Distinguishability in EIT using a hypothesis-testing model

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Abstract: In this paper we propose a novel formulation for the distinguishability of conductivity targets in electrical impedance tomography (EIT). It is formulated in terms of a classic hypothesis test to make it directly applicable to experimental configurations. We test to *distinguish* conductivity distributions $\boldsymbol{\sigma}_2$ from $\boldsymbol{\sigma}_1$, from which EIT measurements are obtained with added white Gaussian noise with covariance $\boldsymbol{\Sigma}_n$. In order to *distinguish* the distributions, we must reject the null hypothesis H_0: $\hat{\text{x}} = 0$, which has a probability based on the z-score: $z = \frac{\bar{x}}{\sigma_x}$. This result shows that distinguishability is a product of the impedance change amplitude, the measurement strategy and the inverse of the noise amplitude. This approach is used to explore different current stimulation strategies.

1 Introduction

Electrical impedance tomography (EIT) attempts to reconstruct the impedance distribution within a body from electrical stimulation and measurement at a series of electrodes attached to body surface. One key figure of merit for a given EIT system is its distinguishability, which measures an index of the feasibility with which two different impedance distributions ($\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$) may be distinguished. This measure incorporates several factors: the size, impedance difference between $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, and the characteristics of the EIT systems such as signal to noise ratio (SNR), the position, number of electrodes, and stimulation and measurement patterns on the electrodes.

In this paper we propose a novel formulation for the distinguishability of conductivity targets in EIT. This work compliments the classic formulation of the problem by Isaacson[1], which calculated the measurement precision needed to distinguish between two different conductivity distributions. A similar formulation was proposed by Lionheart *et al* to find optimal current patterns considering electrical safety [2]. In this work, we propose the formulation of distinguishability in terms of a classic hypothesis test, which makes it easier to apply to experimental configurations. We develop the formulation for the distinguishability and demonstrate its applicability to a simulated EIT system.

2 Image Reconstruction

An EIT system, characterized by $F(\cdot)$, is used to make a set of measurements represented by an $M \times 1$ vector

$$\mathbf{v} = F(\boldsymbol{\sigma}) + \mathbf{n} \tag{1}$$

from an impedance distribution characterized by an $N \times 1$ parameter vector $\boldsymbol{\sigma}$, where $F(\cdot)$ describes the EIT measurement process. In all measurement systems there is a contribution of random noise $\mathbf{n}$, which we characterize as zero-mean, independent and Gaussian. This characterization is justified as follows: the noise has zero-mean because any average bias can theoretically be measured and incorporated into the EIT model $F(\cdot)$; the independent noise means that there is no correlation between the noise and the two samples we wish to distinguish. This is a reasonable assumption in the most EIT hardware designs if measurements are separated in time by several frames. Correlations between individual measurements with the measurement vector $\mathbf{v}$ may exist and be modelled with this approach. The assumption of Gaussian noise is designed to make the statistical computations easier; however, there is some evidence to indicate that EIT noise is not Gaussian[3]. Specifically, large errors occur far more often than would be predicted by a Gaussian with a time-invariant covariance $\mathbf{\Sigma}_n$.

Problem Formulation: we wish to *distinguish* conductivity distributions $\boldsymbol{\sigma}_2$ from $\boldsymbol{\sigma}_1$ through which EIT measurements $\mathbf{v}_1 = F(\boldsymbol{\sigma}_1) + \mathbf{n}_1$, and $\mathbf{v}_2 = F(\boldsymbol{\sigma}_2) + \mathbf{n}_2$ are performed, where $\mathbf{n}_1$ and $\mathbf{n}_2$ represent the instantiations of zero-mean white Gaussian noise with covariance $\mathbf{\Sigma}_n$. Since we generally want to distinguish small changes in conductivity, we linearize around $\boldsymbol{\sigma}_1$ to obtain a conductivity change $\mathbf{x} = \boldsymbol{\sigma} - \boldsymbol{\sigma}_2$ which is a linear function of measurements

$$\mathbf{y} = \mathbf{v} - \mathbf{v}_1 = \mathbf{J}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{J}$ is the Jacobian (sensitivity) defined as

$$[\mathbf{J}]_{i,j} = \left.\frac{\partial F_i(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}_j}\right|_{\boldsymbol{\sigma}=\boldsymbol{\sigma}_1}. \tag{3}$$

This linearization assumes that reasonably small impedance changes $\mathbf{x}$ occur, which is generally a reasonable assumption if we are investigating the distinguishability limits for most EIT systems.

From measurements $\mathbf{y}$, an impedance change image estimate $\hat{\mathbf{x}}$ is reconstructed, from a linearized difference EIT reconstruction algorithm as $\hat{\mathbf{x}} = \mathbf{R}\mathbf{y}$, defined from the norm

$$\hat{\mathbf{x}} = arg\,\underset{\mathbf{x}}{min}\, \|\mathbf{y} - \mathbf{J}\mathbf{x}\|_{\mathbf{\Sigma}_n^{-1}} + P(\mathbf{x}), \tag{4}$$

where $P(\cdot)$ represents a penalty or regularization term. Such a linear reconstruction matrix can describe the majority of difference EIT reconstruction algorithms, such as the Sheffield Backprojection and many regularization based schemes.

In general, we are interested in investigating the image output within a specific region of interest (ROI), defined by $\mathbf{I}_R$, where $\mathbf{I}_R$ is a vector of the size of the image parameter space $N \times 1$ in which each element $[\mathbf{I}_R]_i$ is zero, if i represents a region outside of the ROI, and the area of the parameter region i, if i is in the ROI. If a fraction of region i is in the ROI, $[\mathbf{I}_R]_i$ is weighted by the fraction of membership. Based on the ROI, we define a scalar impedance change x_R in the ROI as the weighted average of the impedance change $\mathbf{x}$ in the ROI, or $x_R = \mathbf{I}_R\mathbf{x}$. The best estimate of parameter x based on measurements $\mathbf{y}$ is defined as $\hat{\mathrm{x}}$.

$$\hat{\mathrm{x}} = \mathbf{I}_R\hat{\mathbf{x}} = \mathbf{I}_R\mathbf{R}\mathbf{y} = \mathbf{R}_R\mathbf{y}, \tag{5}$$

where $\mathbf{R}_R = \mathbf{I}_R\mathbf{R}$ is defined to be the reconstruction matrix of the ROI of size $1 \times M$. Since $\hat{\mathrm{x}}$ is a single parameter, no regularization term $P(\cdot)$ is required, and the maximum likelihood solution matrix, $\mathbf{R}_R$, is given by:

$$\mathbf{R}_R = \left(\mathbf{J}_R^t\mathbf{\Sigma}_n^{-1}\mathbf{J}_R\right)^{-1}\mathbf{J}_R^t\mathbf{\Sigma}_n^{-1}, \tag{6}$$

where $\mathbf{J}_R = \mathbf{J}\mathbf{I}_R^t$.

3 Formulation of Distinguishability

In order to *distinguish* $\boldsymbol{\sigma}_1$ from $\boldsymbol{\sigma}_2$ we must reject the null hypothesis H_0: $\hat{\mathrm{x}} = 0$. The probability of H_0 is based on the z-score:

$$z = \frac{\bar{x}}{\sigma_x}, \tag{7}$$

where

$$\bar{x} = E[\hat{\mathrm{x}}] = E[\mathbf{R}_R(\mathbf{y} + \mathbf{n})] = \mathbf{R}_R\mathbf{y} = \mathbf{R}_R\mathbf{J}\mathbf{x} = \mathbf{I}_R\mathbf{x},$$

and

$$\begin{aligned} \mathrm{Var}\ (x) &= \sigma_x^2 = E[\|\hat{\mathrm{x}} - \bar{x}\|^2] = E[\|\mathbf{R}_R\mathbf{n}\|^2] \\ &= E[\mathbf{R}_R\mathbf{n}\mathbf{n}^t\mathbf{R}_R^t] = \mathbf{R}_R E[\mathbf{n}\mathbf{n}^t]\mathbf{R}_R^t = \mathbf{R}_R\boldsymbol{\Sigma}_n\mathbf{R}_R^t. \end{aligned}$$

Given the maximum likelihood solution for $\hat{\mathrm{x}}$, we further simplify σ_x^2 as

$$\begin{aligned} \sigma_x^2 &= \mathbf{R}_R\boldsymbol{\Sigma}_n\mathbf{R}_R^t \\ &= \left(\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\right)^{-1}\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\boldsymbol{\Sigma}_n\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\left(\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\right)^{-1} \\ &= \left(\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\right)^{-1}\left(\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\right)\left(\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\right)^{-1} = (\mathbf{J}^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J})^{-1}. \end{aligned} \tag{8}$$

The z score may be calculated as

$$z = \frac{\bar{x}}{\sigma_x} = \frac{\hat{\mathrm{x}}}{\left(\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R\right)^{-\frac{1}{2}}} = \hat{\mathrm{x}}\sqrt{\mathbf{J}_R^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J}_R}. \tag{9}$$

This result shows that the distinguishability is a product of the target ($\mathbf{x}$), the measurement strategy ($\mathbf{J}$) and the inverse of the noise amplitude ($\boldsymbol{\Sigma}_n$).

For a practical algorithm, we need to define the ROI of the reconstruction algorithm. This area should be large enough to include the most of the amplitude of the reconstructed response, but to avoid areas which show large reconstruction artefacts.

4 Distinguishability of Current Patterns

In order to illustrate use of this approach, we perform a test to determine the stimulation protocol with the largest distinguishability. Given a 16 electrode Sheffield-type pair drive EIT system, it is traditional to stimulate across adjacent pairs of electrodes (which we label [0-1]), opposite pairs (labelled [0-8]) or any other offset. Using a 2D circular Finite Element Model (FEM), we simulate each scenario for target positions from the medium centre to boundary. Results (Fig. 1) suggest that distinguishability varies with the target position, but a stimulation offset between 3 and 5 is a good compromise.

This formulation may be extended to separate the choice of simulation patterns, the medium geometry and current propagation. Most EIT systems apply a set of current patterns represented as the columns of $\mathbf{C}$ to make a set of p voltage measurements $\mathbf{V} = \mathbf{T}(\boldsymbol{\sigma})\mathbf{C}$, where $\mathbf{T}(\boldsymbol{\sigma})$ is the *transfer impedance* matrix of the medium with impedance distribution $\boldsymbol{\sigma}$ (in units of Ω). For the small change $\mathbf{x} = x_R\mathbf{I}_R$, we can define a transfer impedance change

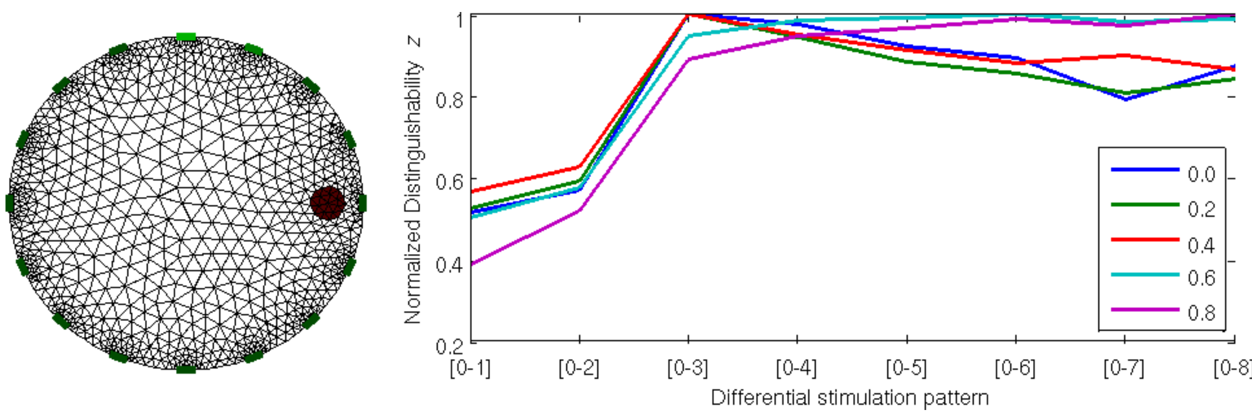

Figure 1: *Left:* FEM of a small conductive target in a 2D circular medium with 16 electrodes (green). The target moves from centre to edge (shown). *Right:* Normalized distinguishability z as a function of target position for pair drive stimulations with electrode offsets (X *axis*). Thus, $[0-1]$ indicates adjacent stimulation, while $[0-8]$ indicates opposite stimulation.

$\mathbf{T}_\Delta = \mathbf{T}(\boldsymbol{\sigma}+\mathbf{x}) - \mathbf{T}(\boldsymbol{\sigma})$. From $\mathbf{T}_\Delta$, the elements of the Jacobian $\mathbf{J}_R$ may be calculated from $\mathbf{M}\mathbf{T}_\Delta\mathbf{C}$, where $\mathbf{M}$ represents the measurement scheme (which is typically the difference between adjacent electrodes). In this case, $\mathbf{J}_R$ is a vector $M \times 1$ while $\mathbf{M}\mathbf{T}_\Delta\mathbf{C}$ is $\frac{N}{S} \times S$. If the noise covariance can be assumed to be the same for each current pattern (which we represent as $\mathbf{S}_n$), then

$$\boldsymbol{\Sigma}_n = \mathbf{S}_n \otimes \mathbf{I}_S, \tag{10}$$

where S is the number of stimulation patterns, $\mathbf{I}_S$ is the identity matrix. In this case

$$\mathbf{J}^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J} = tr\ (\mathbf{C}^t{\mathbf{T}_\Delta}^t\mathbf{M}^t\mathbf{S}_n^{-1}\mathbf{M}\mathbf{T}_\Delta\mathbf{C}), \tag{11}$$

where $\otimes$ represents the Kronecker product. Note that ${\mathbf{T}_\Delta}^t = \mathbf{T}_\Delta$ due to reciprocity. Thus, each diagonal element $[\mathbf{W}]_{ii} = [\mathbf{C}^t\mathbf{T}_\Delta\mathbf{M}^t\mathbf{S}_n^{-1}\mathbf{M}\mathbf{T}_\Delta\mathbf{C}]_{ii}$ represents the contribution from the i^{th} current stimulation, and

$$z = \hat{\mathrm{x}}\sqrt{\mathbf{W}_{11}^2 + \mathbf{W}_{22}^2 + \cdots \mathbf{W}_{pp}^2}, \tag{12}$$

This approach may then be extended to calculate the optimal current patterns $\mathbf{C}$ for a particular target position.

In summary, we propose a formulation for the distinguishability in EIT in terms of a hypothesis test, and show that distinguishability is a product of the impedance change amplitude, the measurement strategy and the inverse of the noise amplitude.

5 References

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