Electrode Models under Shape Deformation in Electrical Impedance Tomography

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Abstract. Electrical Impedance Tomography (EIT) applies current and measures the resulting voltage on the surface of a target. In biomedical applications, this current is applied, and voltage is measured through electrodes attached to the surface. Electrode models represent these connections in the reconstruction, but changes in the contact impedance or boundary relative to the electrode area can introduce artifacts. Using difference imaging, the effects of boundary deformation and contact impedance variation were investigated.

The Complete Electrode Model (CEM) was found to be affected by conformal deformations. Contact impedance variability was found to be a significant source of artifacts in some cases.

In the context of Electrical Impedance Tomography (EIT), the effect of shape deformation on electrode models is considered and it is shown that under certain conditions significant artifacts can occur. The initial proofs of solution existence and uniqueness used a Continuum Model for the electrodes, implying complete knowledge of all boundary data.[1, 2] More recently, models allowing for regular gaps in the boundary data (Gap Model), or more physically realistic models such as the Shunt Electrode Model (SEM) and Complete Electrode Model (CEM) have also been utilized. The CEM adds a complex impedance for each electrode which models the metal electrode, conductive gel and chemical interaction at the skin-electrode interface.[3, 4] (Figure 1)

The Finite Element Method (FEM), used in the numerical solution of EIT images, requires boundary conditions based on these mathematical models. The simplest FEM boundary condition to implement is the Point Electrode Model (PEM), applying current and measuring voltage at single nodes on the boundary. An alternative electrode model implements the mathematical SEM, forcing all nodes associated with an electrode to the same voltage. The SEM is appropriate when contact impedances are so small that the matrices become ill-conditioned.[5] To reconstruct accurate images from in-vivo data, an implementation of the CEM is generally preferred.[3] In the EIT inverse problem, under homogeneous conductivity conditions, solving for the CEM's contact impedances has been successful.[6, 7, 8]

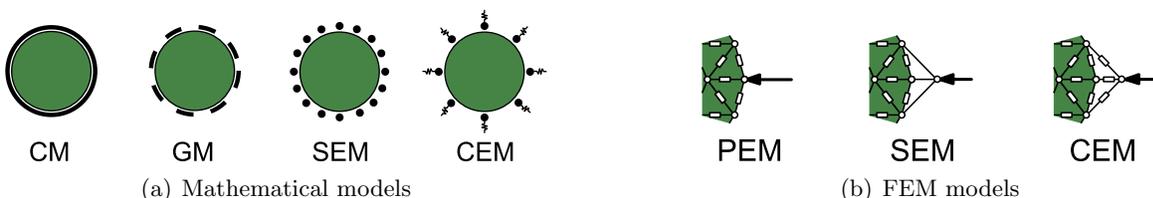(a) Mathematical models   (b) FEM models

Figure 1: Electrode models: CM - Continuous Model, GM - Gap Model, PEM - Point Electrode Model, SEM - Shunt Electrode Model, CEM - Complete Electrode Model

Typically, electrode related image-reconstruction issues do not arise in simulation because a common electrode model is used in the forward problem and its inverse solution. The electrode contact impedance and area is assumed constant throughout and, therefore, cancels in difference imaging. If in-vivo deformations affect electrode contact impedance and area, simulations that do not apply appropriate variation are likely to get optimistic results.

1. Contact Impedance and Electrode Area

Electrode contact impedance is commonly defined in units of impedance and length or area ($\Omega \cdot$ m in 2D, $\Omega \cdot$ m^2 in 3D). For an electrode model spanning multiple edges on the FEM mesh's boundary, there must be a distribution of the contact impedance amongst those edges. One method is to use a linear shape function that assigns the contact impedance based on the length between nodes in two-dimensions, or area of a boundary element in three-dimensions, as implemented in EIDORS.[9] The location of the FEM nodes for the electrode must accurately reflect the total area of the electrode to achieve the correct overall impedance.

With chest EIT, in-vivo deformations occur when a patient breaths.[10, 11] These deformation can be decomposed into independent conformal and not-conformal components. The electrodes themselves, though, are generally fabric or plastic backed and can not stretch to match the deformation exactly. As a result, an error in the area of the electrodes is likely if a conformal deformation such as a dilation occurs. The result will be artifacts in the image due to a change in the boundary's definition.

To investigate this, the behavior of the Point and Complete Electrode Models were explored under two types of conformal deformation: a 10% dilation, and a more complex deformation defined in Figure 2. Simulations were performed on a two-dimensional circular tank with homogeneous conductivity (1m initial tank radius, 33439 elements, 16 0.2m dia. electrodes, background conductivity of 1 S/m). A circular and rectangular target were simulated (conductivity 2 S/m), appearing in the second frame, and the measurements were reconstructed using a course mesh (7207 elements, 16 0.2m dia. electrodes, Gauss-Newton inverse solver μ =1e-5, Tikhonov image prior). Image artifacts were quantified using the Artifact Amplitude Measure (AAM), the sum of the squared normal error of an image's node voltages. Comparison of the deformation simulations is possible because reconstructions occur on identical undeformed meshes. (CEM and PEM models require different meshes.)

$$\mathrm{AAM}_n = \sum \left[\frac{\text{image} - \text{no-noise image}}{\text{no-noise image}} \right]^2 \tag{1}$$

It was found that the PEM was not affected by conformal changes, whether the electrode model fixed the area or it changed to match the boundary. For the CEM, changes that were symmetric, i.e. dilation, did not result in significant artifacts if the electrode deformed with the boundary change. When the CEM was deformed in a complex conformal manner, deformation of the reconstructed image was observed as a form of artifact. When the area of the CEM was fixed and a dilation occurred, "ringing" artifacts were observed.(Table 1)

Table 1: Electrode Model Behavior under Deformation

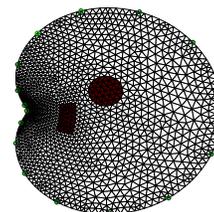| Model	Deformation Domain	Deformation Electrode	AAM$_n$	Comment
PEM	dilation	matching	0	
	complex		0.0807	
	dilation	fixed	0	
CEM	dilation	matching	0.0010	
	complex		2.013	artifacts (deformed)
	dilation	fixed	5.5	artifacts (ringing)

Figure 2: Complex conformal deformation: $z = x + \mathrm{i}y; z \to z + 0.4z^2$

2. Contact Impedance Variation

Depending on current injection patterns, there can be a significant requirement for correct specification of electrode impedance when an absolute image reconstruction is to be attempted.[12] Difference imaging reconstructs an image from two "frames" of data which removes the need to resolve the component of both contact and internal conductivity common to the two "frames." It is commonly assumed in the reconstruction process that the contact impedance remains constant, and thus, all measurement changes are due to internal conductivity changes. As electrodes age, their contact impedance changes, but changes that are not correlated will be minimized by difference imaging with frames taken at short intervals. (Typical EIT systems, such as the Goë-MF II EIT system, acquire at 13 frames-per-second.) Changes that are correlated do not affect the reconstruction with PEMs, while for CEMs, ringing was observed when impedances decreased, analogous to the fixed area electrodes under a dilation deformation. (Impedance increases have a minimal effect.) Independent of the electrdoe model, a large change in contact impedance for a single electrode will appear in the reconstructed image as an artifact near the electrode. The effects are less clear where contact impedance changes in a manner that is uncorrelated amongst the electrodes. For the purposes of this work, this will be termed "electrode noise."

Simulations of the effect of uncorrelated electrode contact impedance changes, as might be found with in-vivo electrodes, were performed. For comparison, a reconstruction with no electrode noise or measurement noise and a similar reconstruction with only measurement noise (-50dB SNR) were simulated. (Figure 3a)

These initial reconstructions were compared to reconstructions with only electrode noise at various levels (Figure 3b–d). Electrode contact impedance z_c [$\Omega \cdot$ m] was assigned using an exponential Gaussian distribution

$$z_c = 10^{\mathcal{N}(\mu, \sigma^2)} \tag{2}$$

where $\mathcal{N}$ is a Gaussian distribution with a given mean μ and variance σ^2.

With electrode noise $\mu = 0$ and $\sigma^2 = 0.25$ ($\simeq 1 \ \Omega \cdot$ m), no noticeable reconstruction artifacts were observed, similar to the no noise reconstruction. With an increase in electrode contact impedance variability, $\mu = -0.75$ and $\sigma^2 = 1.5$, image artifacts resembling those seen at -50dB SNR measurement noise were observed. Electrode impedances varying such that $\mu = -1.5$ and $\sigma^2 = 3$ resulted in significant artifacts throughout the image.

Both the mean of the electrode noise and the variance were varied to observe their affect on the reconstructed image. Reconstructions with an increase in the mean of the electrode noise, irrespective of variance, did not exhibit an increase in artifacts. (Figure 3e)

3. Discussion and Summary

A method was developed for understanding and quantifying the effect of errors in electrode area and contact impedance that occur in two-dimensional reconstructions when the boundary is deformed. Results show that the CEM produces artifacts when conformal deformations are applied. The results obtained for contact impedance variation simulations generally agree with previously published results [13] which indicated that variation of as little as 20% can result in an image that has artifacts significant enough to render the image "almost meaningless." The results also show that, in most cases, applying some level of measurement noise may have the same effect as electrode contact impedance and area variation and should be an appropriate approximation.

It would be reasonable to expect that, as electrode contact impedance variability increases, the magnitude and quantity of image artifacts increases whether the electrode's impedance has increased or decreased. Somewhat unexpectedly, increases in contact impedances result in no observable artifacts in the reconstructed image. This behavior might be explained by
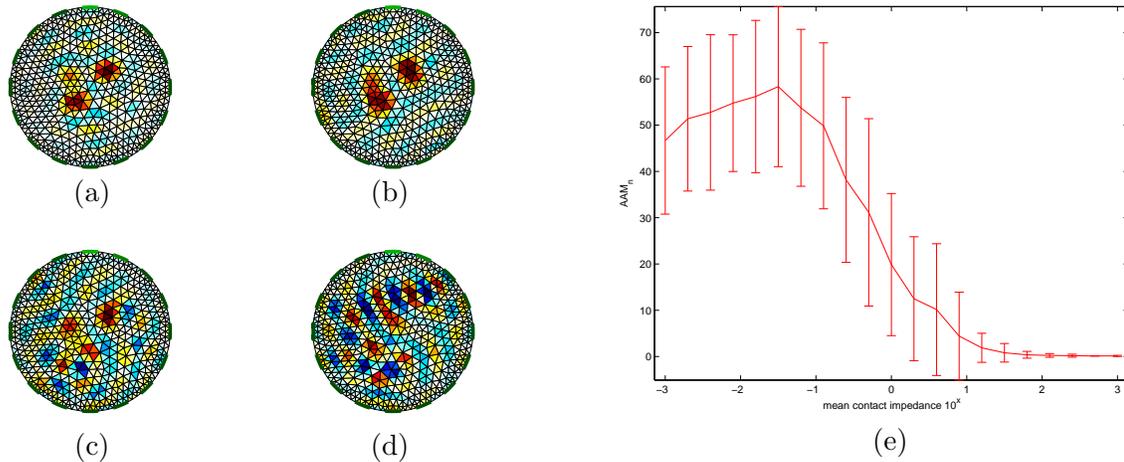
Figure 3: Electrode noise: electrodes form 1/3 of the boundary where the domain is 1 meter in radius (a) reconstruction, -50dB SNR, $\text{AAM}_n = 7.0$; (b) $\mu = 0$, $\sigma^2 = 0.25$, $\text{AAM}_n = 5.9$; (c) $\mu = -0.75$, $\sigma^2 = 1.5$, $\text{AAM}_n = 24.5$; (d) $\mu = -1.5$, $\sigma^2 = 3$, $\text{AAM}_n = 74.1$; (e) contact impedance ($-3 \leq \mu \leq 3$, $\sigma^2 = 1$) versus artifact amplitude (AAM_n) with results over 100 different electrode contact impedance configurations per-contact impedance mean

considering the electrode model as a resistor network attached to the FEM, itself a low impedance resistor network. Large contact impedances mean that the voltage measured at the electrode is approximately the average of the boundary voltage connected to the electrode; however, a small contact impedance will result in an electrode voltage that is highly dependent on the surrounding conductivity. In this environment, electrode voltages measured across small contact impedances will be heavily affected by reconstructed conductivity artifacts near the boundary and are therefore more likely to introduce these artifacts in the inverse problem.

In general biomedical and industrial applications, achieving a minimal contact impedance is desirable to maximize measurement sensitivity. These EIT simulations show that, with contact impedance variability, reconstruction artifacts can be a significant factor in image quality as contact impedance is reduced. To diminish this effect, one potential avenue would be to increase the electrode contact impedance. Another alternative would be to simultaneously reconstruct the electrode contact impedance and interior conductivity distribution, employing some form of regularization. Further in-vivo studies of electrode contact impedance under boundary movement are required to determine if electrode impedances do in fact vary by a sufficient amount to warrant further attempts at mitigating the effect of impedance variability.

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