Validity of Using the Sheffield Algorithm for the Sussex EIT MK4

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Abstract: This paper introduces the image reconstruction algorithm from Sheffield group and the validity of this algorithm to the Sussex MK4.

1 Introduction

The Sussex MK4 electrical impedance mammography (EIM) is developed for breast cancer detection[1][2]. This paper is focusing on the validity analysis of using the Sheffield algorithm for the MK4.

2 Methods

The widely used equation to explain the relationship between the change of the conductivity and the change of the boundary voltage measurements is:

$$\Delta V = V_m - V_{ref} = S(C - C_{ref}) = S\Delta C \quad (1)$$

where S is the Jacobin matrix. $\partial V_j / \partial C_i = S_{ij}$. Vector V_m denotes the real voltage measurement corresponding to the real conductivity C. Vector V_{ref} denotes the reference voltage measurements corresponding to the reference conductivity C_{ref}. S is a function of C. As C changes, S changes. Eq. (1) is based on the assumption that the changes of C are small, so that the changes of S can be ignored. However Eq. (1) was proven by us to have a poor noise tolerance for the MK4, thus the Sheffield method using the voltage ratio rather than the difference is employed [3]: (For details, please read [3], Page 368-371)

$$\Delta \ln V = F \Delta \ln C \quad (2)$$

where $\Delta \ln V_i = \ln(V_{mi}/V_{refi})$, $\Delta \ln C_i = \ln(C_i/C_{refi})$.

$$\frac{\partial \ln(V_j)}{\partial \ln(C_i)} = \frac{\partial \ln(V_j)}{\partial V_j} \cdot \frac{\partial V_j}{\partial C_i} \cdot \frac{\partial C_i}{\partial \ln(C_i)} = \frac{1}{V_j} S_{ij} C_i = F_{ij} \quad (3)$$

The image reconstruction algorithm is:

$$\begin{cases} \Delta \ln(C) = (F^* F + \alpha^2 I)^{-1} F^* \left(\ln(V_m) - \ln(V_{ref})\right) \\ \ln(C_{ref}) = \ln(C) + \Delta \ln(C) \end{cases} \quad (4)$$

where α is the regularization parameter, I is the identity matrix. Let's see Eq. (3). As V and S are both determined by C, basically F is determined by C. As C changes, F changes. So this algorithm is based on the assumption that the changes of F are ignored when the changes of the conductivity are sufficiently small. However how much changes of the conductivity will make the assumption invalid? According to Eq. (3), if V_j is equal or close to 0, F_{ij} will go to infinity, which will make the algorithm unavailable. In practice, the measurements from the 0.5 mS/cm saline are used as the reference measurements and in each excitation, we only use at most 12 strongest measurements which are collected parallel to the electric field [1][2], therefore V_j won't be close to 0. However if there are big changes of conductivity, V_j may be close to 0, then Eq. (2)-(4) becomes invalid.

This section discuss how much changes of the conductivity will make Eq. (2)-(4) invalid. See Figure 1. The positive pole of the current source is at the yellow dot S+ and the negative pole of the current is at the yellow dot S-. The electric potential at P1, P2, P3, P4 is denoted by: $\emptyset_1, \emptyset_2, \emptyset_3, \emptyset_4$ and the voltage measurements between P2 and P1, P4 and P3 are denoted by V_{21}, V_{43}. For an uniform field (0.5 mS/cm Saline), according to our study, V_{21}, V_{43} are approximately 300mv when the tank height is 4.5cm. For a significant changes of the field, $\emptyset_1$ and $\emptyset_3$ may get close to $\emptyset_2$ and $\emptyset_4$, which means V_{21}, V_{43} may become 0 even negative. then Eq. (2)-(4) will be invalid. Thus, we conclude that the changes of the conductivity which cause $V_{21} \le 0$ or $V_{43} \le 0$ will make Eq. (2)-(4) invalid. If $V_{21} \le 0$, there must be a high conductivity path between S+ and P1, so that most of the current flows through this path and brings up $\emptyset_1$. See Figure 1. The most likely condition to make $V_{21} \le 0$ or $V_{43} \le 0$ is that the high conductivity path needs 1) the shortest distance between S+ and P1 without covering P2; 2) a big volume of the path to reduce the resistance between P1 and S+; 3) a much higher conductivity than the surrounding background. We made such a path shown in Figure 1. W=5.5cm, L=5.2cm, H=0.9cm. The straight-line distance between P1 and S+ is 4.5 cm. σ_1 and σ_2 indicate the conductivity of the background and the path. According the our studies, only if $\sigma_2/\sigma_1 > 44$ is $V_{21} \le 0, V_{43} \to 0^+$ Therefore in a real case, if the tumour size is smaller than 4.5cm, which means the high conductive path can't form and the conductivity contrast of the whole tank is smaller than 40, Eq. (2)-(4) will be valid. Practically, a 4.5cm tumour can be found easily, and it is not usual that the conductivity contrast of a breast is bigger than 40.

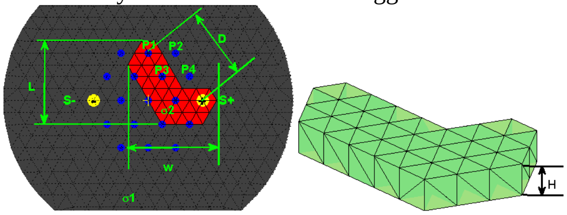

Figure 1: Voltage measurements reverse analysis

3 Conclusions

The Sheffield algorithm is not valid for every condition. For the MK4 system, it is available, for a real breast is too far from the conditions which make the boundary voltage measurements close to 0, hence invalidating Eq. (2)-(4).

References

[1] B. Nevis, "An investigation into combining electrical impedance mammography with 3D ultrasound for breast cancer detection," Doctoral thesis, Engineering and Design, University of Sussex, 2013.

[2] G. Sze, "Detection of breast cancer with electrical impedance mammography," Doctoral thesis, Engineering and Design, University of Sussex, 2012.

[3] D. C. Barber, "EIT: The view from Sheffield," Electrical impedance tomography : methods, history and applications, D. S. Holder, ed., pp. 348-371, Bristol: Institute of Physics, 2005.