# Anomaly depth detection in trans-admittance mammography: a formula independent of anomaly size or admittivity contrast

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**Abstract: TAM(trans-admittance mammography) aims to determine the location and size of any anomaly from the multi-frequency. A formula is proposed here that can estimate the depth of an anomaly independent of its size and the admittivity contrast. This depth estimation can also be used to derive an estimation of the size of the anomaly. Numerical simulation shows that the proposed method also works well in general settings.**

## 1 Introduction

Bioimpedance techniques such as electrical impedance tomography (EIT) are potential supplemental tools for breast cancer detection [1–5]. This paper considers a TAM system [6, 7] in which the breast is compressed between two electrical plates (see Figure 3). One plate is a large solid voltage driving electrode, and the other is composed of current-sensing point electrodes. The system employs constant sinusoidal voltage at frequencies of 50 Hz-500 kHz. The voltage is applied through the voltage driving electrical plate; a grounded voltage is maintained on the other plate. The voltage difference induces time harmonic electrical current that is determined by the conductivity distribution in the breast. The current-sensing electrodes can be used to measure exit currents to obtain two-dimensional transadmittance maps at multiple frequencies. The corresponding inverse problem is to detect the conductivity anomalies that denote breast tumor tissue from the multiple-frequency trans-admittance maps.

## 2 Methods

When a sinusoidal voltage $V_0 \sin \omega t$ with an angular frequency $\frac{\omega}{2\pi}$ is applied through $\Upsilon$(see Figure 3), potential $u_\omega$ satisfies:

$$\begin{cases} \nabla\cdot(\gamma_\omega(\mathbf{r})\nabla u_\omega(\mathbf{r}))=0, & \mathbf{r}\in\Omega \\ u_\omega(\mathbf{r})=V_0, & \mathbf{r}\in\Upsilon \\ u_\omega(\mathbf{r})=0, & \mathbf{r}\in\Gamma \\ \mathbf{n}\cdot(\gamma_\omega(\mathbf{r})\nabla u_\omega(\mathbf{r}))=0, & \mathbf{r}\in\partial\Omega\backslash(\Upsilon\cup\Gamma), \end{cases} \tag{1}$$

where $\gamma_\omega(\mathbf{r}) = \begin{cases} \gamma^n_\omega(\mathbf{r}) := \sigma^n_\omega(\mathbf{r})+i\omega\varepsilon^n_\omega(\mathbf{r}), & \mathbf{r}\in\Omega\backslash\overline{D} \\ \gamma^a_\omega(\mathbf{r}) := \sigma^a_\omega(\mathbf{r})+i\omega\varepsilon^a_\omega(\mathbf{r}), & \mathbf{r}\in D \end{cases}.$

**Theorem 2.1** *Let* $g_{\omega_j} = -(\sigma_{\omega_j}+i\omega_j\varepsilon_{\omega_j})\frac{\partial u_{\omega_j}}{\partial \mathbf{n}}$ *be the measured Neumann data at frequency* $\omega_j$ *for* $j=1,2$ *and let* $\alpha=\gamma^n_{\omega_1}/\gamma^n_{\omega_2}$. *Then we have depth formula:*

$$z_D = 2\sqrt{3}\left|\frac{(g_{\omega_1}-\alpha g_{\omega_2})(\mathbf{r}^*)}{(\partial_x^2+\partial_y^2)(g_{\omega_1}-\alpha g_{\omega_2})(\mathbf{r}^*)}\right|^{\frac{1}{2}}\left|\frac{1+r_1(\mathbf{r}^*)}{1+r_2(\mathbf{r}^*)}\right|^{\frac{1}{2}}, \tag{2}$$

*where* $C_{k1}, C_{k2}$ *are depending only on* $\rho,\delta,d_0/V_0,|D|,|\Omega|,\gamma^n_{\omega_1}$, $\gamma^a_{\omega_1},\gamma^n_{\omega_2}$ *and* $\gamma^a_{\omega_2}$, $\mathbf{r}^*=(x_D,y_D)$ *and* $u_0$ *is the solution of (1) in the absence of anomaly.*

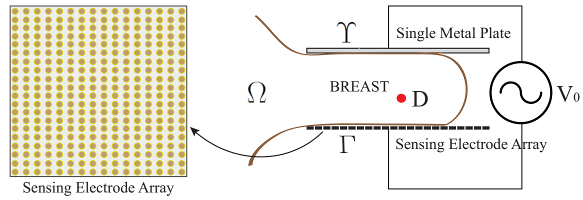


**Figure 1:** Trans-admittance mammography (TAM). (L): the sensor electrode array. (R):Schematic cross-section

**Observation 2.2** *Under the same assumption given in theorem 2.1, we define a connected set S containing* $(x_D, y_D, 0)$ *such that*

$$S := \{\,(x,y,0)\in\Gamma \,:\, \nabla^2_{xy}G(x,y)\cdot\nabla^2_{xy}G(x_D,y_D) > 0\,\}, \tag{3}$$

*where* $\nabla^2_{xy} = \partial_x^2+\partial_y^2$ *and* $G(x,y) = [Real(g_{\omega_1}-\alpha g_{\omega_2})](\mathbf{r}), \mathbf{r}\in\Gamma$. *Then we have*

*(1) the set S is independent on the size of anomaly D and the admittivity contrast*

*(2) depth* $z_D$ *can be determined by* $z_D \approx 1.6938\cdot$ *(the radius of S).*

## 3 Numerical simulations

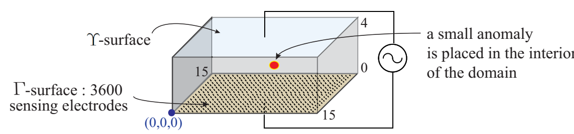


**Figure 2:** Numerical simulation domain

| depth | $z_D$ (theorem 2.1) | $z_D$ (observation 2.2) |
|---|---|---|
| 0.5 cm | 0.87 | 0.63 |
| 1 cm | 1.17 | 1.08 |
| 1.5 cm | 1.61 | 1.55 |
| 2 cm | 2.12 | 2.04 |

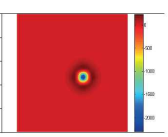

**Figure 3:** Anomaly depth estimated by using theorem 2.1 and observation 2.2

## 4 Conclusions

Throughout this paper, we gave a rigorous mathematical analysis for the proposed algorithms to estimate the size and position of anomaly and presented a successful numerical simulation results supporting the theories. In the future, we plan to make use of the suggested algorithm in experiments.

## References

[1] Assenheimer M, Moskovitz O, Malonek D, et al. *Physiological Measurement* **22**:1–8, 2001

[2] Mueller JL, Isaacson D, Newell JC. *IEEE Trans BiomedEngrg* **46**:1379–1386, 1999

[3] Cherepenin V, Karpov A, Korjenevsky A, et al. *Physiol Meas* **22**:9–18, 2001

[4] Kim S, Lee J, Seo JK, et al. *SIAM J Appl Math* **69**:22–36, 2008

[5] Ammari H, Kwon O, Seo JK, et al. *SIAM J Appl Math* **65**:252–266, 2004

[6] Zhao MK, Liu Q, Woo EJ, et al. In *J. Phys.: Conf. Ser.* 2010

[7] Zhao MK, Wi H, Kamal AHM, et al. *BioMedical Engineering Online* **11**:751, 2012