Fine-tuning of the Complete Electrode Model

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Abstract: The Complete Electrode Model (CEM) is a realistic measurement model for Electrical Impedance Tomography. We present a non-uniform discretization of the conductivity space based on its sensitivity to boundary data and an adaptive adjustment of electrode parameters leading to improved reconstructions of Newton-type solvers. We demonstrate the performance of this concept when reconstructing with incorrect geometry assumptions from noisy data.

1 Sensitivity-based conductivity discretization

The Neumann-to-Dirichlet (ND) map Λ_σ of the CEM with L electrodes is an $L \times L$ matrix that maps the applied currents to the resulting (measured) potential vectors, where σ is the conductivity on a domain $\Omega \subset \mathbb{R}^2$, cf. [1]. By

$$\lambda_\sigma = \|\Lambda_\sigma - \Lambda_1\| / \|\Lambda_1\|,$$

we define the *sensitivity* for distinguishing a conductivity $\sigma \in L^\infty_+(\Omega)$ from the homogeneous case $\sigma \equiv 1$ by boundary measurements. For $\Omega = B_1(0)$, we can determine λ_σ analytically for conductivities of the form $\sigma = 1 + \delta\chi_D(x)$, where D is a disk inside Ω. With this information, we discretize the conductivity space such that the ND map is equally sensitive to perturbations δ in each segment. This is achieved by filling the disk with non-overlapping circles resulting in equal sensitivity for perturbations and applying Voronoi tessellation afterwards to get a partition of the entire disk. Motivated by the similarities between the CEM and the continuum boundary model of EIT, we derive a simple heuristic to generate sensitivity-based conductivity discretizations for non-circular domain geometries. A sensitivity-based discretization for a setting with 16 electrodes and a heuristic approximation for a non-circular domain are shown in fig. 1.

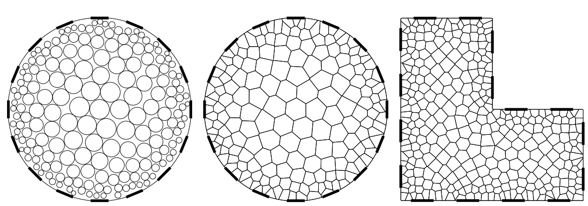

Figure 1: Left: Sizes of circular perturbations resulting in a sensitivity $\lambda_\sigma = 0.02$. Center: Corresponding Voronoi tessellation. Right: Heuristic approx. of a sensitivity-based discretization.

The advantage over generic triangulations is that each conductivity coefficient is equally sensitive to measurement noise, thus regularization during inversion can effectively be applied by a single parameter, i.e. the estimated noise level of the data, free of additional priors. Fig. 2 shows reconstructions on uniform and sensitivity-based discretizations with the same number of coefficients for simulated data Λ_σ with 1% artificial noise.

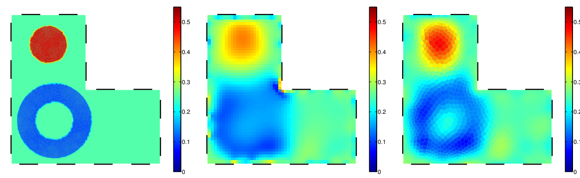

Figure 2: Left: True setting. Center: Reconstruction on a uniform mesh, convergence after 33 Newton-iterations with 23% rel. error. Right: Reconstruction on a sensitivity mesh (13 it., 18% error).

2 Adjustment of the electrode parameters

Most EIT applications involve non-circular domain geometries. Even for circular domains, the electrode parameters (location, contact impedance) are usually not known exactly which can cause severe reconstruction artifacts. To account for these model uncertainties, we incorporate the reconstruction of the electrode parameters into the reconstruction process of the conductivity. This is done by adding the Fréchet derivative of the ND map with respect to the contact impedance, see e.g. [2], and the Fréchet derivative with respect to the electrode location, see [3], to the inexact Newton-type algorithm REGINN [4]. Moreover, the adaptive adjustment of the electrode locations can be helpful in dealing with non-circular domain geometries. According to the Riemann mapping theorem, any simply connected domain in $\mathbb{R}^2$ can be mapped onto the unit disk conformally. For the CEM, this means that the electrode parameters change. When the original domain is not too far from a circle (e.g. an ellipse), we observe that the reconstructed image is a conformally mapped solution of the true domain without additional artifacts. This is shown in fig. 3.

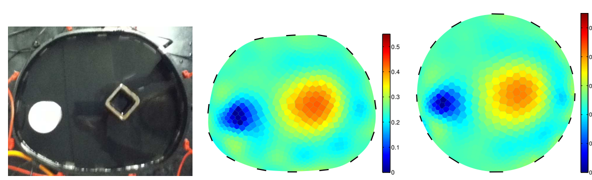

Figure 3: Left: Measurement setting with a resistive inclusion (left) and a conducting inclusion (right). Data kindly provided by Aku Seppänen, University of Eastern Finland. Center: Reconstruction on the estimated domain. Right: Reconstruction on a disk.

3 Conclusions

With a sensitivity-based conductivity discretization and an adaptive adjustment of the domain geometry, we introduced a reconstruction scheme for EIT which considers effects of measurement noise and is robust to geometry inaccuracies, resulting in improved reconstructions over generic solvers.

References

[1] Somersalo E, Cheney M, Isaacson D. *SIAM J Appl Math* 1992
[2] Vilhunen T, Kaipio JP, Vauhkonen PJ, et al. *Measurement Science and Technology* **13**(12):1848, 2002
[3] Darde J, Hakula H, Hyvönen N, et al. *Inverse Problems and Imaging* **6**:399–421, 2012
[4] Rieder A. *Inverse Problems* **15**(1):309–327, 1999