Homework 1

September 24, 2010

1. Show that a process with independent increments has the Markov property. The reverse is not true! (Provide a counter-example)

The following are problems related to Markov Chains

1. Prove the Chapman-Kolmogorov equation: $\mathbf{P^{(m+n)}} = \mathbf{P^m} \cdot \mathbf{P^n}$ $m, n = 1, 2, 3...$
2. Prove the renewal equation of page 6 (in the notes)
3. Show that any state in an irreducible Markov Chain with finitely many states must be positive recurrent
4. To look backwards in time, write $q_{ij} = P(X_n = j \mid X_{n+1} = i)$. Show that if the chain starts at time 0 in a state according to the stationary probability distribution $\underline{\pi}$ (i.e. $P(X_0 = i) = \pi_i$) then $q_{ij} = \frac{\pi_i p_{ij}}{\pi_i}$

The following problems refer to Markov Processes

1. Define the Poisson process and show that is a Markov Process
2. Show that for a Poisson process the interarrival times are i.i.d variables exponentially distributed. Furthermore, show the inverse i.e. if the interarrival times of a process are independent and exponentially distributed then the process is a Poisson process.
3. Work out and clean up the proof of the *independent observer property* of the Poisson process as was presented on page 11 of the notes.
4. Show that the residence time at a state i of a Markov process is a random variable that is exponentially distributed.

Note: Here are some instructions for installing the latex software. Do not install any other package except the ones listed below!

1. First run the installer setup 2.4.1705 file. It will ask you where it should intall Miktex from. Point it to the Miktex directory included in the CD I gave you. The installation process is slow. Be warned.

2. After installing Miktex you may wish to install ghostview (gsv48w32) and ghostscript(gs850w32-gpl).

3. Install the WinTex XP package

4. Using WinTex XP load the example file in C:/texmf/tex/latex/base/sample2e by pressing the "LATEX" box on the upper left of the screen. Pressing "PDFLATEX" will give you a PDF file. From this point you are on your own to explore and learn LATEX!