Fast D-bar reconstructions of ventilation and perfusion on a pairwise current injection system

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Abstract: A fast implementation of the D-bar method for reconstructing conductivity changes on a 2-D chest-shaped domain is described. Cross-sectional difference images of ventilation and perfusion in a healthy human subject are presented. The images constitute the first D-bar images from EIT data on a human subject collected on a pairwise current injection system.

1 Introduction

D-bar methods are a class of direct (noniterative) reconstruction algorithms that make use of complex geometrical optics (CGO) solutions to PDEs known as D-bar, or $\bar{\partial}$, equations. D-bar equations are of the form $\bar{\partial} u = f$, where f may depend on u, and the $\bar{\partial}$ operator is defined by $\bar{\partial} = 0.5\,(\partial_x + i\partial_y)$. The common threads in these methods are (1) a direct relationship between the CGO solutions and the unknown conductivity, (2) a nonlinear Fourier transform, also known as the scattering transform, providing a link between the data and the CGO solution, and (3) a D-bar equation to be solved for the CGO solutions with respect to a complex-frequency variable.

These steps have been generally regarded as computationally intensive, but through parallelization and careful optimization of the computational routines, we present a fast implementation capable of providing real-time images from the pairwise current injection system at CSU.

2 Methods

The D-bar method used here is based on the uniqueness proof [1] and subsequent results and implementations [2, 3].

2.1 Fast implementation

The first step of the method is to compute a matrix approximation to the current-to-voltage map. This can be accomplished efficiently with inner products, as explained in [4].

The fast algorithm uses the approximation to the scattering transform known as $\mathbf{t}^{\exp}$, which linearizes the scattering transform, but not the entire method, by replacing the CGO solution ψ that depends on the unknown conductivity σ by another CGO solution independent of σ representing the asymptotic behaviour of ψ. Introducing the function $\mu(z,k) = e^{-ikz}\psi(z,k)$, the conductivity can be determined directly from μ by solving

$$\frac{\partial \mu(z,k)}{\partial \bar{k}} = \frac{\mathbf{t}^{\exp}(k)}{4\pi\bar{k}} e^{-i(zk+\bar{z}\bar{k})}\overline{\mu(z,k)},$$

and computing, for each z in the domain, $\sigma(z) = \mu^2(z,0)$. The computational solution of this equation comprises by far the bulk of the run time. The equation is formulated as an integral equation and solved using a fast implementation of the method in [5]. This equation was solved in a truncated region of the k-plane in parallel using optimized Matlab code and multiple processors.

2.2 Results

To demonstrate the D-bar method on data from this EIT system and to time the algorithm, we consider data sets collected using pairwise current excitation on 32 electrodes evenly spaced around the chest of a healthy male subject sitting upright and holding his breath. 360 frames of data were collected at 16 frames/s at 125 kHz and current amplitude 0.823 mA. Runtimes on a 12 core Mac Pro with two 2.66 GHz 6 core Intel Xeon processors and Matlab's parallel computing toolbox are listed in Table 1. Utilizing 7 cores in parallel results in a runtime of 0.0621 s/frame, which is less than the data acquisition time of 0.0625 s/frame.

Two difference images in the sequence of 360 frames are presented in Figure 1.

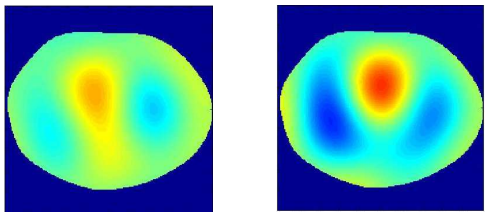

Figure 1: Changes due to perfusion in the chest of a healthy human subject. The heart is at the top, and red represents high conductivity and blue low conductivity. The images are displayed on the same scale.

Table 1: Runtimes on a 562 element grid with n cores in parallel

Number of cores	Total runtime (s)	s/frame
1	71.74	0.1998
2	49.02	0.1365
4	26.23	0.0731
7	19.27	0.0621
8	20.25	0.0647

3 Conclusions

The results presented here show for the first time the D-bar method applied to human chest data collected on a pairwise current injection system. Conductivity changes due to perfusion are clearly visible in the images. The fast implementation demonstrates the clinical potential of the D-bar algorithm as a reconstruction algorithm for real-time bedside imaging.

References

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