V - V(0)$.

$$V(T) - V(\infty) = (V(0) - V(\infty))e^{-T/\tau}$$

$$10 - V(0) - 0V = (V(0) - 0V)e^{-T/\tau}$$

$$10 - V(0) = V(0)e^{-T/\tau}$$

$$10V = V(0)e^{-T/\tau} + V(0) = V(0)(1 + e^{-T/\tau})$$

$$V(0) = 10V\left(\frac{1}{1+e^{-T/\tau}}\right)$$

As $-T/\tau \rightarrow 0$,

$$V(0) = 10V\left(\frac{1}{1+1}\right) = 5V$$

As $-T/\tau \rightarrow \infty$,

$$V(0) = 10V\left(\frac{1}{1+0}\right) = 10V$$