Advances in EIT reconstruction through Simulated Annealing

Thiago C. Martins[1] and Marcos Sales Guerra Tsuzuki[1]

[1]Escola Politécnica, University of São Paulo, São Paulo, Brazil

Abstract: EIT reconstruction can be solved as an optimization problem through Simulated Annealing (SA), but often at a high computational cost. This paper presents new techniques of EIT reconstruction through SA, including partial evaluation of the objective function, alternate objective functions and multi-objective optimization. Reconstructions of experimental impedance data using the techniques exposed were successfully performed.

1 Introduction

EIT is a imaging technique for determining the electrical conductivity distribution inside an object from boundary measurements. A set of electrodes is attached to the object surface, electrical current is injected through these electrodes and electrical potentials are measured. This work is focused on the reconstruction of static conductivity images.

EIT image reconstruction can be performed by a Finite Element Model (FEM) parameter estimation and a maximum likelihood formulation. The resulting optimization problem may be solved using SA. It is a probabilistic optimization meta-heuristic of local exploration that requires only the variation of the objective function between two consecutive solutions [1]. As a drawback, it requires the evaluation of many solutions, thus making it problematic for objective problems with computationally expansive objective functions such as EIT.

2 SA Applied to EIT

This work presents some advances when SA is used to reconstruct conductivity distribution in EIT.

2.1 Partial Evaluation of the Objective Function

One objective function is the Euclidean distance between the measured electric potentials and the calculated potentials for all the applied current patterns for a given conductivity distribution. To reduce the reconstruction computational cost a *partial* evaluations of the objective function can be performed that is, at each SA iteration, an estimate $\tilde{E}$ and upper and lower boundaries E_{max} and E_{min} are obtained [2]. It can be shown that whenever the variation of those estimates $\Delta\tilde{E}$, ΔE_{max}, ΔE_{min}. satisfy

$$P_{err} \geq \min(e^{-\Delta\tilde{E}/kt}, 1) - \min(e^{-\Delta E_{max}/kt}, 1) \quad (1)$$

$$P_{err} \geq \min(e^{-\Delta E_{min}/kt}, 1) - \min(e^{-\Delta\tilde{E}/kt}, 1) \quad (2)$$

then the probability of SA at that iteration deviating of an SA with full objective function evaluation is less than P_{err}.

Those estimates may be determined by iteratively solving the FEM linear systems while obtaining an upper limit on the norm of the error at each iteration using a technique described in [3]. This error norm is pertinent to *all* equations of the FEM linear systems, while only the uncertainty on the electrode nodes really contribute to the uncertainty of the objective function. This leads to an overestimation of the required CG iterations to satisfy (1,2). This overestimation gets worse for denser FEM meshes.

2.2 Least squares error as an objective function

By taking the FEM linear systems of the simulated domain and imposing that the measured potentials are identical to the simulated ones, one obtains an overdetermined linear system whose total least squares error can be used as an objective function in EIT image reconstruction. This new objective function is quite suitable for the partial evaluation described in sec. 2.1, as a variation of the Lanczos Algorithm can be used to obtain increasingly better boundaries for its upper and lower values[4].

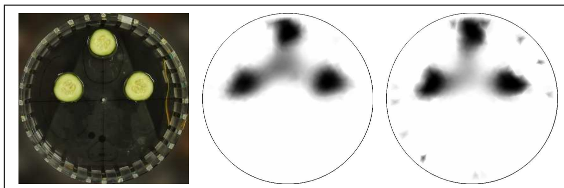
2.3 Multi-Objective

Regularization of inverse problems posed as optimization processes often appears as new terms in the objective function. The determination of the appropriate weight for those terms is difficult problem on itself.

An alternate approach is to consider both the original objective function and the regularization terms as concurrent objective functions to be minimized. This optimization category is called “multi-objective optimization”. Multi-objective optimization problems do not admit a single solution, having instead a set of mutually non-dominating solutions.

3 Experimental Results

Experimental phantoms (Fig. 1a) were constructed with cucumeber slices. Impedance images were reconstructed using the methods proposed in sections 2.1 (Fig. 1b) 2.2 (Fig. 1c) and 2.3.



(a) (b) (c)

Figure 1: Experimental phantom and its reconstructions.

4 Conclusion

EIT images can be reconstructed using SA. The high computational cost of Simulated Annealing can be mitigated by the adoption of *partial* evaluation of the objective function. An alternative objective function, based on total least squares errors of overdetermined FEM linear systems, provide superior scalabiltiy with mesh density. Regularization with *a posteriori* weights can be obtained through multi-objective SA.

References

[1] Kirkpatrick S, Gellat CD, Vecchi MP. *Science* **220**:671–680, 1983

[2] Martins TC, Camargo EDLB, Lima RG, et al. *IEEE T Biomed Eng* (59):1867–1870, 2012

[3] Meurant G. *Numer Algorithms* **40**:157–169, 2005

[4] Golub GH, Meurant G. *Matrices, Moments and Quadrature with Applications*. Princeton University Press, 2010