A Primal Dual – Interior Point Framework for EIT Reconstruction and Regularization with 1-Norm and 2-Norm

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1. Image Reconstruction and Regularization

Image Reconstruction in Electric Impedance Tomography (EIT) is an ill-posed problem. Ill-posedness affects the reconstruction in two ways: 1) the image is very sensitive to measurement noise, instrumentation and modeling errors 2) in order to reconstruct meaningful images regularization technique need to be used. This limits the ability to reconstruct sharp images. Traditionally image reconstruction is formulated as:

$$\sigma_{rec} = \underset{\sigma}{argmin} \frac{1}{2}\|V(\sigma) - V^{meas}\|^2 + \alpha\frac{1}{2}\|L(\sigma - \sigma^*)\|^2 \qquad \text{2-norm/2-norm} \tag{1}$$

where: σ is the conductivity or admittivity of the media, $V(\sigma)$ is the forward operator, V^{meas} are the experimentally collected electric potentials, L is a regularization matrix, typically a discrete representation of a first or second order differential operator, α is the Tikhonov factor, and σ^* a prior conductivity distribution (in our case the homogenous distribution that best fits the data). In this framework the 2-norm is used both for measuring the mismatch between predicted and measured electric potentials, and for measuring the likelihood that the image presents with respect to the prior that the regularization matrix L expresses. The use of the 2-norm in the data term makes the reconstruction sensitive to data outliers, as the differences between predicted and actual measurements are squared. Similarly, the use of the 2-norm on the regularization term enforces over-smooth solutions, as 'conductivity jumps' are weighted quadratically. We propose three alternative reconstruction formulations, that mix the 1-norm and 2-norm. These formulations are useful for solving problems with sharp transitions in the conductivity image and for diminishing the sensitivity to data outliers.

$$\sigma_{rec} = \underset{\sigma}{argmin} \sum_j |V_j(\sigma) - V_j^{meas}| + \alpha\frac{1}{2}\|L(\sigma - \sigma^*)\|^2 \qquad \text{1-norm/2-norm} \tag{2}$$

$$\sigma_{rec} = \underset{\sigma}{argmin} \frac{1}{2}\|V(\sigma) - V^{meas}\|^2 + \alpha\sum_i |L_i(\sigma - \sigma^*)| \qquad \text{2-norm/1-norm} \tag{3}$$

$$\sigma_{rec} = \underset{\sigma}{argmin} \sum_j |V_j(\sigma) - V_j^{meas}| + \alpha\sum_i |L_i(\sigma - \sigma^*)| \qquad \text{1-norm/1-norm} \tag{4}$$

In (2) and (4) we use of the 1-norm on the data term, making the reconstruction more robust to outliers, as discrepancies between the model and the actual data are not squared. In (3) and (4) we use of the 1-norm on the regularization term, allowing the reconstruction of sharper profiles, as fast spatial conductivity transitions are not overly-penalized by a quadratic regularization term.

The actual numerical solution of (2), (3), and (4) poses difficulties, as the absolute value function is not differentiable when the argument is zero. Formulations (2), (3), and (4) result from the sum of several absolute value functions, presenting several points of non-differentiability. Primal Dual – Interior Point Methods (PD–IPM) (Andersen, Christiansen, Conn & Overton 2000) have proven efficient at minimizing sums of 1-norms and have successfully been applied to EIT reconstruction with 1-norm regularization (Borsic, Graham, Adler & Lionheart 2009). In this work we have extended the PD–IPM framework of (Borsic et al. 2009) to treat the all 4 combinations of 1-norm and 2-norm on the data and regularization terms (the 2-norm/2-norm is of course trivial).

2. Numerical Experiments

Figure 1 shows a numeric phantom used for generating simulated data. The phantom presents two sharp inclusions in the upper and lower regions. In a first series of experiments illustrated in Figure 2, we added a Gaussian noise to the simulated forward measurements, to give an SNR of 10. Although the 2-norm on the data term is optimal for reconstructing data affected by Gaussian noise (Tarantola 1987), all reconstructions for the 4 different formulations (1)-(4) are able to identify well the inclusions in the simulated data. Figures 2(c) and 2(d) show how the use of the 1-norm on the regularization term allows sharper reconstructions.

Figure 3 shows numerical experiments in the presence of a single outlier in the data: additive noise, with and amplitude of 700%, was added on a single measurement in the dataset. The reconstructions that use the 2-norm on the data term

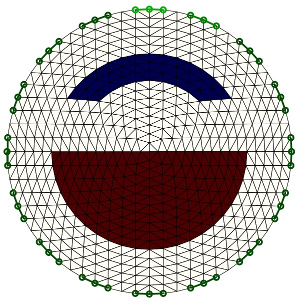
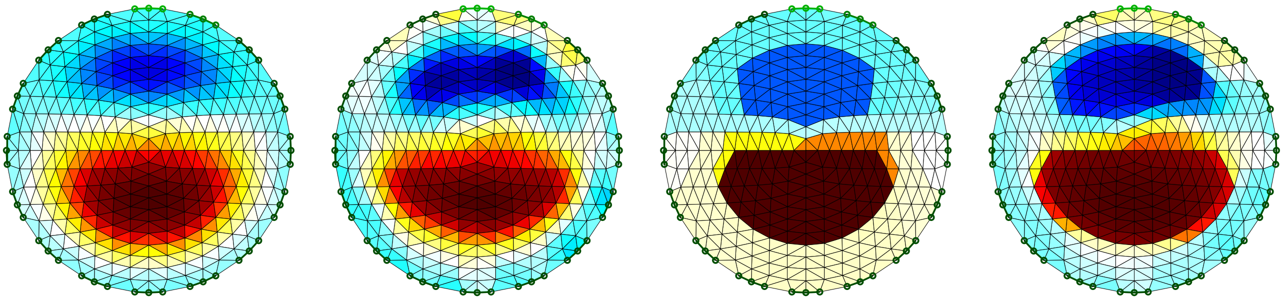
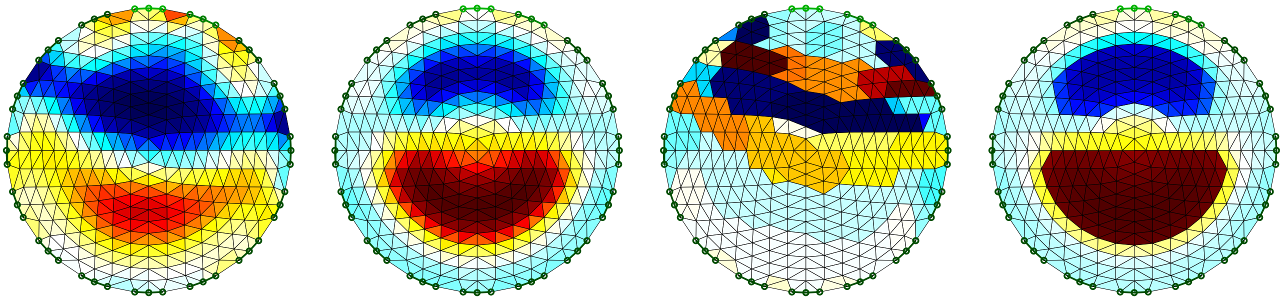


Figure 1. Image of the numerical phantom used for generating the simulated data. The background conductivity value is 1 S m^{-1}, the top inclusion presents a value of 0.5 S m^{-1} and the bottom inclusion a value of 1.5 S m^{-1}.



(a) 2-norm/2-norm (b) 1-norm/2-norm (c) 2-norm/1-norm (d) 1-norm/1-norm

Figure 2. Reconstructions with the four different formulations (1)-(4) in the presence of Gaussian noise on the measurements (SNR=10). All methods present a similar immunity to noise, and methods that use the 1-norm on the regularization term (Figures 2(c) and 2(d)) show shaper conductivity profiles, closer to the test conductivity distribution.



(a) 2-norm/2-norm (b) 1-norm/2-norm (c) 2-norm/1-norm (d) 1-norm/1-norm

Figure 3. Reconstructions with the four different formulations (1)-(4) in the presence of one single strong outlier in the dataset. Methods that use the 2-norm on the data term (Figures 3(a) and 3(c)) show a strong sensitivity to the outlier, while methods that use the 1-norm (Figures 3(b) and 3(d)) are almost unaffected by the presence of this noise.

(Figure 3(a) and 3(c)) are significantly more sensitive to the presence of the outlier, compared to reconstructions that use the 1-norm on the data term, which allows a robust reconstruction (Figure 3(b) and (d)).
We believe these results can be important in clinical applications of EIT, where noise and modeling errors can significantly affect the use of EIT. The ability of using the 1-norm on the data term should render the reconstruction more tolerant to data errors. The use of the 1-norm on the regularization term has been successfully demonstrated on clinical data in (Borsic et al. 2009), and allows the reconstruction of sharp image transitions. This can be useful in applications that involve imaging inter-organ boundaries, where there are step changes in the conductivity, or in applications like cancer detection, where a lump of tissue can present a sharp change in electrical properties with respect to the background.

References

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