# Solution of Inverse Problem by Infinite Boundary Elements and the Level Set Method

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**Abstract:** In this paper the inverse problem for the electric field is investigated. In order to solve the forward part of such problem we use the boundary element method coupled with infinite elements. The inverse problem is based on the gradient technique and the level set method. Such task can be considered as application of the electrical impedance tomography. Investigated structure is given in Fig. 1. We want to detect the closed curve localised on upper part of this plot.

## 1 Introduction

Boundary element method (BEM) is well known and effective numerical technique used to solve partial differential equations [3]. In literature we have a lot of extensions of BEM. For example a lot of effort has been put into combining BEM and finite element method. Another example is coupling BEM with infinite elements [1,2]. It gives us possibility to solve equations with boundaries described by open curves.

## 2 Theoretical Model

In the forward problem we start our considerations from following formula (proper for all boundary points) [3]:

$$\frac{1}{2}u(\vec{r}_i)+\sum_{j=1}^{N}\int_{\Gamma_j}u(\vec{r})\,q^*(\vec{r},\vec{r}_i)\,d\gamma_j=\sum_{j=1}^{N}\int_{\Gamma_j}q(\vec{r})\,u^*(\vec{r},\vec{r}_i)\,d\gamma_j. \qquad (1)$$

Symbols u represents electrical potential, whereas q defines his normal derivative. The Green function and its normal derivative are denoted by u* and q* respectively. In equation (1) we have N finite boundary elements.

Next, we have introduced infinite boundary elements and the governing equation (2) has been derived. The derivation is quite long, and will be present in the full version of article. The governing integral equation is given by:

$$\begin{aligned}&\frac{1}{2}u(\vec{r}_i)+\sum_{j=2}^{N-1}u_j\int_{\xi=-1}^{\xi=+1}q^*(\vec{r_j}(\xi),\vec{r}_i)\,d\gamma_j+\\&+u_1\int_{\xi\to-\infty}^{\xi=+1}S_\infty(\xi)\,q^*(\vec{r_1}(\xi),\vec{r}_i)\,d\gamma_1+u_N\int_{\xi=-1}^{\xi\to+\infty}S_\infty(\xi)\,q^*(\vec{r_N}(\xi),\vec{r}_i)\,d\gamma_N\\&=\sum_{j=2}^{N-1}q_j\int_{\xi=-1}^{\xi=+1}u^*(\vec{r_j}(\xi),\vec{r}_i)\,d\gamma_j+\\&+q_1\int_{\xi\to-\infty}^{\xi=+1}S_\infty(\xi)\,u^*(\vec{r_1}(\xi),\vec{r}_i)\,d\gamma_1+q_N\int_{\xi=-1}^{\xi\to+\infty}S_\infty(\xi)\,u^*(\vec{r_N}(\xi),\vec{r}_i)\,d\gamma_N.\end{aligned} \qquad (2)$$

Symbol S denotes the sum of the interpolation functions with exponential decay along infinite elements. One should notice that in our model there is only one open boundary curve. However generalisations of formula (2) can be easy done. In mathematical model we assume that in N – 2 nodes the normal derivatives q equal zero. Only in two nodes we set the electrical potential (see Fig. 1).

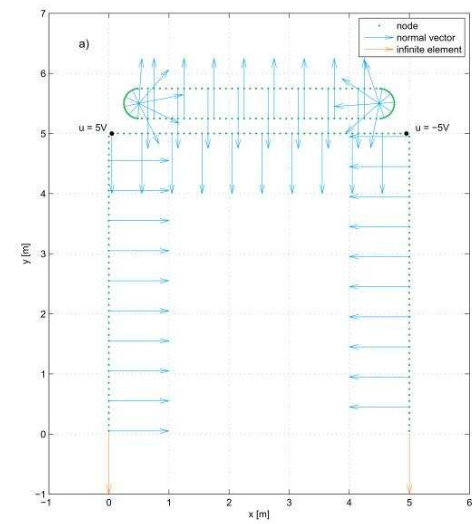


**Figure 1:** Geometrical model used in our calculations. Nodes, normal vectors and positions of infinite elements are indicated. The boundary of the domain is indicated by green dots.

Very important part of our research is the level set method. The equation of motion takes the form:

$$\frac{\partial\Phi}{\partial t}+\vec{V}\cdot\nabla\Phi=0, \qquad (3)$$

where $\phi$ is the level set function. Function $\phi$ is transformed under influence of the velocity field $\vec{V}$. This field is given by the gradient technique [4].

## 3 Conclusions

Altogether during our studies three different geometrical models have been verified. It turns out that solving the forward problem through external approach is the best way in numerical analysis. Solutions of the inverse problem give us good results in all three cases.

## References

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