Correcting for variability in mesh geometry in Finite Element Models

Andy Adler[1], and William R.B. Lionheart[2]
[1]Systems and Computer Engineering, Carleton University, Ottawa, Canada
[2]School of Mathematics, University of Manchester, UK

Abstract: We show that variability in the finite element model (FEM) geometry in the EIT forward model can result in image artefacts in the reconstructed images. We explain these artefacts as the result of changes in the projection of the anisotropic conductivity tensor onto the FEM system matrix. This introduces anisotropic components into the simulated voltages, which cannot be reconstructed onto an isotropic image, and appear as artefacts. In order to address this problem, we show that it is possible to incorporate a FEM vertex movement component into the formulation of the inverse problem. These results suggest that it may be important to consider artefacts due to FE mesh geometry in the formulation of EIT image reconstruction.

1 Introduction

In this paper, we describe the image reconstruction artefacts which occur in electrical impedance tomography (EIT) images due to limitations in the finite element models, and show an algorithmic approach to limit such effects. The earliest approaches to EIT image reconstruction were based on 2D circular approximations of the thorax[6]. However, since such analytical models cannot describe electrical propagation in realistic body shapes, finite element models were used. Other numerical models, such as those based on finite differences [9] have been used, but are less popular, primarily because FE mesh elements can be easily refined in regions of high electric field, typically near to electrodes. While the FEM literature is rich in terms of variety of model structure, most EIT research has used the simplest FEM implementations. Simplex elements are chosen (triangles in 2D and tetrahedrons in 3D) and conductivity is modelled as piecewise constant (so that changes in conductivity occur only at element boundaries). Such choices are reasonable: all FE meshing packages provide good support for simplex elements, and anatomically realistic conductivity changes do occur abruptly at organ boundaries. The most common models use first order elements. Physically, such elements may be modelled by a resistor network[8], which has been used as the basis for a physical resistor network model of the medium [4]. On the other hand, FEM model errors decrease linearly with element size for first order FEMs, while the rate of error decrease is larger for higher order models.

The most common approach to image reconstruction in EIT has been to parametrize the conductivity distribution, $\mathbf{x}$, based on the FEM element (or vertex) geometry. The conductivity (or conductivity change) distribution, $\hat{\mathbf{x}}$ is calculated from data (or difference data), $\mathbf{y}$ as

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{y} - F(\mathbf{x})\|^2_{\mathbf{\Sigma}_n^{-1}} + \|\mathbf{x} - \mathbf{x}_0\|^2_{\mathbf{\Sigma}_x^{-1}} \quad (1)$$

where $F(\mathbf{x})$ represents the FEM, $\mathbf{\Sigma}_n$ the noise covariance, and $\mathbf{x}_0$, and $\mathbf{\Sigma}_x$ the prior expected mean and covariance. In this way, reconstructed image is the one which, by varying the FEM conductivity parameters, best fits the data.

In this paper, we report that this approach to image reconstruction is very sensitive to any geometrical variability in the FEM. For example, small changes in the positions of internal nodes, such that the same conductivity distribution is represented, can result in large image reconstruction artefacts. This effect was unexpected, since the voltage distribution simulated by the FEM is accurate even for moderately dense models. This effect may explain a few puzzling results. For example, studies of electrode movement have often used FEM simulations to show that unacceptable artefacts occur for very small electrode displacements (under 1%) (eg. [2]); while, experimentally, electrode movements an order of magnitude greater still permit usable images.

2 Geometry variability example

In this section, we show how re-meshing around a circular object can introduce large artefacts into the reconstructed difference images. The easiest (and most common) way to simulate a target in a medium is to use a single FEM to select and then interpolate which elements are part of the target. There is no change to the underlying FEM, and thus no model noise in the images. A more accurate representation of a target is to create a target region within the FEM and to remesh around it. This means that the mesh changes between each target position, not only near the target, but throughout the FEM due to the propagation of changes in triangularization.

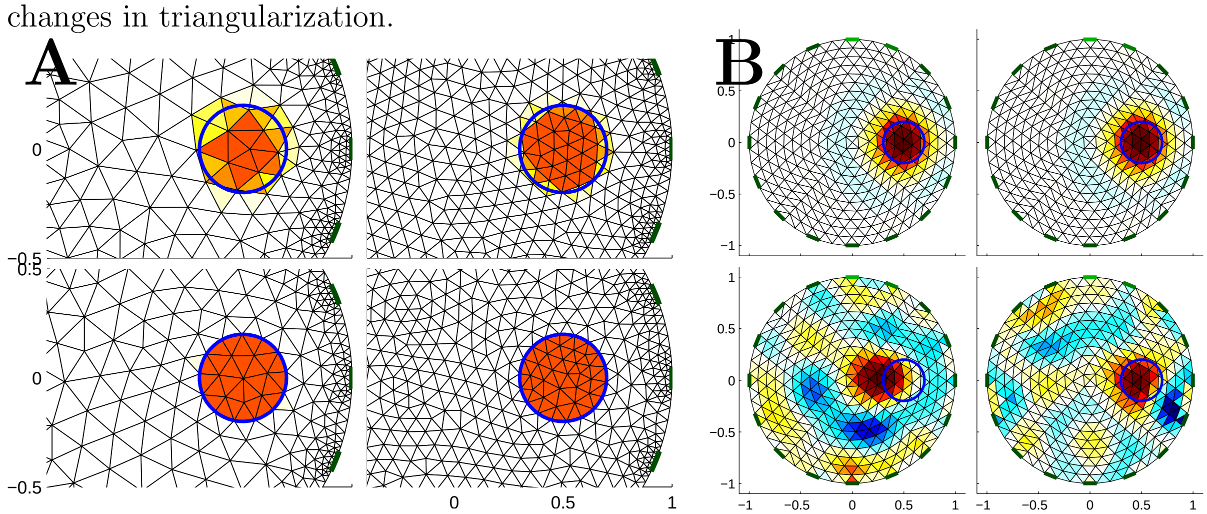

Figure 1: **A:** Simulation FEMs and simulated conductivity target positions in blue circle. Electrode nodes are indicated in green. An inner region surrounding a target is shown. *Left:* Coarse meshes (maximum mesh size of 0.16, 1441 triangles) *Right:* Fine meshes (maximum mesh size of 0.07, 1941 triangles) *Top:* Meshes with no adaptation for target. Element conductivity is defined by region membership. *Bottom:* Meshes adapted to target region. **B:** Reconstructed difference images calculated from the simulation FEMs (A) on a 576 element 2D mesh. In each case, the homogeneous data were simulated on the unadapted mesh. The target position is indicated by the blue circle.

To illustrate this process, Fig. 1 shows 2D circular FEMs with 16 electrodes with local refinement of the FEM near each electrode. Coarse and fine meshes are shown by controlling the maximum permissible element size. Two different strategies to specify the region of a simulated conductivity target region are shown. On top, the mesh is not adapted to the target. The conductivity of each element is selected based on the membership in the target region (an element with 50% of its area in the region will have a target conductivity of the average of the background and target region). On the bottom, the FEM is adapted to the target region, resulting in mesh geometry changes which propagate throughout the FEM. EIT data were simulated using a Sheffield-type adjacent stimulation and measurement protocol, and images are reconstructed on a simple regular mesh geometry. A one-step Gauss

Newton reconstruction is used with a scaled diagonal image prior[3], and the regularization parameter is chosen such that the noise figure is 1.0[1].

Images in which the simulation mesh geometry matched exactly (top) show the expected location and shape. However, if the mesh geometry changes between difference data simulations (bottom) artefacts occur throughout the images. As shown in Fig. 1, reconstruction artefacts reduce as the FEM density increases. This is largely due to the increase in accuracy of the FEM model with decreasing mesh size. Similar artefacts occur for 3D simulations; however, it the effect is considerably larger because many more FEM elements are required to achieve the same level of refinement in 3D compared to 2D.

3 Compensation for image reconstruction errors

We hypothesize that these image reconstruction errors result largely from an effect equivalent to anisotropic conductivity in the simulated EIT measurements. A selection of a particular FEM geometry is equivalent to a particular representation of the (potentially anisotropic) conductivity tensors on each tetrahedron onto the FEM system matrix. Any change in the FEM geometry projects this conductivity tensor differently, resulting in slightly different anisotropic "content" in the simulated voltages. During image reconstruction, such anisotropic content cannot be explained by an inverse model parametrized only for isotropic conductivity. These differences then appear as noise, which is projected as artefacts into the reconstructed image.

Since a first-order FE model is equivalent to a resistor on each FE mesh edge, the maximum number of degrees of freedom in the FE model is the number of edges. Since there are more edges than elements (by approx $\frac{3}{2}$ in 2D), not all FEM matrices correspond to isotropic conductivities, and any distortion of the mesh will (typically) be consistent with an anisotropic conductivity on the original mesh.

This hypothesis suggests that the origin of these image errors is the geometrical constraints of the inverse model. The reconstruction process is not allowed to vary the FEM geometry to "explain" the measurements, but must only use isotropic conductivities on each element. Therefore, we can address this issue by parametrizing both the element conductivities and the positions of each vertex. This allows the image reconstruction to "jiggle" the node locations to avoid pushing artefacts into the conductivity images.

4 Image Reconstruction Formulation

We consider EIT difference measurements, $\mathbf{y} = F(\Delta\boldsymbol{\sigma})$, originating from isotropic conductivity changes $\Delta\boldsymbol{\sigma}$ and a FEM model, $F(\Delta\boldsymbol{\sigma}, \Delta\mathbf{p})$, with vertex positions $\Delta\mathbf{p}$. Based on this model, we calculate a conductivity Jacobian, $\mathbf{J}_c$ – the sensitivity of measurements to conductivity changes; and a movement Jacobian, $\mathbf{J}_m$ – the sensitivity of measurements to vertex position movements.

$$[\mathbf{J}_c]_{ij} = \frac{\partial F_i(\Delta\boldsymbol{\sigma}, \Delta\mathbf{p})}{\partial[\Delta\boldsymbol{\sigma}]_j}, \text{ and } [\mathbf{J}_m]_{ij} = \frac{\partial F_i(\Delta\boldsymbol{\sigma}, \Delta\mathbf{p})}{\partial[\Delta\mathbf{p}]_j}, \tag{2}$$

The standard linear solution, $\hat{\mathbf{x}}$, to (1) is

$$\hat{\mathbf{x}} = \left(\mathbf{J}^t\boldsymbol{\Sigma}_n^{-1}\mathbf{J} + \boldsymbol{\Sigma}_x^{-1}\right)^{-1}\mathbf{J}\boldsymbol{\Sigma}_n^{-1}\mathbf{y} = \boldsymbol{\Sigma}_x\mathbf{J}^t\left(\mathbf{J}\boldsymbol{\Sigma}_x\mathbf{J}^t + \boldsymbol{\Sigma}_n\right)^{-1}\mathbf{y}, \tag{3}$$

assuming $\mathbf{x}_0 = 0$, as normally done for difference EIT.

Normally, the parameters, $\hat{\mathbf{x}}$, represent only the conductivity change $\Delta\boldsymbol{\sigma}$. Instead, we represent our scheme using the conductivity change and vertex movement as parameters and: $\hat{\mathbf{x}} = [\Delta\boldsymbol{\sigma}^t|\Delta\mathbf{p}^t]^t$ and $\mathbf{J} = [\mathbf{J}_c|\mathbf{J}_m]$. In this case, the prior $\boldsymbol{\Sigma}_x$ may be decomposed into a conductivity changes $\boldsymbol{\Sigma}_c$ and movement $\boldsymbol{\Sigma}_m$ parts with no correlation assumed between these parameters (following [7]), yielding:

$$\hat{\mathbf{x}} = [\boldsymbol{\Sigma}_c\mathbf{J}_c|\boldsymbol{\Sigma}_m\mathbf{J}_m]^t \left(\mathbf{J}_c\boldsymbol{\Sigma}_c\mathbf{J}_c^t + \mathbf{J}_m\boldsymbol{\Sigma}_m\mathbf{J}_m^t + \boldsymbol{\Sigma}_n\right)^{-1}\mathbf{y} \tag{4}$$

Since we are only interested in displaying the reconstructed conductivity change, $\Delta\hat{\boldsymbol{\sigma}}$, the standard and proposed reconstruction may be shown:

$$\Delta\hat{\boldsymbol{\sigma}} = \begin{cases} \boldsymbol{\Sigma}_c\mathbf{J}_c^t\left(\mathbf{J}_c\boldsymbol{\Sigma}_c\mathbf{J}_c^t + \boldsymbol{\Sigma}_n\right)^{-1}\mathbf{y} & \text{standard} \\ \boldsymbol{\Sigma}_c\mathbf{J}_c^t\left(\mathbf{J}_c\boldsymbol{\Sigma}_c\mathbf{J}_c^t + \mathbf{J}_m\boldsymbol{\Sigma}_m\mathbf{J}_m^t + \boldsymbol{\Sigma}_n\right)^{-1}\mathbf{y} & \text{proposed} \end{cases} \tag{5}$$

5 Results and Discussion

The standard and proposed image reconstruction approaches (5) were implemented for the simulated data of Fig. 1, and are shown in Fig. 2. The proposed approach (bottom row) shows a significant reduction in image artefacts, but roughly the same resolution of the conductivity target. $\boldsymbol{\Sigma}_m$ was chosen as the scaled identity matrix with an amplitude such that the noise figure[1] reduced from 1.0 (standard) to 0.8 (proposed).

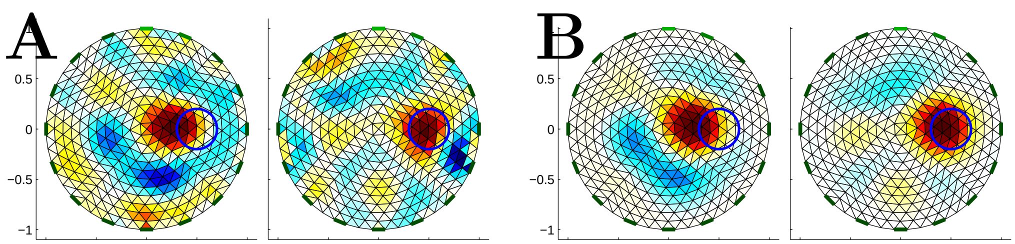

Figure 2: Reconstructed difference images calculated as in Fig. 1 for changes in FEM geometry. The target position is indicated by the blue circle. **A:** Standard approach using *Left:* Coarse Mesh, *Right:* Fine Mesh **B:** Proposed approach using *Left:* Coarse Mesh, *Right:* Fine Mesh.

Based on these results, we note the following observations. First, these results explain the strange effect that EIT simulations from adapted target meshes can show large artefacts. It also explains why simulations of electrode movement show much larger effects than are observed. Second, these results suggest that EIT image reconstruction should allow for anisotropic behaviour in the measured signals. Thus, this work calls for consideration of FE models and the assumption of isotropic conductivity in EIT.

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