Four-dimensional electrical capacitance tomography imaging using experimental data

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Abstract: Electrical capacitance tomography (ECT) is a relatively mature non-invasive imaging technique that attempts to map dielectric permittivity of materials. ECT has become a promising monitoring technique in industrial process tomography especially in fast flow visualization. One of the most challenging tasks in further development of ECT for real applications are the computational aspects of the ECT imaging. Recently 3D ECT has gained interest because of its potential to generate volumetric images. Computational time of image reconstruction in 3D ECT makes it more difficult for real time applications. In this paper we present a robust and computationally efficient 4D image reconstruction algorithm applied to real ECT data. The method takes advantage of temporal correlation between 3D ECT frames to reconstruct movies 4D of dielectric maps, which enhance the noise performance of and its computational efficiency, improves the speed of ECT image reconstruction. The 4D image reconstruction results are presented for experimental data from fast moving object.

Keywords: Electrical capacitance tomography, 4D image reconstruction

1. Introduction

Electrical capacitance tomography (ECT) is a relatively mature imaging method in industrial process tomography (Yang 2006, Yang et al 2002). The aim of ECT is to image materials with a contrast in dielectric permittivity by measuring capacitance from a set of electrodes. Applications of ECT include the monitoring
of oil-gas flows in pipelines, gas-solids flows in pneumatic conveying and imaging flames in combustion, gravitational flows in silo (Romanowski et al).

There has been a great deal of progress in image reconstruction methods, especially applied to 2D ECT; however, 3D ECT presents especially challenging numerical issues (Wajman et al, Warsito et al 2003, Warsito et al 2007, Soleimani 2006, Soleimani et al. 2007). 3D ECT is valuable for imaging the volumetric distribution of electrical permittivity. 3D ECT image reconstruction presents a similar inverse problem to electrical impedance tomography (EIT) which has been extensively studied, so ECT will naturally benefit from progress in EIT image reconstruction. Similarly to EIT, ECT has potential to generate images with high temporal resolution and relatively poor spatial resolution. The spatial resolution is limited as a result of the inherent ill-posedness of the inverse problem and the existence of modeling and measurement errors and limited number of independent measurements.

Among non-invasive imaging techniques, ECT has a much higher temporal resolution than others such as MRI, CT etc. This makes ECT a good candidate for 4D imaging technique which is capable of long term monitoring on fast-varying industrial process applications. ECT image reconstruction is ill-conditioned, and is typically solved by adding a priori information using a regularized matrix $\mathbf{R}$, which represents underlying image probability distribution. Conventional single-step ECT reconstruction algorithms treat each data frame and each frame of the 3D image independently, thus interframe temporal prior are ignored. This paper proposes 4D ECT image reconstruction for fast ECT measurements. The approach directly accounts for correlations between images in successive data frames of 3D images. The 4D algorithm creates movie images of electrical permittivity directly from multi-frame ECT data, so it is not a post processing 4D image reconstruction.

2. Experimental setup.

A typical three-dimensional capacitance sensor comprises an array of conducting plate electrodes, which are mounted on the outside of a non-conducting pipe, and surrounded by an electrical shield. For a metal wall pipe/vessel, the sensing electrodes must be mounted internally, with an insulation layer between the electrodes and the metal wall and using the metal wall as the electrical shield. Other components in the sensor include radial and axial guard electrodes, which are arranged differently to reduce the external coupling between the electrodes and to achieve improved quality of measurements and hence images. As
usually the electrodes do not make physical contact with the materials to be measured, ECT provides a non-intrusive and non-invasive means, avoiding the risk of contamination.

In this paper a 32-electrodes ECT sensor has been introduced with 4 planes and 8 electrodes on each plane. The first and fourth plane consists of 58 mm of width and 70 mm of height copper plates. The planes in the middle consist of 58 mm of width and 30 mm of height copper plates. This approach improves an uniformity effective field of imaging for the whole volume of the sensor. A copper shield with a 15 mm distance from electrode array has been used for the screening purpose. For the on-line measurement purpose a 32-channel ET3 system, developed in Poland, has been applied. The numerical sensor simulation and the on-line 3D image reconstruction has been carried out using Matlab code, VTK scientific visualization library and Intel Xeon quad core based workstation with 16GB of RAM. The electrode arrangement used in this study is 32 electrodes 3D ECT system that has been depicted in figure 1.

3. Forward modelling

The forward problem is the simulation of measurement data for give value of excitation and material (permittivity) distribution and the inverse problem is the imaging result for a given set of measurement data. Before we solve the inverse problem the forward problem needs to be solved.

At this stage we assume there is no wave effect and use low frequency approximation to the Maxwell’s equations. If we need to we can develop a forward model that takes into account the wave effect with high computational costs.

With high frequency a more complicated model is needed. In a simplified mathematical model, the electrostatic approximation \( \nabla \times E = 0 \) is taken, effectively ignoring the effect of wave propagation. Let’s take \( E = -\nabla u \) and assume no internal charges. Then the following equation holds.

\[
\nabla \cdot (\mathcal{E} \, \nabla u) = 0 \quad \text{in} \quad \Omega
\]

where \( u \) is (complex) the electric potential, \( \mathcal{E} \) is complex conductivity and \( \Omega \) is the region containing the field.
The potential on each electrode is known as

\[ U = V_k e_k \]  

(2)

where \( e_k \) is the \( k \)-th electrode held at the potential \( v_k \).

To solve equation (1) together with equation (2) numerically, the domain is partitioned into finished amount of tetrahedrals with a total of \( n \) vertices. The conductivity coefficients are each approximated by a piecewise constant function on that mesh. Given a standard nodal basis \( \{ \phi_i \}_{i=1}^n \) for the set of piecewise linear functions, a potential is sought in the form,

\[ u_k = \sum_{i=1}^n u_i \phi_i \]  

(3)

Multiplying the Laplace equation (1) by an arbitrary but sufficiently smooth test function \( v \) and integrating over \( \Omega \) gives

\[ \int_{\Omega} v \nabla \cdot (\epsilon \nabla u_k) dx^3 = 0 \]  

(4)

By partial integration, the following equation is obtained.

\[ \int_{\Omega} \epsilon \nabla u_k \cdot \nabla dx^3 = \int_{\Gamma_1} \epsilon \nabla u_k \cdot n \, d\tilde{x} + \int_{\Gamma_2} \epsilon \nabla v \cdot \tilde{u}_k \, d\tilde{x} \]  

(5)

Assuming \( K(i, j) = \int_{\Omega} \epsilon \nabla \phi_i \cdot \nabla \phi_j \, dx^3 \quad i, j = 1 : n \), a linear system of equations is obtained as

\[ K(\epsilon)U = B \]  

(5)

where the matrix \( K \) is the discrete representation of the operator \( \nabla \cdot \epsilon \nabla \) and the vector \( B \) is the boundary condition term and \( U \) is the vector of electric potential solution. The electric current on the \( k \)-th electrode is given by

\[ I_k = \int_{E_k} \epsilon \frac{\partial u}{\partial n} \, dx^2 \]  

(6)
where \( n \) is the inward normal on the \( k \)-th electrode. The Jacobian matrix is calculated using an efficient method (Soleimani 2006).

4. 4D inversion

Instead of calculating an image based on the sequence of past frames, we propose a temporal image reconstruction algorithm which uses a set of data frames nearby in time (Adler et al.). The data frame sequence is treated as a single inverse problem, with regularization prior to account for both spatial and temporal correlations between image elements. Given a vertically concatenated sequence of capacitance measurements (normalized capacitance data) frames \( \widetilde{C} = [C_{-d}, \ldots, C_0, \ldots, C_d] \) and the corresponding relative permittivity images \( \widetilde{\varepsilon} = [\varepsilon_{-d}, \ldots, \varepsilon_0, \ldots, \varepsilon_d] \), the direct temporal forward model \( C = S\varepsilon + n \) is rewritten as

\[
\begin{bmatrix}
C_d \\
\vdots \\
C_0 \\
\vdots \\
C_d
\end{bmatrix} =
\begin{bmatrix}
S & & & 0 \\
& \ddots & & \\
& & S & \\
0 & & & S
\end{bmatrix}
\begin{bmatrix}
\varepsilon_d \\
\vdots \\
\varepsilon_0 \\
\vdots \\
\varepsilon_d
\end{bmatrix} +
\begin{bmatrix}
n_d \\
\vdots \\
n_0 \\
\vdots \\
n_d
\end{bmatrix} \tag{7}
\]

and also as

\[
\widetilde{C} = \widetilde{S}\widetilde{\varepsilon} + \mathbf{n} \tag{8}
\]

where \( \mathbf{n} = [n_{-d}; \ldots; n_0; \ldots; n_d] \) is the noise in the measured data. We assume \( S \) to be constant, although this formulation could be modified to account for a time variation in \( S \). Based on this approximation \( \widetilde{S} = I \otimes S \), where the identity \( I \) has size 2d+1, and \( \otimes \) is the Kronecker product.

The correlation of corresponding elements between adjacent frames (delay \( t=1 \)) can be evaluated by an inter-frame correlation \( \gamma' \), which has value between 0 (independent) and 1 (fully dependent). As the frames become separated in time, the inter-frame correlation decreases; for an inter-frame separation \( t \), the inter-frame correlation is \( \gamma^t \). Frames with large time lag, \(|t| > d\), can be considered independent. Image reconstruction is then defined in terms of minimizing the augmented expression:

\[
\begin{bmatrix}
C_d \\
\vdots \\
C_0 \\
\vdots \\
C_d
\end{bmatrix} =
\begin{bmatrix}
S & & & 0 \\
& \ddots & & \\
& & S & \\
0 & & & S
\end{bmatrix}
\begin{bmatrix}
\varepsilon_d \\
\vdots \\
\varepsilon_0 \\
\vdots \\
\varepsilon_d
\end{bmatrix} +
\begin{bmatrix}
\mathbf{z} \\
\vdots \\
\mathbf{z} \\
\vdots \\
\mathbf{z}
\end{bmatrix} \tag{9}
\]

and the inversion can be written as
where $\tilde{W} = I \otimes W$. $\tilde{W}$ is diagonal since measurement noise is uncorrelated between frames. 

$\tilde{R} = \Gamma^{-1} \otimes R$ where $\Gamma$ is the temporal weight matrix of an image sequence $\tilde{\varepsilon}$ and is defined to have the form as

$$
\Gamma = \begin{bmatrix}
1 & \gamma & \ldots & \gamma^{d-1} & \gamma^{d} \\
\gamma & 1 & \ldots & \gamma^{d-2} & \gamma^{d-1} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\gamma^{d-2} & \gamma^{d-3} & \ldots & 1 & \gamma \\
\gamma^{d-1} & \gamma^{d-2} & \ldots & \gamma & 1
\end{bmatrix}
$$

From (10) and (11),

$$
\tilde{B} = \left[ \Gamma \otimes \left( PS^T \right) \left[ \Gamma \otimes \left( SPS^T \right) + \lambda^2 I \otimes V \right]^{-1} \right] \tilde{C}
$$

(6) is rewritten as

$$
\begin{bmatrix}
\tilde{\varepsilon}_d \\
\vdots \\
\tilde{\varepsilon}_1 \\
\tilde{\varepsilon}_0
\end{bmatrix} = \tilde{B} \tilde{C}
$$

Although this estimate is an augmented image sequence, we are typically only interested in the current image $\tilde{\varepsilon}_0$. It is calculated by $\tilde{\varepsilon}_0 = \tilde{B}_0 \tilde{C}$ where $\tilde{B}_0$ is the rows $n_h \times (d+1)\ldots n_M \times (d+1)$ of $\tilde{B}$.

5. Experimental results

The proposed algorithm has been tested against several experimental example of moving 3D objects (4D). Figure 2 shows the reconstructed images for different moving objects in a tank. The movie includes several frames; here we only show few frames of the movie. The results presented in figure 2 are among the first experimental results from proposed algorithm. In all cases 4D algorithms successfully reconstructed the movement of 3D object. The algorithm were also applied to two more experimental examples, similar results were observed in capturing dynamic of a moving 3D object (i.e. plastic rod) – see figure 2. We have attached to this paper the movies related to each of these 4 experiments.

During the first two experiments we were using a cylindrical object with 150 mm of diameter and a concentrically drilled hole with 50 mm of diameter and a rod of the same diameter respectively. The cylinder and the rod both had been made from Ertalon with relative permittivity of about 3.2. Firstly the rod was being put with a constant velocity of about 2cm/s near the wall of pipe. The rod was being pulled
out from the bigger cylinder as the next experiment. The velocity of rod movement has been well-matched to the measurement abilities of used ET3 system which is now able to measure data in 32 channel mode with up to 15 frames per second (Olszewski et al). It can be observed that 4D algorithm allows for on-line visualization of such objects in the whole volume of the sensor. In the next two experiments we moved two balls inside a pipe. The first ball was about 70 mm in diameter and the second one was bout 100 mm in diameter both filled with plastic granulate (relative permittivity of 2.6). We let these balls move freely through the pipe with constant velocity acquiring measurement data simultaneously. The results we have got using 4D algorithm are promising even though the velocity of balls was too high. It can be easily observed that some axial resolution limitations of a 3D capacitance sensor in the lower and upper part of the imaging volume exist which is obvious because of sensor coverage. The central area of the sensor volume characterizes relatively low sensitivity. Nevertheless the ball movement in that area has been visualized with acceptable quality regardless of poor sensitivity. The main bottleneck in these experiments we had to face with was the ET3 capacitance system which is now relatively slow in 32 channel mode. With faster and less noisy ECT system the quality of 4D images for on-line visualization would be improved.

[Figure 2: Here]

6. Conclusion

A direct temporal image reconstruction has been applied to the ECT that simultaneously reconstructs 4D dielectric permittivity imaging using multi-frame ECT data. Including a temporal correlation term and reconstruction of the 4D ECT images simultaneously improves the noise fidelity of the image reconstruction. We anticipate that the 4D algorithm could potentially improve the spatial resolution of ECT imaging. Study of improvement in image resolution using the 4D method is under way. The efficient implementation of the 4D algorithm is a promising aspect of the method presented here and with further work we are hoping to achieve a fully real time 4D reconstruction in ECT, which will pave the way for many more industrial applications for the ECT.
7. Acknowledgement

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8. REFERENCE


Figure 1: 32 electrodes array for 3D ECT system
Figure 2: 4D ECT visualisation of moving objects